# ECE 342
# Electronic Circuits

# Lecture 7
# DC Analysis of MOSFET

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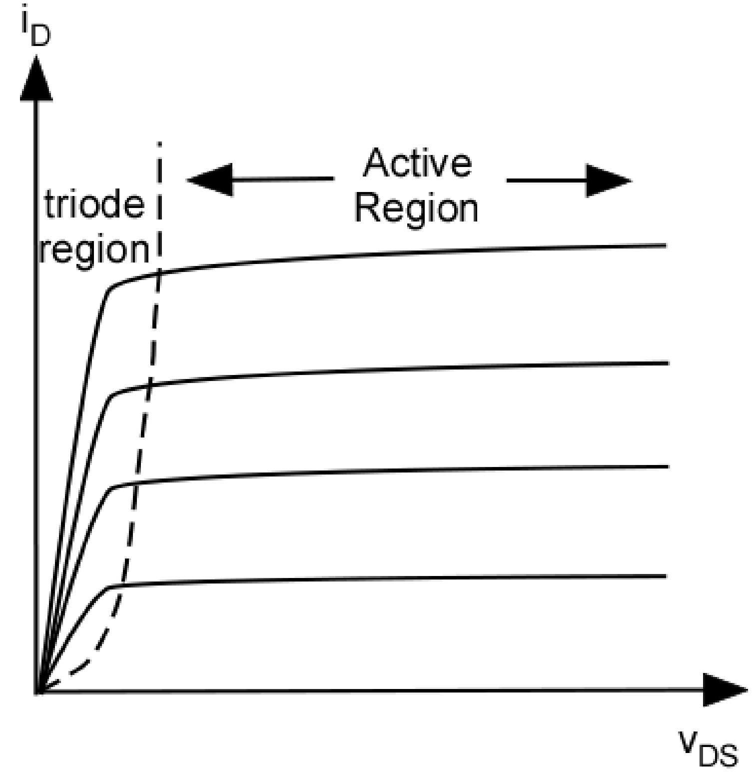
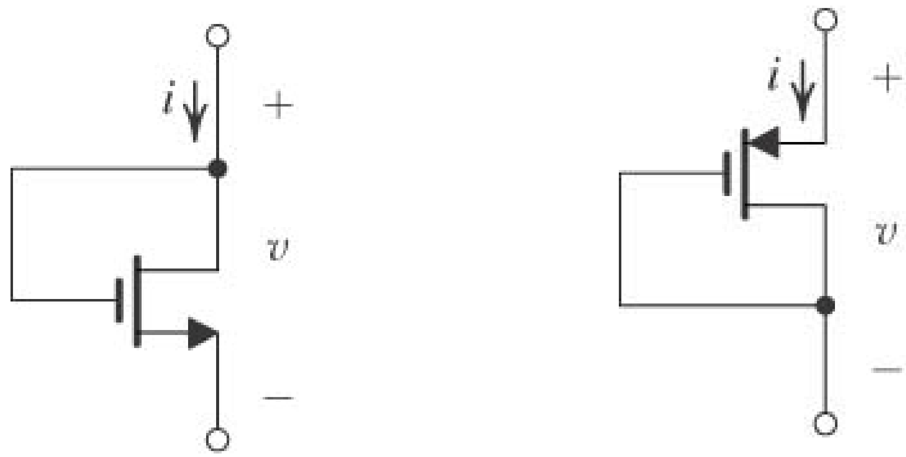
# MOS – Active Region




- **Saturation**
  - Channel is pinched off
  - Increase in $V_{DS}$ has little effect on $i_D$
  - Square-law behavior wrt $(V_{GS}\text{-}V_T)$
  - Acts like a current source

# Diode-Connected Transistor




**When the drain and gate of a MOSFET are connected together the result is a two-terminal device known as a diode-connected transistor**

$$V_{GD} \leq V_T$$

for saturation region. Since $V_{GD}$ is zero, then the device is always in the saturation region.

