ECE 342
Electronic Circuits

Lecture 1
KCL, KVL, Thevenin & Norton

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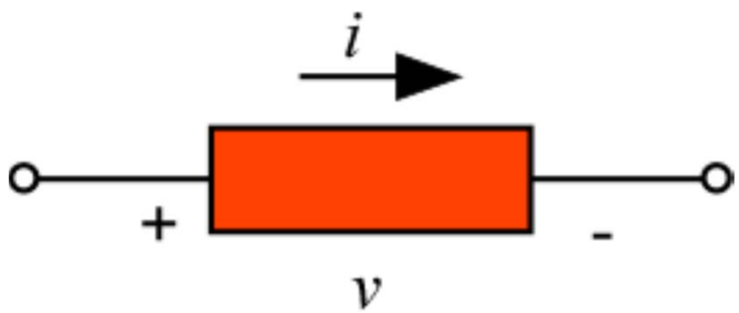
Voltage

- **Voltage: Energy loss per unit charge**

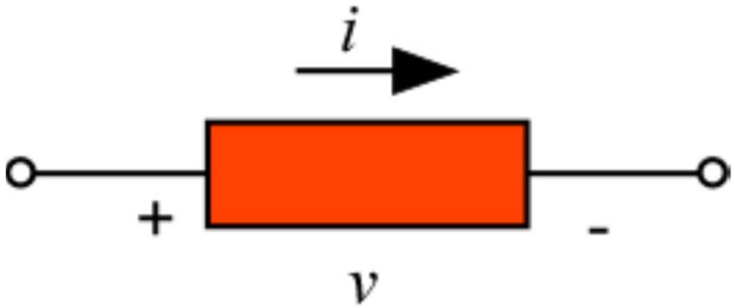


$$Voltage = \frac{Joules(J)}{Coulombs(C)} = Volts(V)$$

- **Charge is transported from "+" terminal to "-" terminal**

- **The electrical potential is higher at the "+" terminal and lower at the "-" terminal**

Voltage



Due to the force of its electrostatic field, an electric charge has the ability to do the work of moving another charge by attraction or repulsion. The ability of a charge to do work is called its *potential*

A battery with a voltage output of 6V means that the potential difference between the two terminals of the battery is 6V

Current

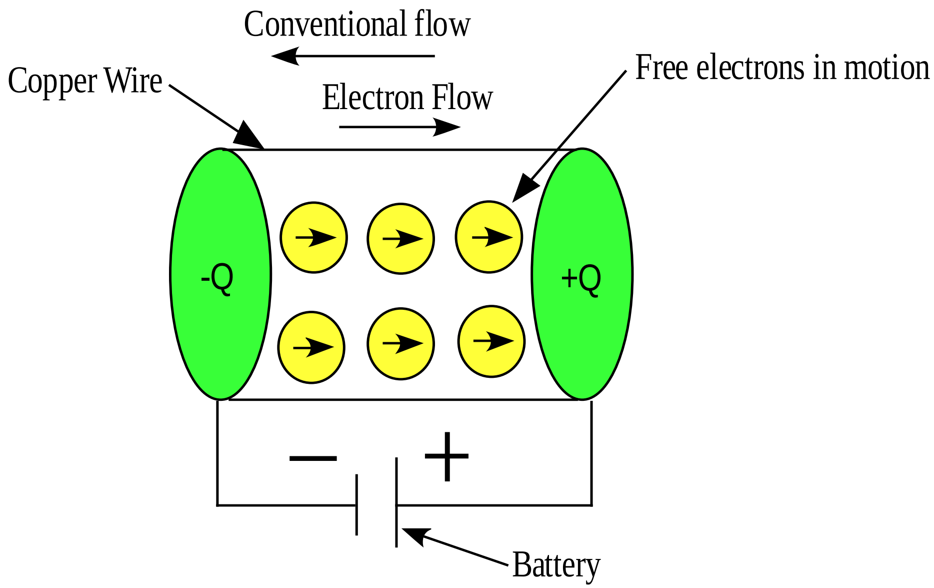
The movement or the flow of electrons in a conductor is called current. To produce current the electrons must be moved by a potential difference. The basic unit of current is the Ampere (A). One ampere is defined as the movement of one coulomb past any point of a conductor during one second of time.

$$current: i = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

Δq denotes the net amount of electrical charge transported in direction $\rightarrow$ during Δt

Current

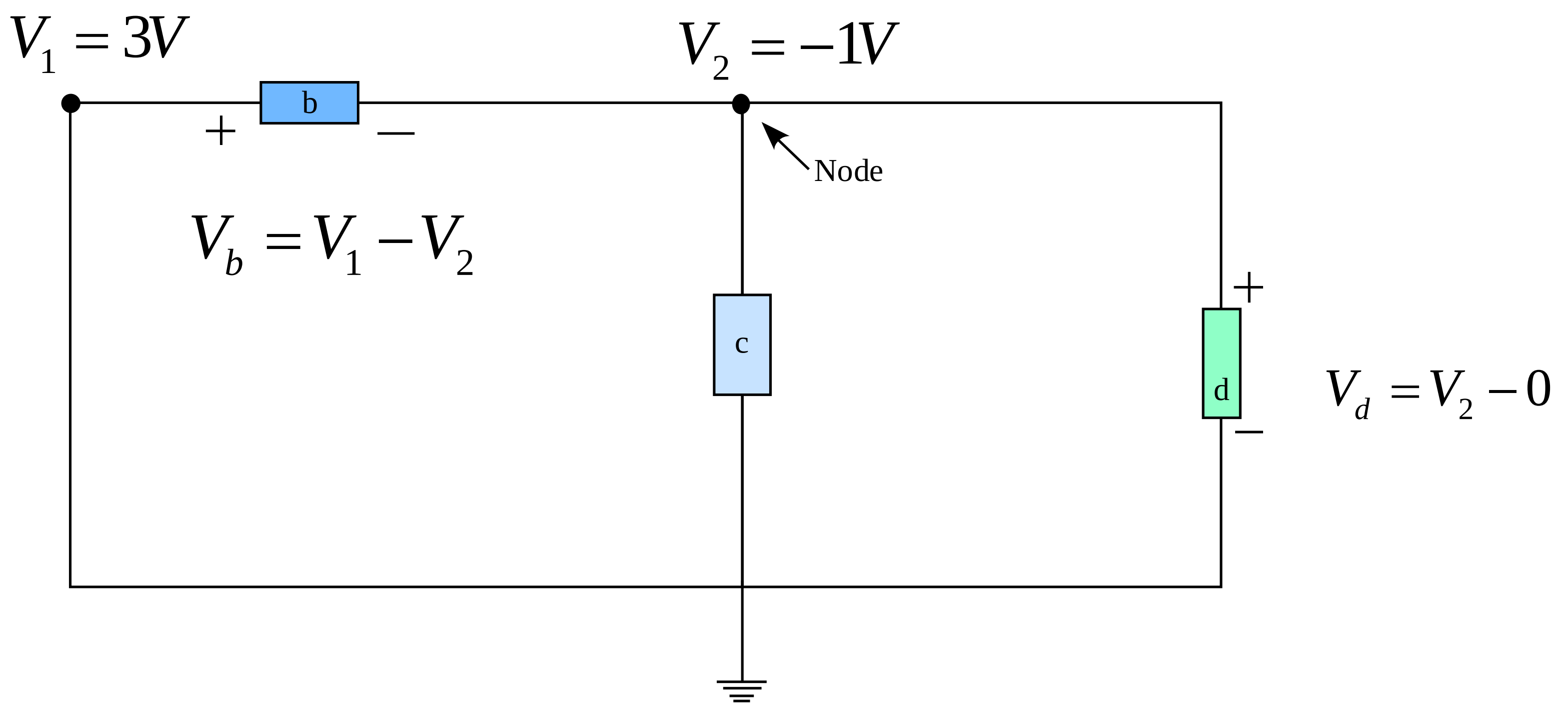
$$i = \frac{coulombs(c)}{second(s)} = Ampere(A)$$



- **Plus and minus signs are essential**
- **They indicate the polarity of the element voltage**
- **Voltage rise is from "–" terminals to "+" terminals**
- **Voltage drop is from "+" terminals to "-" terminals**

Nodes

- **Connection points of element terminals in a circuit.**
- **The node marked by the ground is the reference node.**



Nodes

The voltage across each element in a circuit is the difference of electrical potentials of the nodes of the element terminal

Element c and d are in parallel: they make terminal contacts with the same pair of nodes and they have the same terminal potential.