# Diode-Connected Transistor

$$i_D = i = \frac{1}{2} k_n' \frac{W}{L}\left(V_{GS} - V_t\right)^2$$

If we replace $V_{GS}$ by $V$ and use $k' = \frac{1}{2} k_n'$

$$i = k' \frac{W}{L}\left(V - |V_t|\right)^2$$

***incremental resistance*** →

$$r = \left(\frac{\partial i}{\partial V}\right)^{-1} = \frac{1}{2k' \frac{W}{L}\left(V - |V_t|\right)} = \frac{1}{k_n' \frac{W}{L} V_{ov}}$$

$$V = |V_t| + V_{ov}$$

# Example

**An MOS process technology has $L_{min}$= 0.4 μm, $t_{ox}$= 8 nm, $\mu$ = 450 cm²/V.s, $V_T$ = 0.7V**

**(a) Find $C_{ox}$ and $k_n' = \mu_n C_{ox}$**

**(b) $W/L$ = 8 μm/0.8μm. Calculate $V_{GS}$, $V_{DSmin}$ for operation in saturation with $I_D$= 100 μA**

**(c) Find $V_{GS}$ for the device in (b) to operate as a 1 kΩ resistor for small $v_{DS}$**

# Example - Solution

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45\times 10^{-11}}{8\times 10^{-9}} = 4.32\times 10^{-3}\ F/m^2 = 4.32\ fF/\mu m^2$$

$$C_{ox} = 4.32\ fF/\mu m^2$$

$$k_n' = \mu_n C_{ox} = 450\, cm^2/V.s \times 4.32\ fF/\mu m^2 = 194\ \mu A/V^2$$

For operation in saturation region

$$i_D = \frac{1}{2} k_n' \frac{W}{L}\left(V_{GS} - V_T\right)^2$$

$$100 = \frac{1}{2}\times 194 \times \frac{8}{0.8}\left(V_{GS} - 0.7\right)^2 \Rightarrow V_{GS} - 0.7 = 0.32\,V \Rightarrow V_{GS} = 1.02\,V$$

$$V_{DS\,\min} = V_{GS} - V_T = 0.32\,V$$

$$V_{DS\,\min} = 0.32\ V$$

# Example – (con't)

Triode region with $v_{DS}$ very small

$$r_{DS} = \left.\frac{v_{DS}}{i_D}\right|_{small\ v_{DS}} = \frac{1}{\left[k_n' \frac{W}{L}(V_{GS} - V_T)\right]}$$

$$1000 = \frac{1}{\left[194 \times 10^{-6} \times 10(V_{GS} - 0.7)\right]}$$

$$V_{GS} - 0.7 = 0.52\,V$$

$$V_{GS} = 1.22\,V$$

# Body Effect

- **The body effect**
  - $V_T$ varies with bias between source and body
  - Leads to modulation of $V_T$

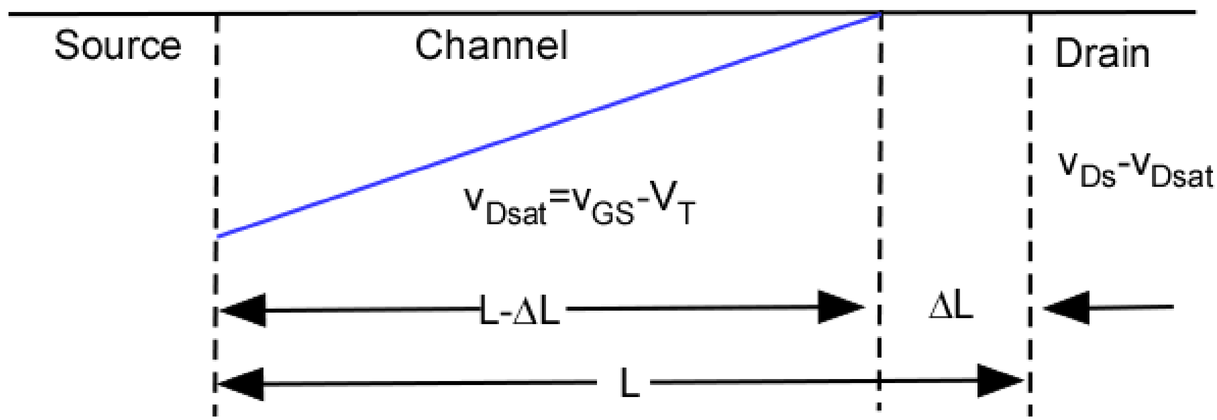
**Potential on substrate affects threshold voltage**

$$V_T(V_{SB}) = V_{To} + \gamma\left[\left(2|\phi_F| + V_{SB}\right)^{1/2} - \left(2|\phi_F|\right)^{1/2}\right]$$

$$|\phi_F| = \left(\frac{kT}{q}\right)\ln\left(\frac{N_a}{n_i}\right)$$ Fermi potential of material

$$\gamma = \frac{\left(2qN_a\varepsilon_s\right)^{1/2}}{C_{ox}}$$ Body bias coefficient

# Channel-Length Modulation



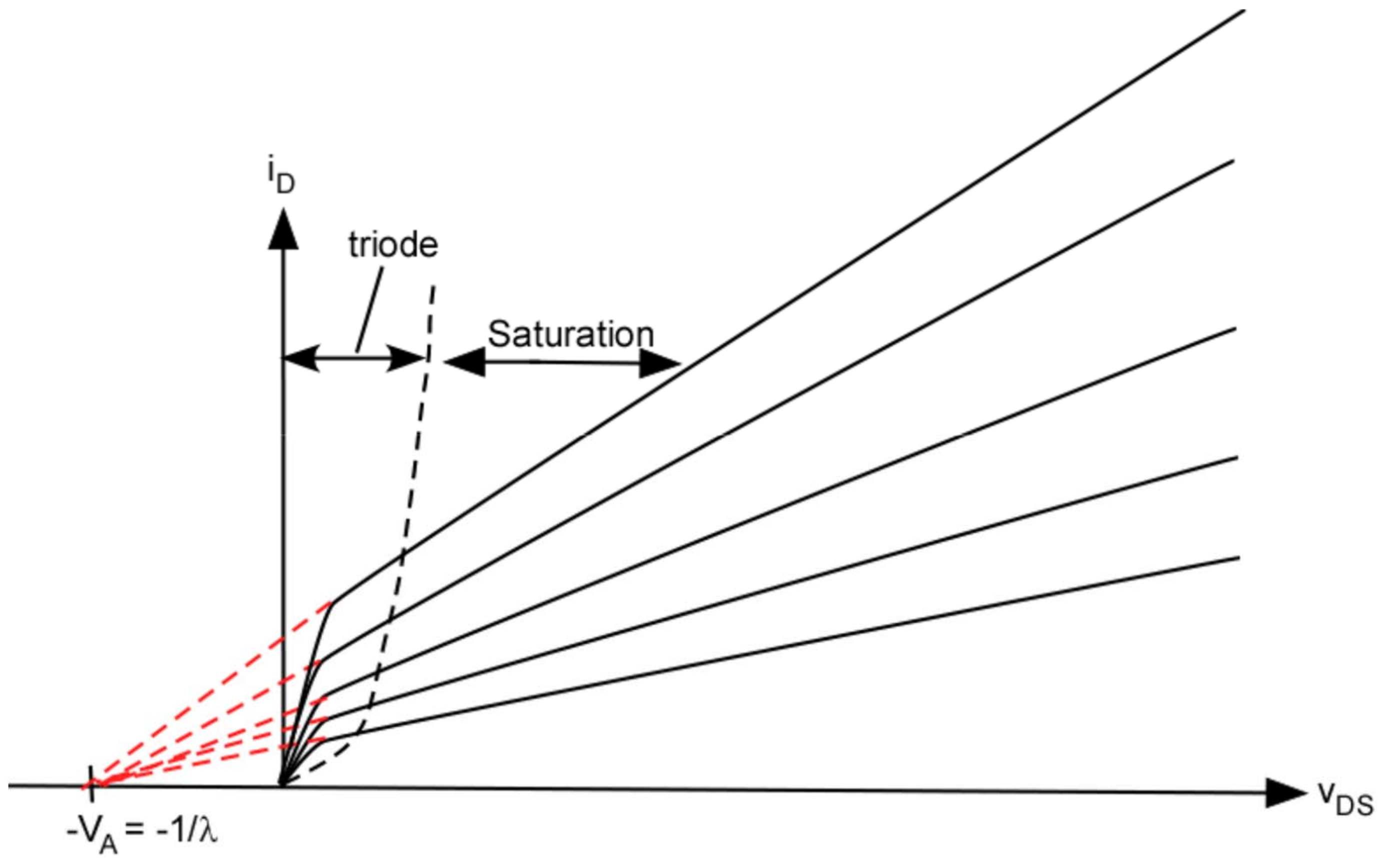

With depletion layer widening, the channel length is in effect reduced from *L* to *L-ΔL* ➔ Channel-length modulation

This leads to the following I-V relationship

$$i_D = \frac{1}{2}k_n' \frac{W}{L}\left(v_{GS} - V_T\right)^2 \left(1 + \lambda v_{DS}\right)$$

Where λ is a process technology parameter

# Channel-Length Modulation




Channel-length modulation causes $i_D$ to increase with $v_{DS}$ in saturation region