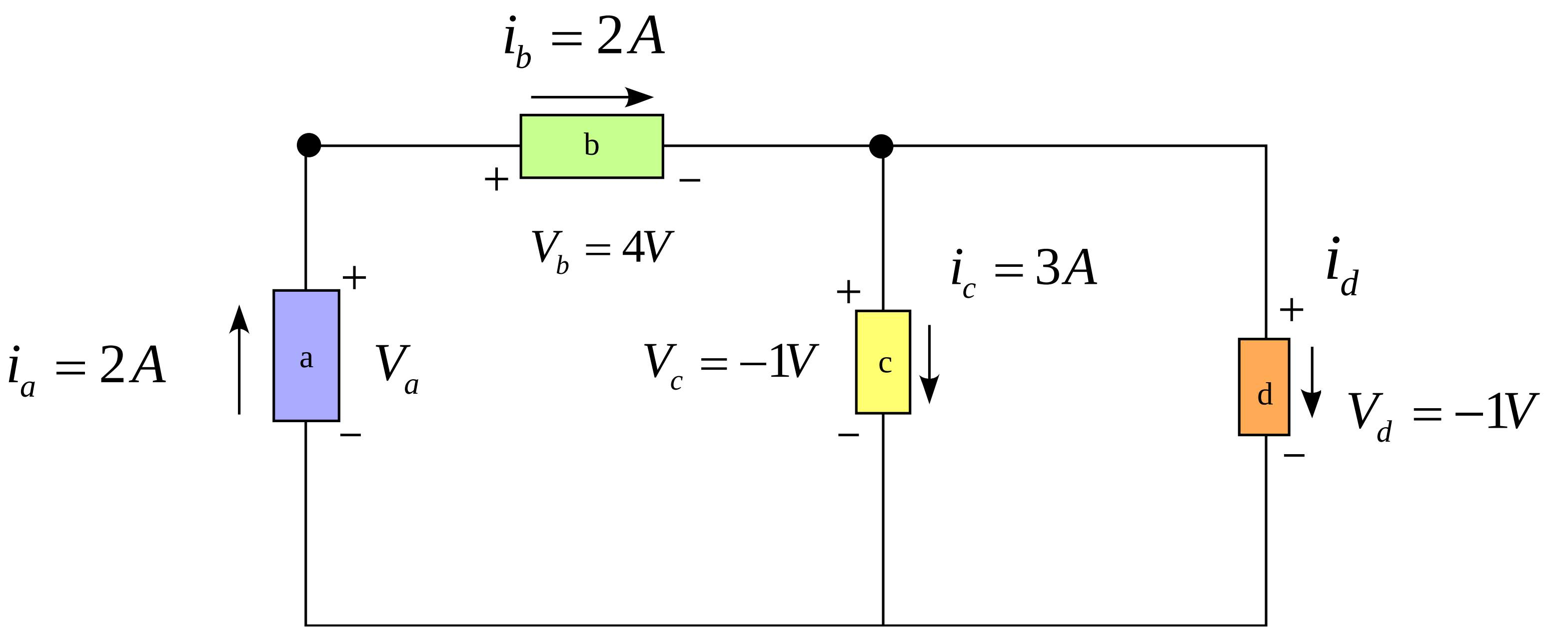
Absorbed Power

For a 2-terminal element, absorbed power is defined as:

$$p = vi$$

(Where p is the absorbed power)

Absorbed Power



For instance $v_b = 4v, \quad i_b = 2A \rightarrow p_b = v_b\ i_b = 8w$ absorbed

Absorbed Power

Circuit elements can:

- **Absorb Power (resistive)**
- **Inject Power (source)**
- **Store Power (reactive)**

Energy conservation requires that at each instant that the sum of all the energy that is absorbed and stored be equal to the energy that is injected into the circuit.

Powers absorbed in a circuit sums to zero

$$p_a + p_b + p_c + p_d = p_a + 8w + (-3w) + p_d = 0$$

Kirchhoff's Voltage & Current Laws (KVL & KCL)

KVL: $$\sum V_{rise} = \sum V_{drop}$$

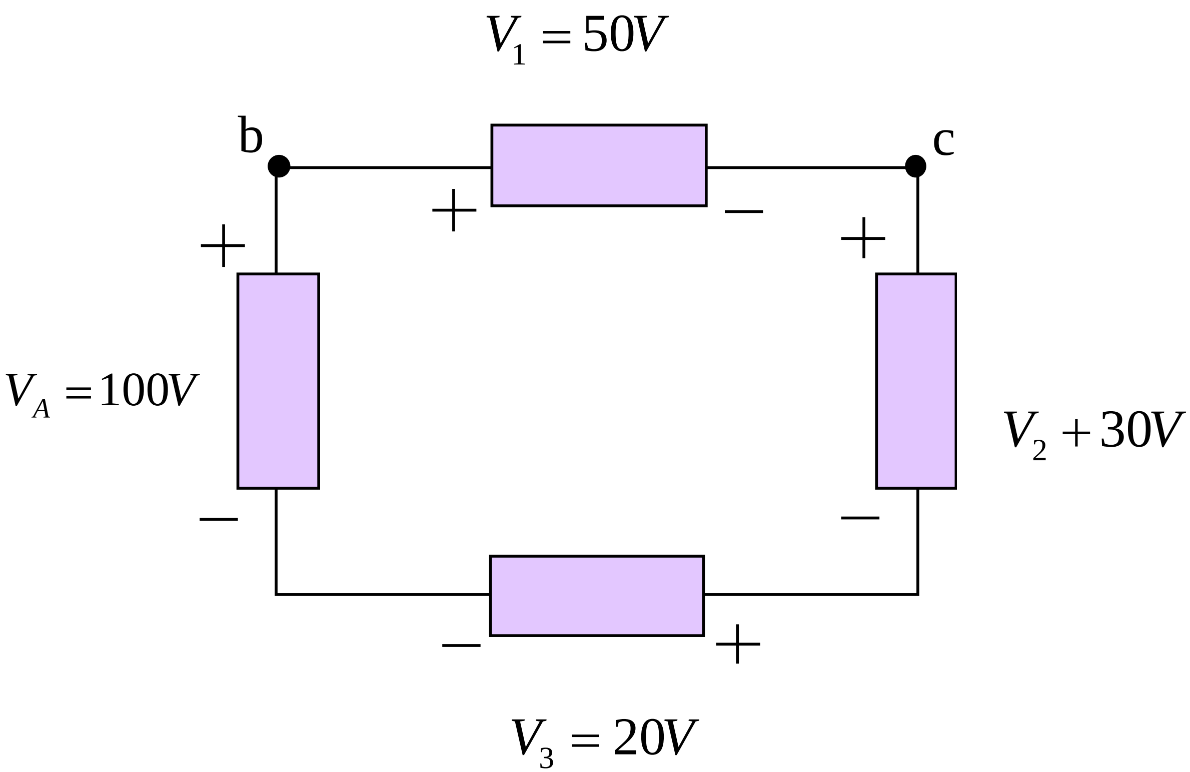
- **Each element voltage applied to a closed circuit equals the sum of the voltage drops in that circuit.**

In an algebraic sense:

$$\sum V = 0$$

KVL & KCL

Example:



$$\sum V_{rise} = 100V$$

$$\sum V_{drop} = V_1 + V_2 + V_3$$

$$= 50 + 30 + 20V$$

$$= 100V$$

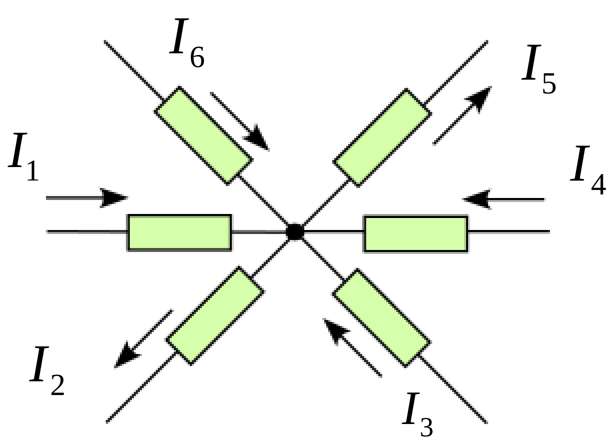
KVL & KCL

KCL: $\sum I_{in} = \sum I_{out}$

(At any node in a circuit)

The sum of all currents flowing into a node equals the sum of all currents flowing out.

Example:



$$I_1 + I_3 + I_4 + I_6 = I_2 + I_5$$

In algebraic notation:

$$\sum I = 0$$

Resistor

An ideal resistor is a two-terminal element which satisfies the relation

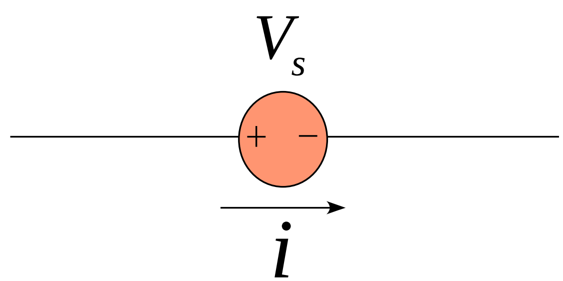
$$V = Ri$$

This is known as Ohm's Law R is $>$0 and is the resistance of the element.

- Resistance is opposition to current flow.
- A resistor is a device whose resistance is a known value.

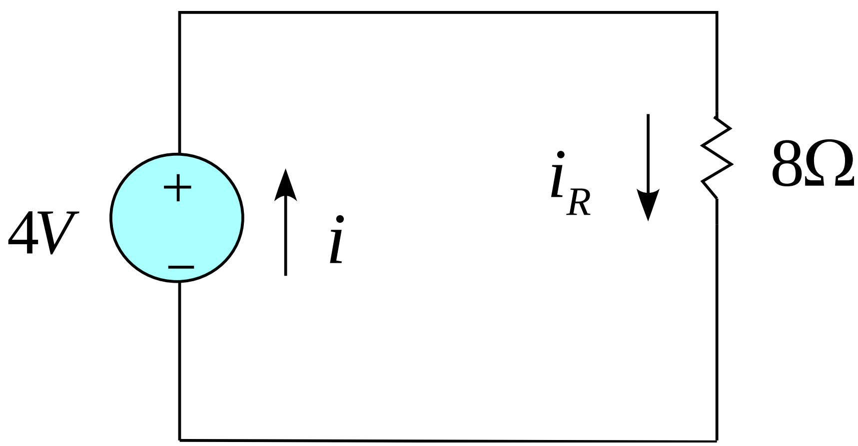
Independent voltage source.

An independent voltage source is an element that maintains a specified potential difference V's between its terminals independent of the current through it



Example:

$$i = i_R$$

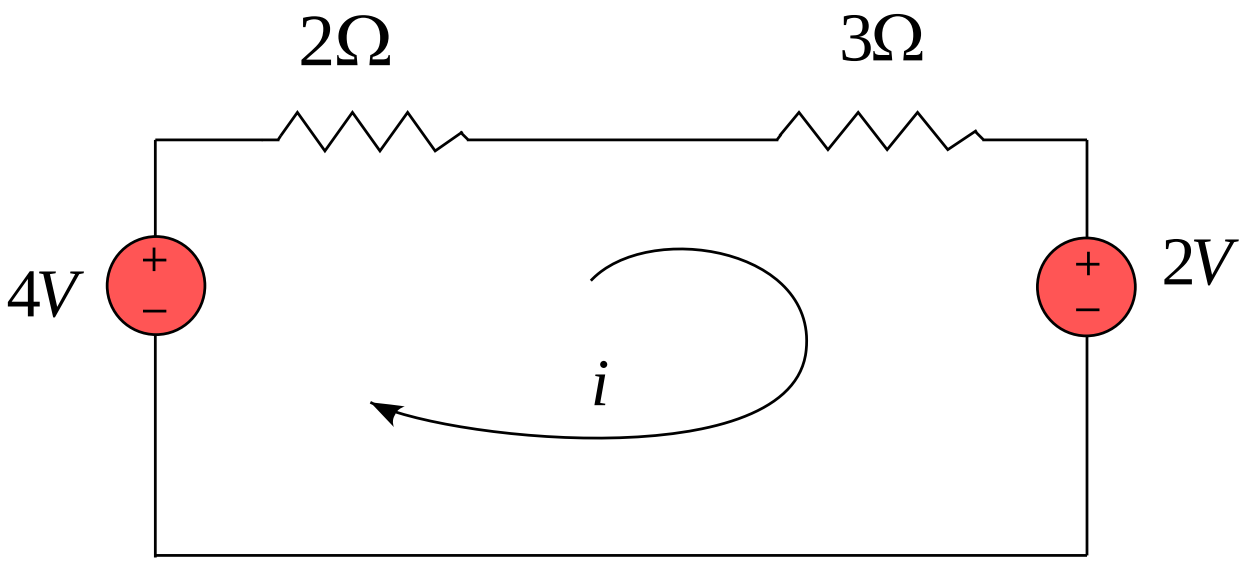


$$4V = 8 \times i_R$$

$$i_R = \frac{4V}{8} = 0.5A$$

Independent voltage source.

Example:



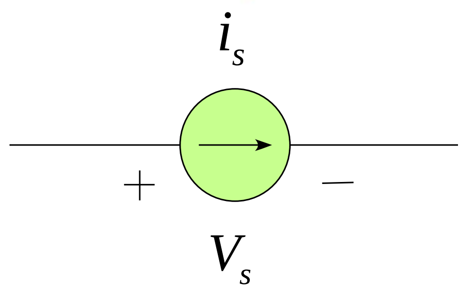
$$4 = 2i + 3i + 2$$

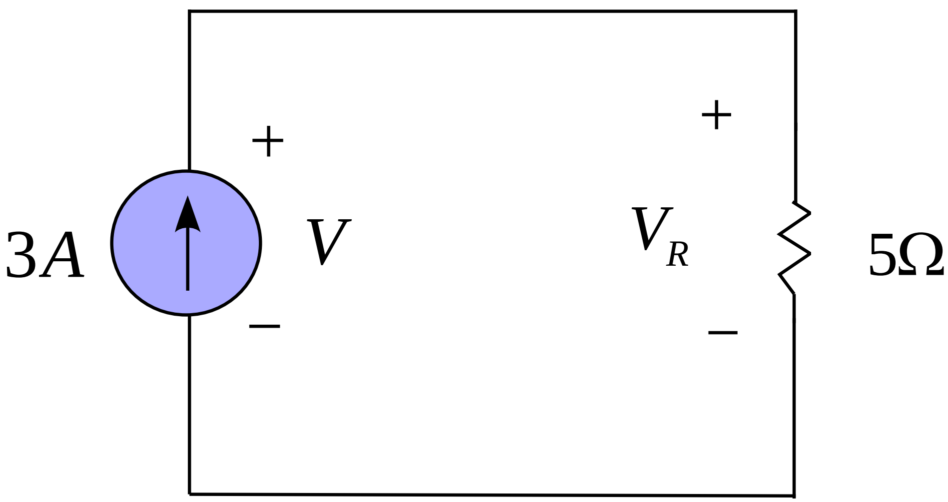
$$i = \frac{4-2}{2+3} = \frac{2}{5}$$

$$i = 0.4A$$

Independent current source.