# Problem

A MOSFET has $V_T$ = 1 V with measured data:

| $V_{GS}(V)$ | $V_{DS}(V)$ | $I_D(\mu A)$ |
|---|---|---|
| 2 | 1 | 80 |
| 2 | 8 | 91 |

Find $\lambda$

$$(a)\ V_{GS} > V_T \qquad V_{DS} = V_{GS} - V_T \Rightarrow \ Pinchoff$$

$$(b)\ V_{GS} > V_T \qquad V_{DS} > V_{GS} - V_T = 1V \Rightarrow \ Active\ region$$

# Problem (cont')

Find $i_D$ at pinchoff $V_{DSP} = V_{GS} - V_T = 1V$

$$I_D = \frac{1}{2} k_n' \frac{W}{L} \left(V_{GS} - V_T\right)^2 \left(1 + \lambda V_{DS}\right)$$

$$I_{D1} = \frac{1}{2} k_n' \frac{W}{L} \left(V_{GS1} - V_T\right)^2 \left(1 + \lambda V_{DS1}\right)$$

$$I_{D2} = \frac{1}{2} k_n' \frac{W}{L} \left(V_{GS2} - V_T\right)^2 \left(1 + \lambda V_{DS2}\right)$$

# Problem (cont')

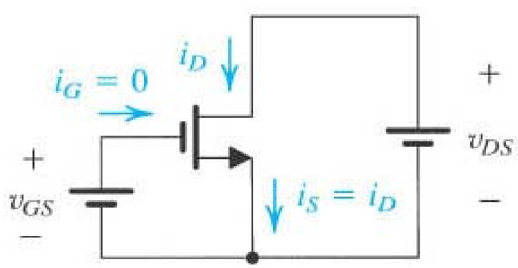
$$R = \frac{1+\lambda V_{DS1}}{1+\lambda V_{DS2}} = \frac{91}{80} = 1.1375$$