An independent current source is an element that maintains a specified current flow i_s between its terminals independent of the voltage across it

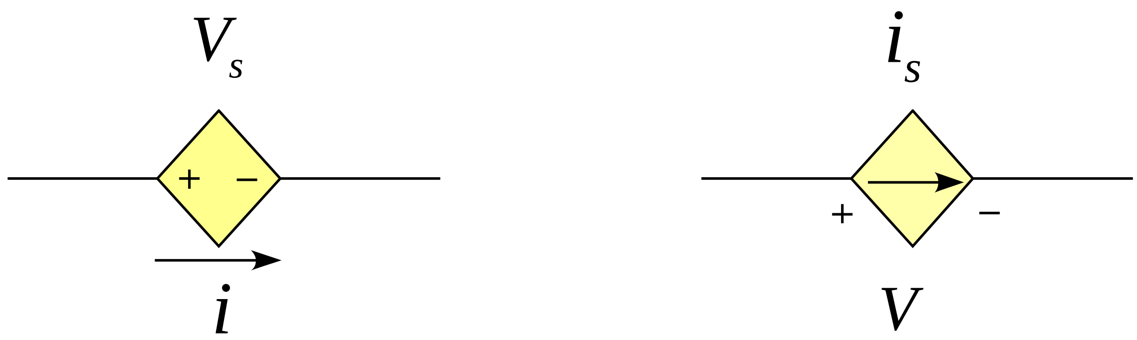




$$3A = \frac{V_R}{5\Omega}$$

$$V_R = 3 \times 5 = 15V$$

Dependent Sources



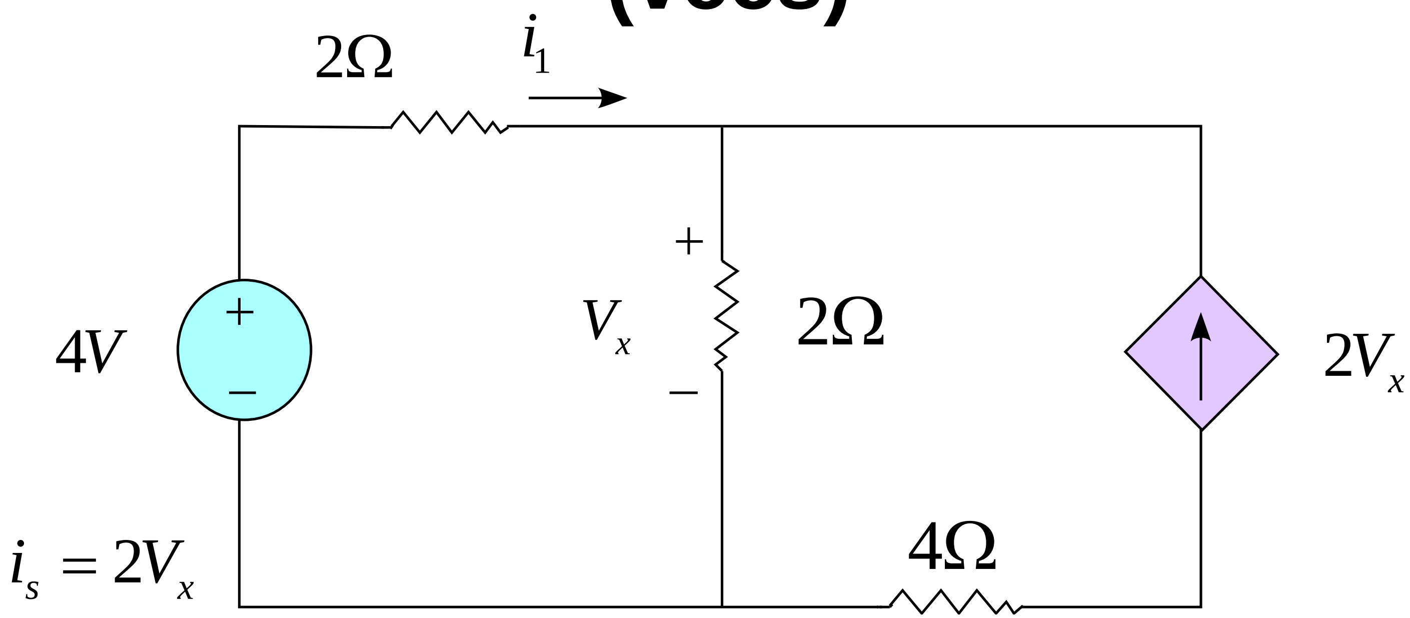
$V_s = AV_x \rightarrow$ **Voltage [controlled dependent] voltage source (VcVs)**

$V_s = Bi_y \rightarrow$ **Current controlled voltage source (ccvs)**

$i_s = CV_x \rightarrow$ **Voltage controlled current source (vccs)**

$i_s = Di_y \rightarrow$ **Current controlled current source (cccs)**

Voltage controlled current source (vccs)



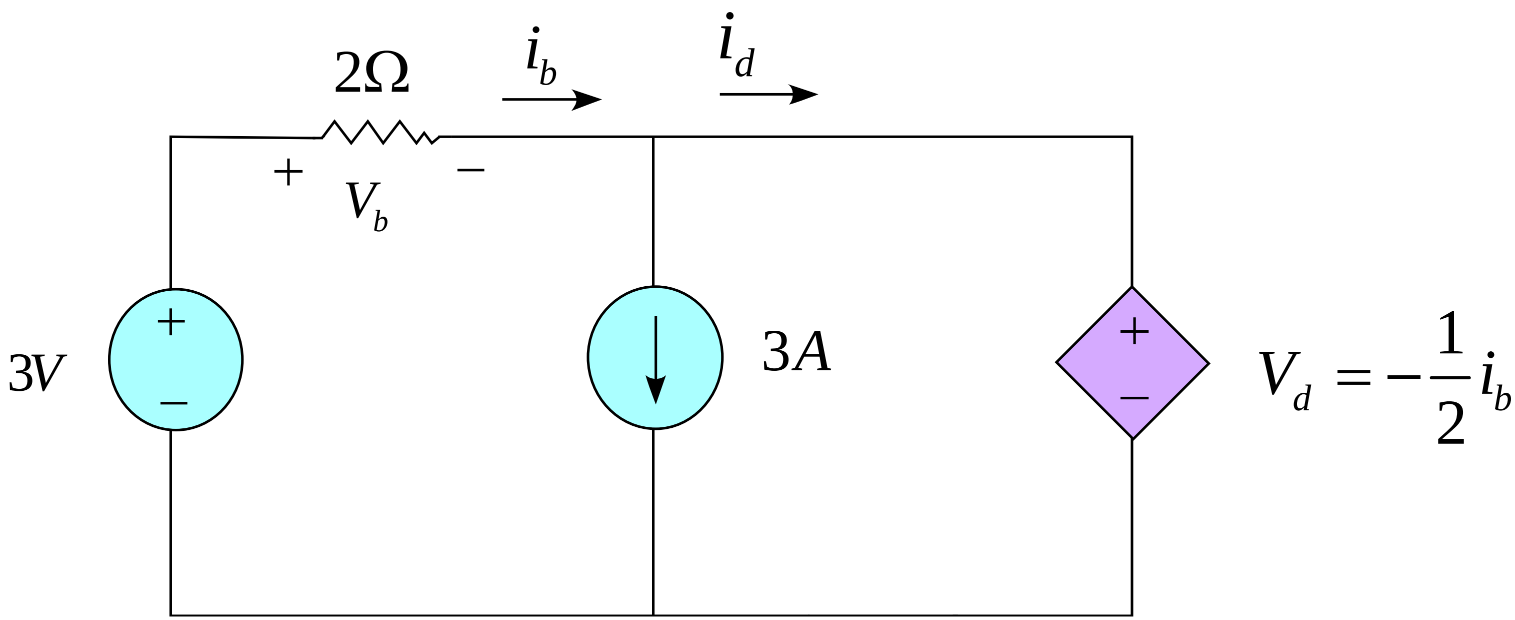
KCL:

$$i_1 + 2V_x = \frac{V_x}{2} \Rightarrow i_1 = -\frac{3}{2}V_x$$

$$4 = 2i_1 + V_x$$

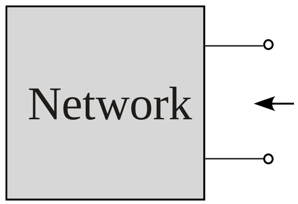
$$V_x = -2V,\ i_1 = 3A$$

Current controlled voltage source (ccvs)

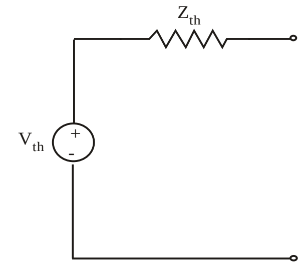


KCL: $i_b = 3 + i_d$

$$i_d = i_b - 3 = 2 - 3 = -1A$$

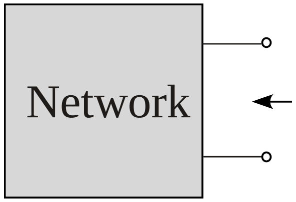


Thevenin Equivalent

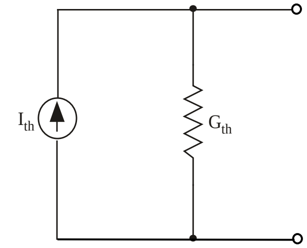


- **Principle**
  - Any linear two-terminal network consisting of current or voltage sources and impedances can be replaced by an equivalent circuit containing a single voltage source in *series* with a single impedance.

- **Application**
  - To find the Thevenin equivalent voltage at a pair of terminals, the load is first removed leaving an open circuit. The open circuit voltage across this terminal pair is the Thevenin equivalent voltage.
  - The equivalent resistance is found by replacing each independent voltage source with a short circuit (zeroing the voltage source), replacing each independent current source with an open circuit (zeroing the current source) and calculating the resistance between the terminals of interest. Dependent sources are not replaced and can have an effect on the value of the equivalent resistance.



Norton Equivalent

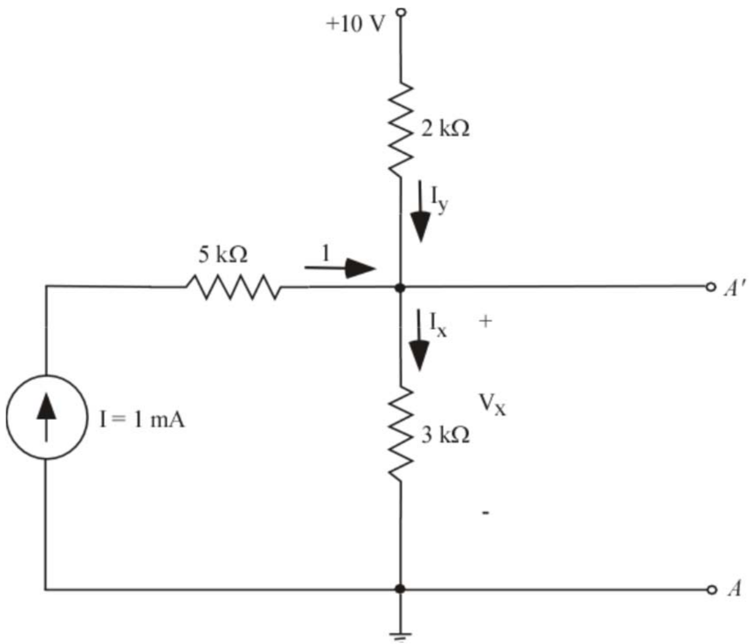


- **Principle**
  - Any linear two-terminal network consisting of current or voltage sources and impedances can be replaced by an equivalent circuit containing a single current source in *parallel* with a single impedance.

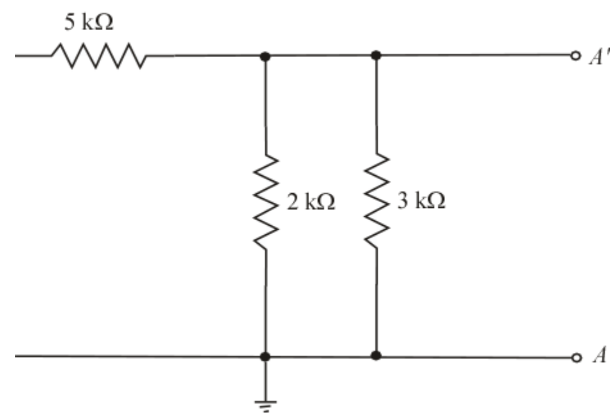
- **Application**
  - To find the Norton equivalent current at a pair of terminals, the load is first removed and replaced with a short circuit. The short-circuit current through that branch is the Norton equivalent current.

  - The equivalent resistance is found by replacing each independent voltage source with a short circuit (zeroing the voltage source), replacing each independent current source with an open circuit (zeroing the current source) and calculating the resistance between the terminals of interest. Dependent sources are not replaced and can have an effect on the value of the equivalent resistance.

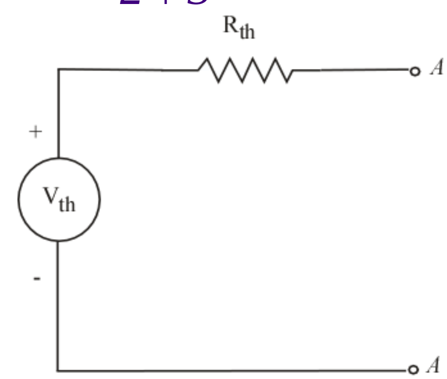
Thevenin Equivalent - Example



Circuit for calculating impedance



$$R_{th} = \frac{(2)(3)}{2+3} = 1.2\ \mathrm{k}\Omega$$



Calculating Thevenin voltage

KCL law at node A' gives $1+I_y = I_x$

Also KVL gives $10=2I_y+3I_x$

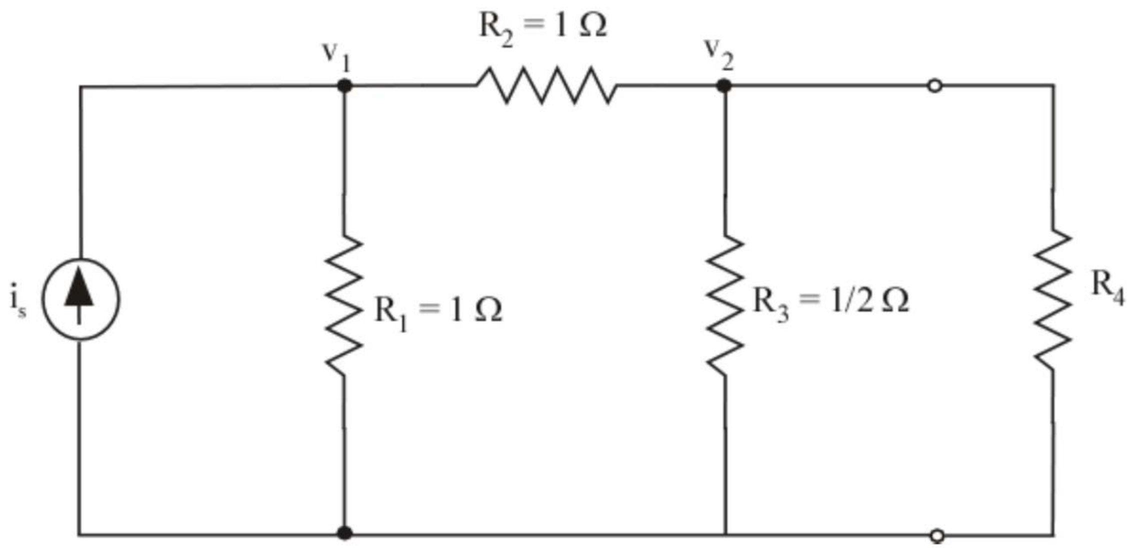
Combining these equations gives $I_x = 2.4$ mA