$$1+\lambda V_{DS2} = R + R\lambda V_{DS1}$$

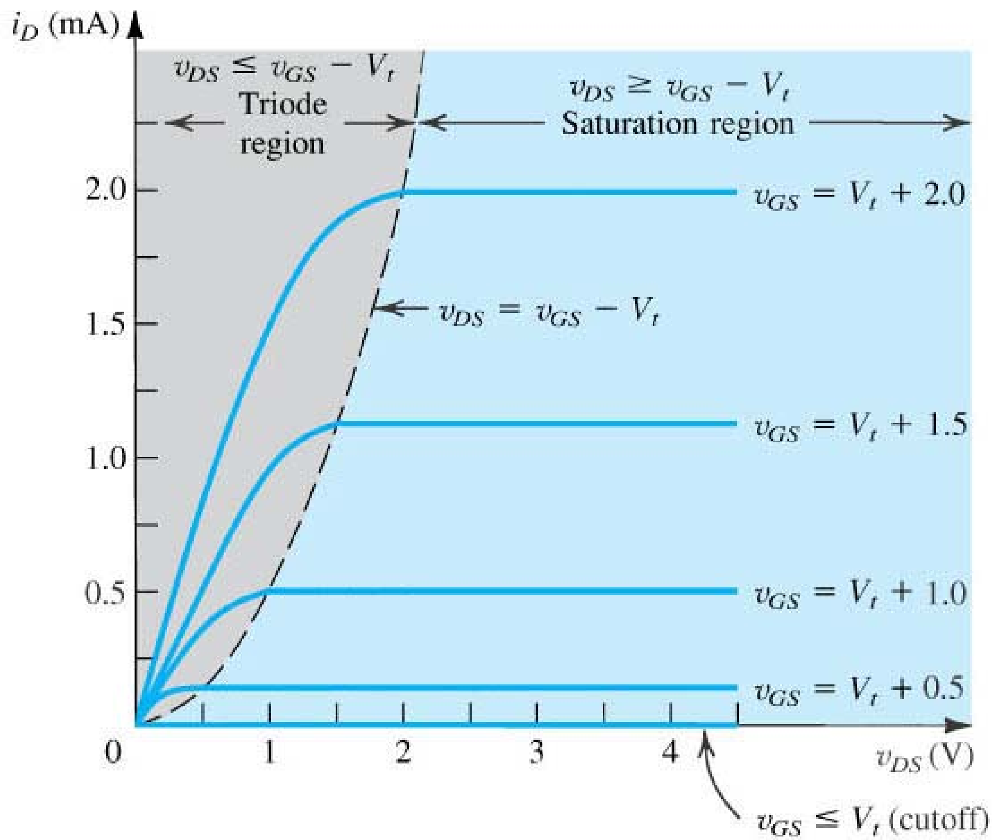
$$\lambda (V_{DS2} - RV_{DS1}) = R - 1$$

$$\lambda = \frac{R-1}{V_{DS2} - RV_{DS1}} = \frac{1.1375-1}{8-1.1375} = 0.02\,V^{-1}$$

# NMOS – IV Characteristics



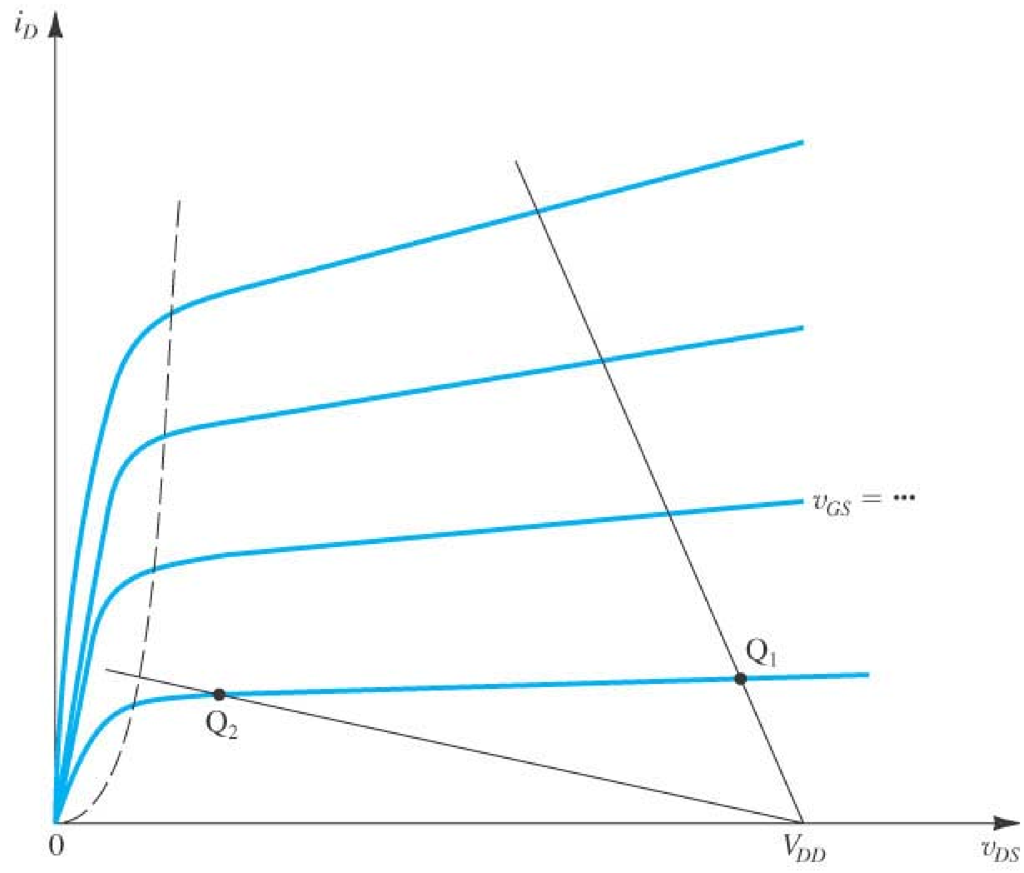
(a)