From which we calculate

$V_x = V_{th}=3(2.4)=7.2$ V

Thevenin Example

Find voltage across R_4



$$Define\ \ G_1 = 1/R_1, G_2 = 1/R_2, and\ G_3 = 1/R_3,$$

$$(G_1 + G_2)v_1 - G_2 v_2 = i_s$$

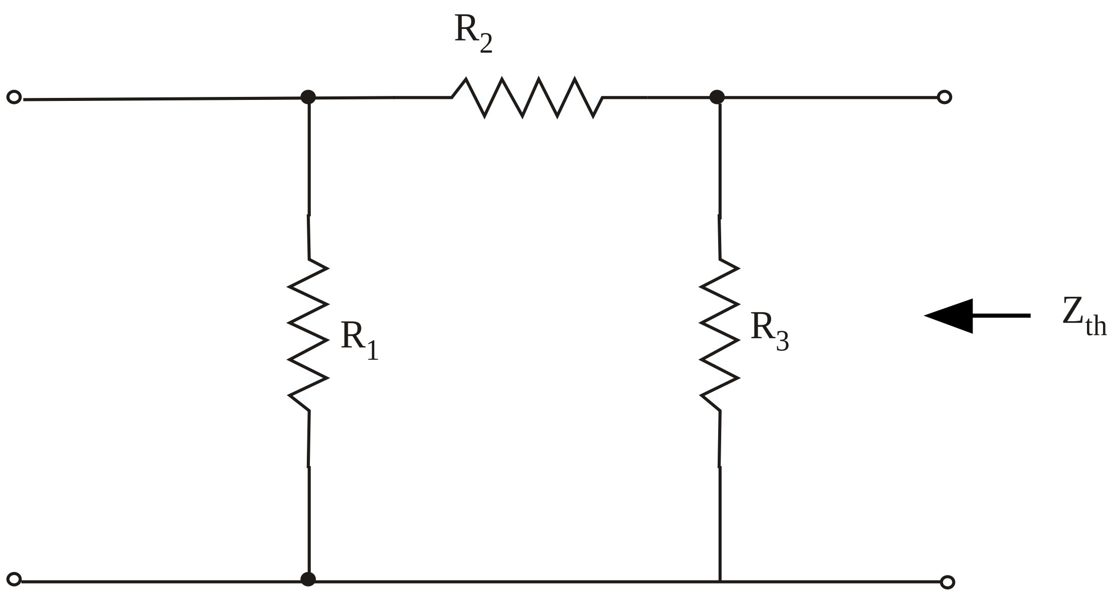
$$-G_2 v_1 + (G_2 + G_3)v_2 = 0$$

From which

$$v_2 = \frac{i_s}{5} = V_{th}$$

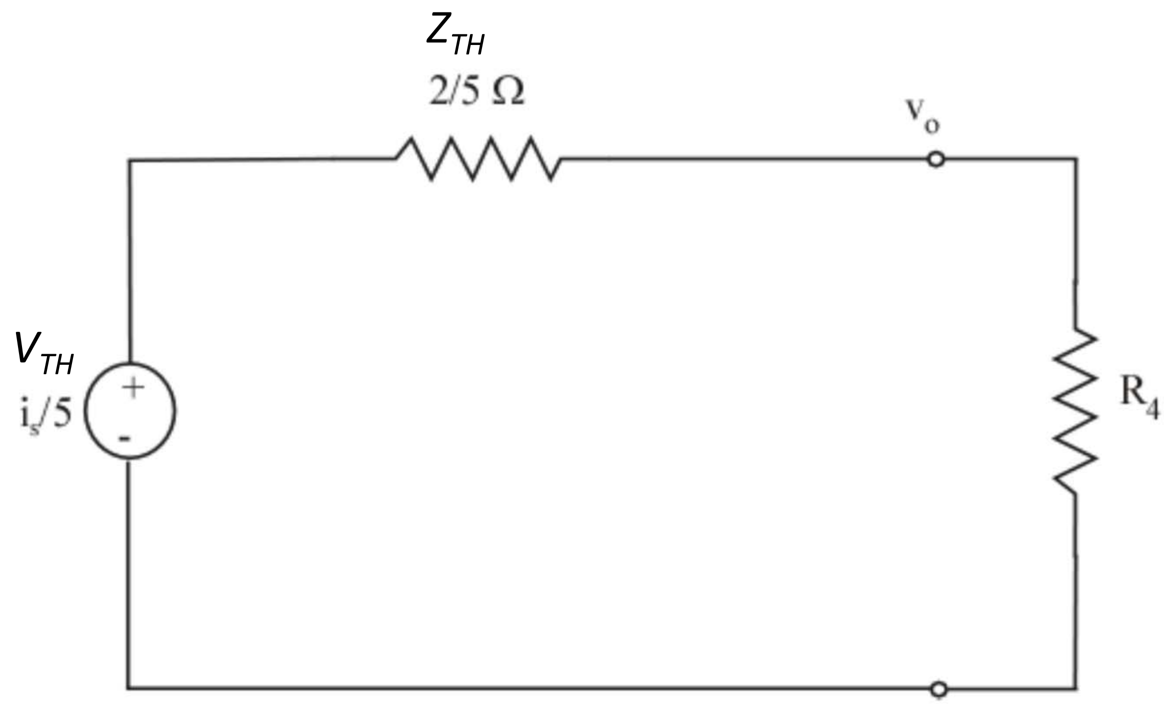
Thevenin Equivalent

Circuit for calculating impedance
(current source is replaced with open circuit)



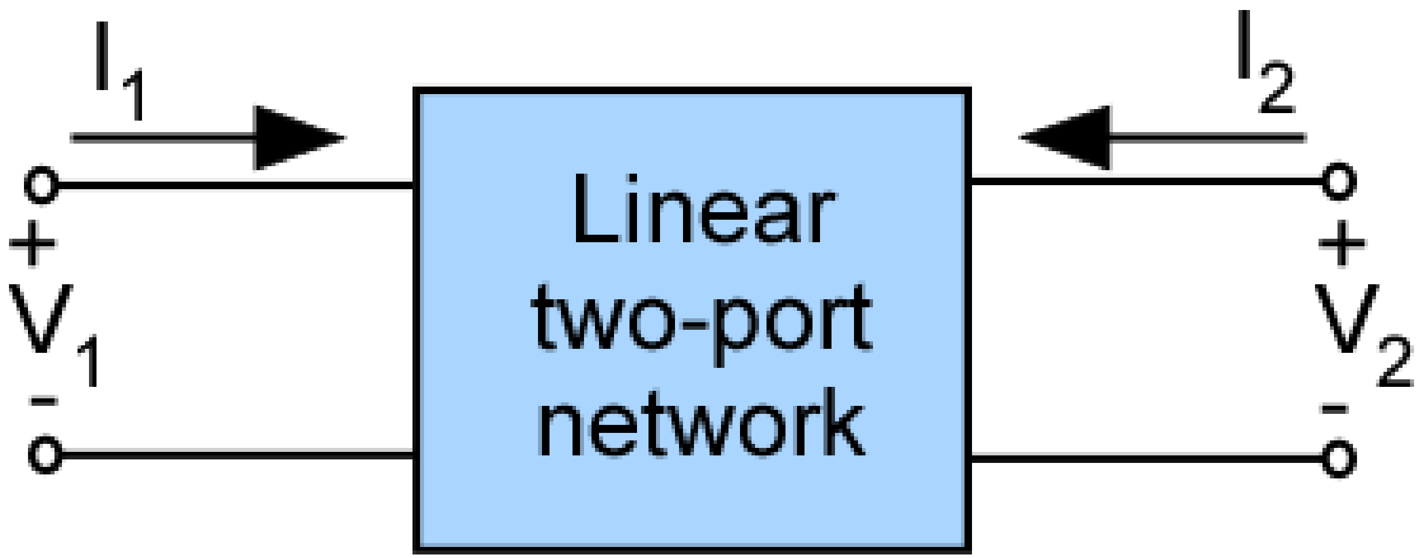
$$R_{th} = (R_1 + R_2) \,//\, R_3 = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} = \frac{2}{5}\,\Omega$$

Circuit with Thevenin Equivalent



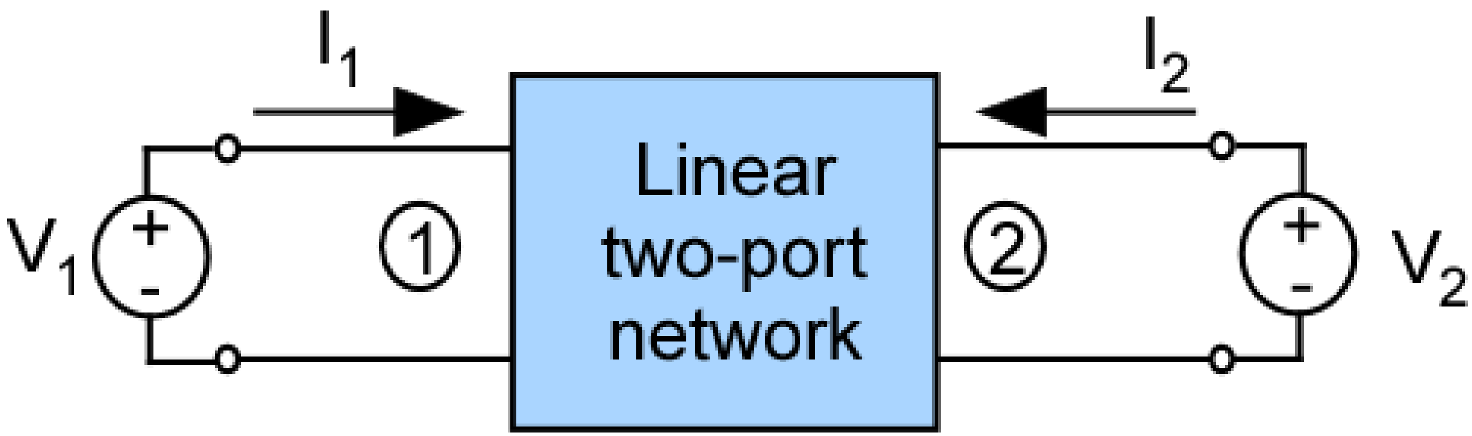
$$v_o(t) = \frac{R_4 i_s / 5}{2/5 + R_4}$$

Transfer Function Representation



Use a two-terminal representation of system for input and output

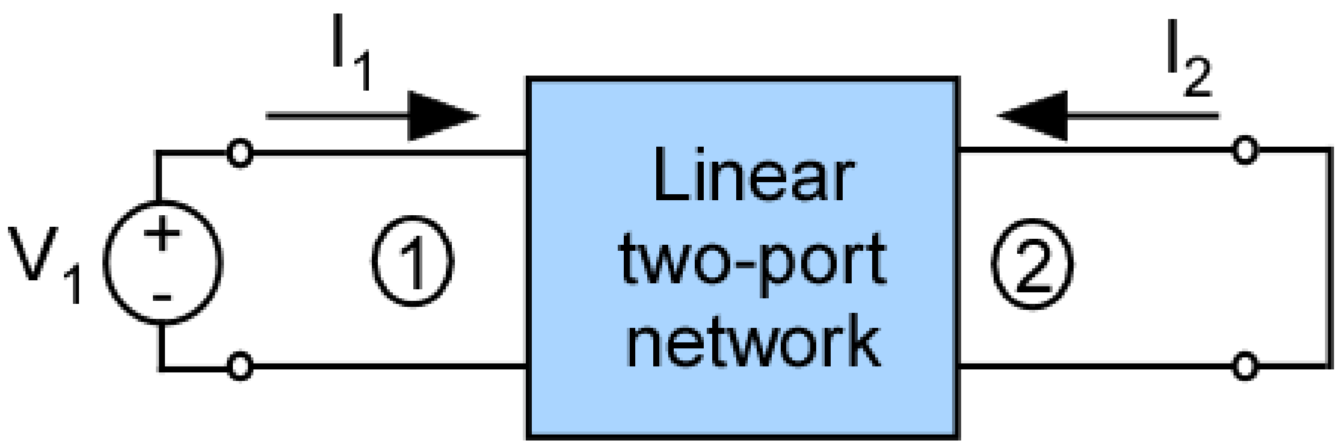
Y-parameter Representation



$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

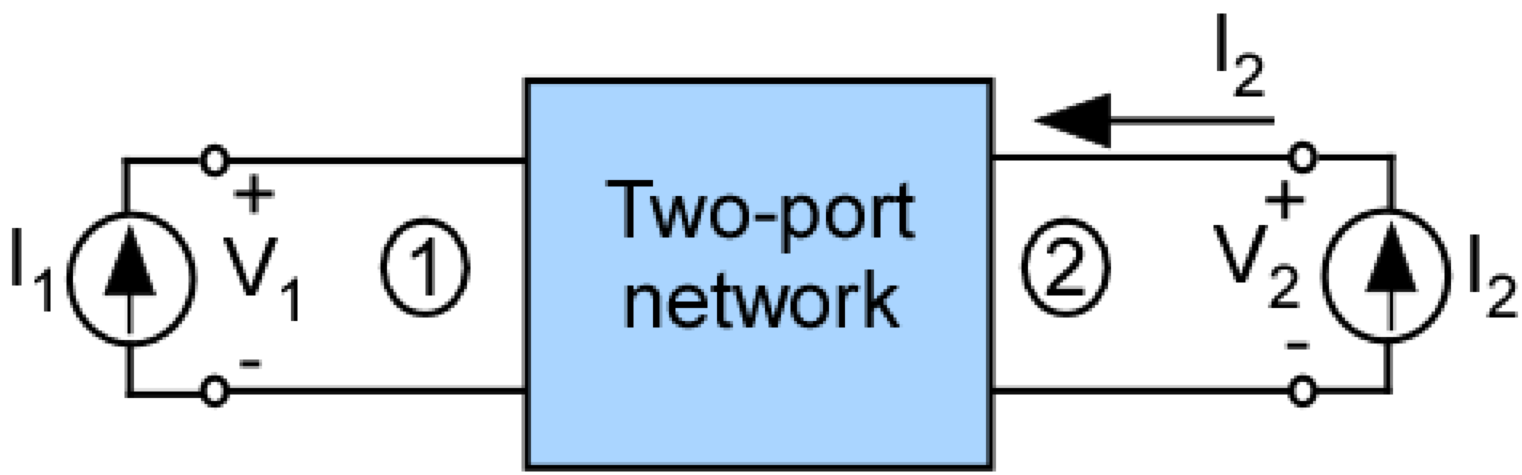
Y Parameter Calculations



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

To make $V_2 = 0$, place a short at port 2

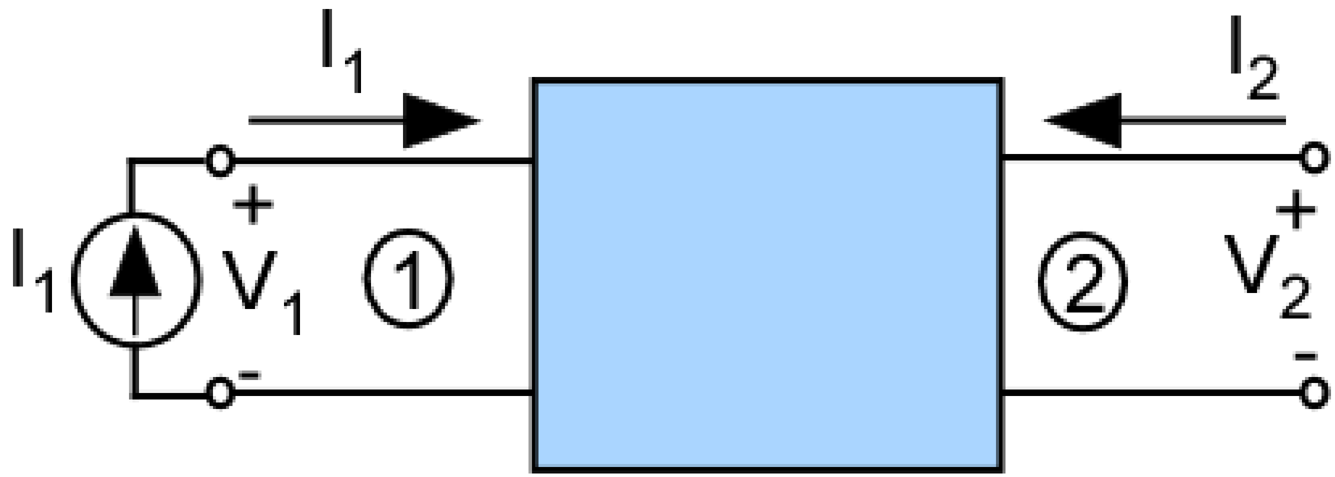
Z Parameters



$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

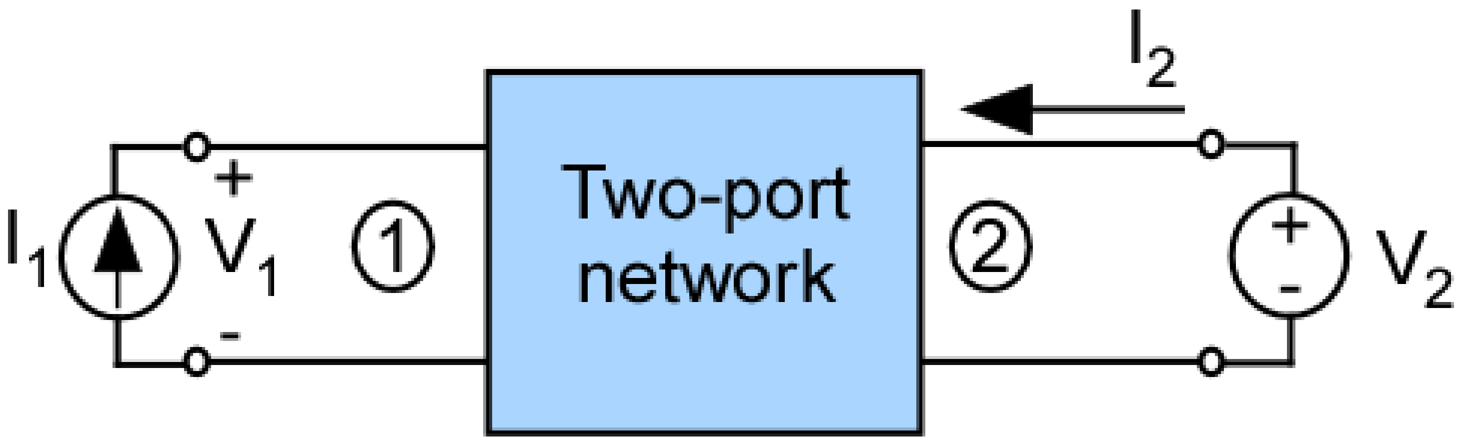
Z-parameter Calculations



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

To make $I_2 = 0$, place an open at port 2

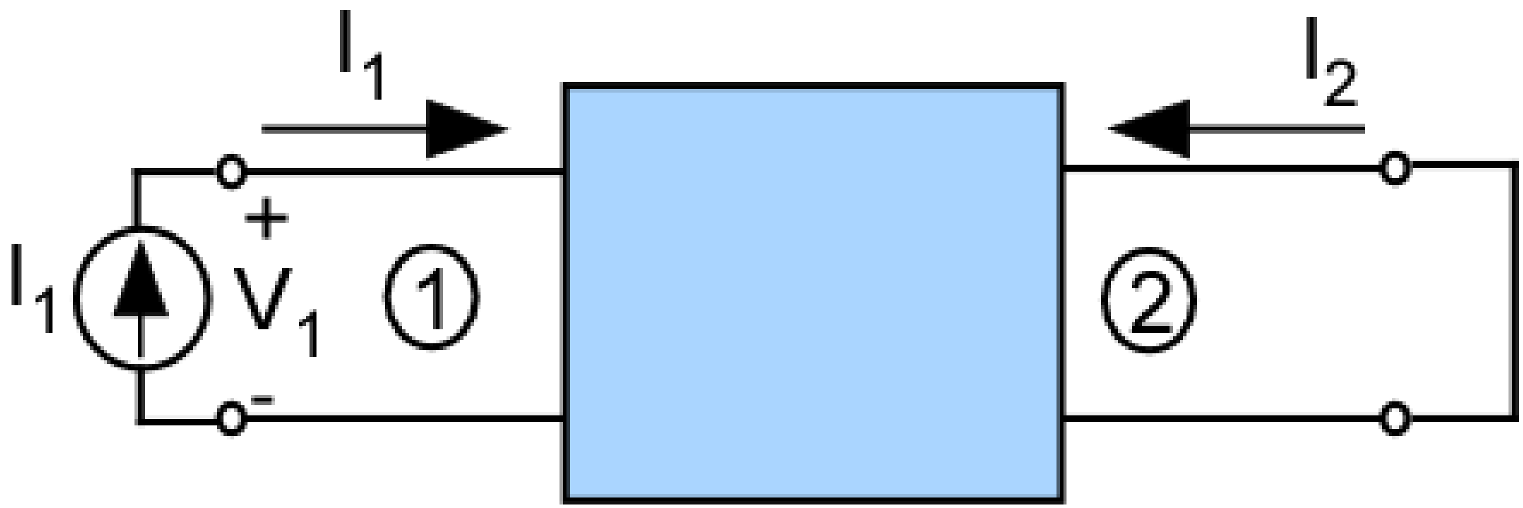
H Parameters



$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

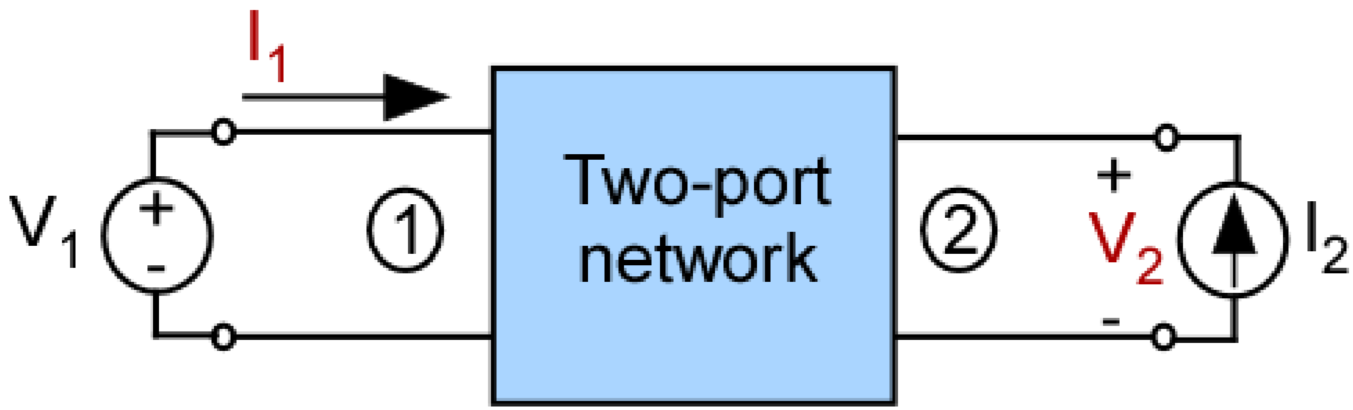
H Parameter Calculations



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

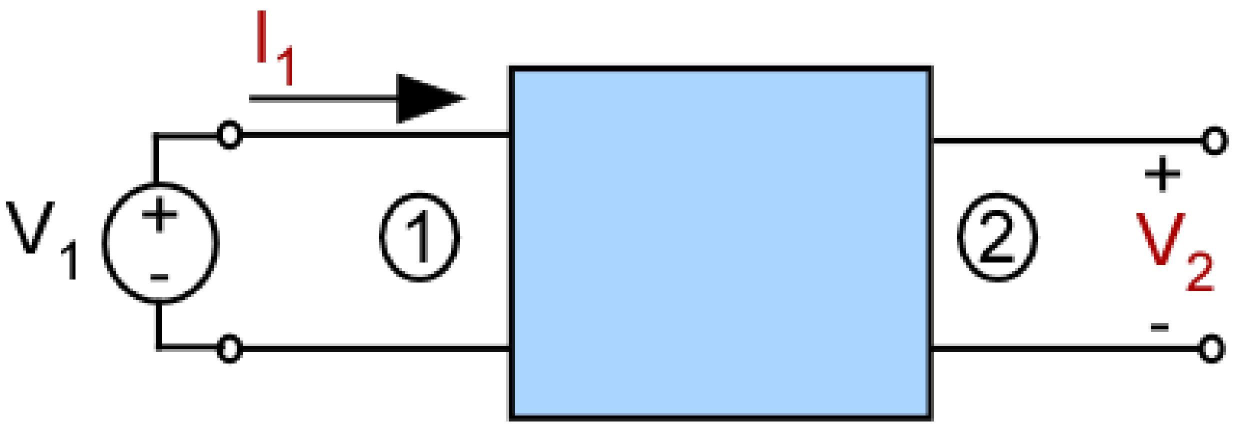
To make $V_2 = 0$, place a short at port 2

G Parameters



$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

G-Parameter Calculations



$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \qquad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$

To make $I_2 = 0$, place an open at port 2