(b)

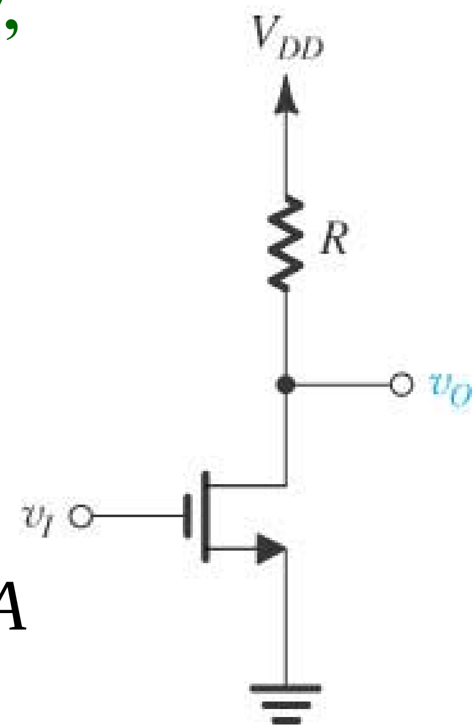
characteristics for a device with $k'_n (W/L) = 1.0$ mA/V$^2$.

# NMOS IV Curves



# MOSFET Circuit at DC – Problem 1

**The MOSFET in the circuit shown has $V_t = 1V$, $k_n' = 100\mu A/V^2$ and $\lambda = 0$. Find the required values of $W/L$ and of $R$ so that when $v_I = V_{DD} = +5$ V, $r_{DS} = 50\ \Omega$ and $v_o = 50$ mV.**



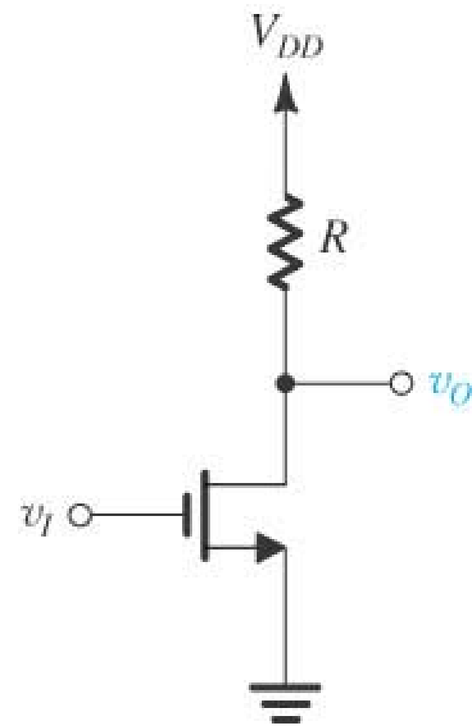

$$v_I = V_{GS} = 5\,V, \quad v_o = V_{DS} = 0.05\,V$$

$$r_{DS} = 50\,\Omega = \frac{V_{DS}}{I_D} \Rightarrow I_D = \frac{0.05}{50} = 0.001\,A = 1\,mA$$

$$R = \frac{V_{DD} - v_o}{I_D} = \frac{5 - 0.05}{1} = 4.95\,k\Omega$$

# MOSFET Circuit at DC – Problem 1 (cont')

$$V_{DS} < V_{GS} - V_t \Rightarrow \text{triode region}$$



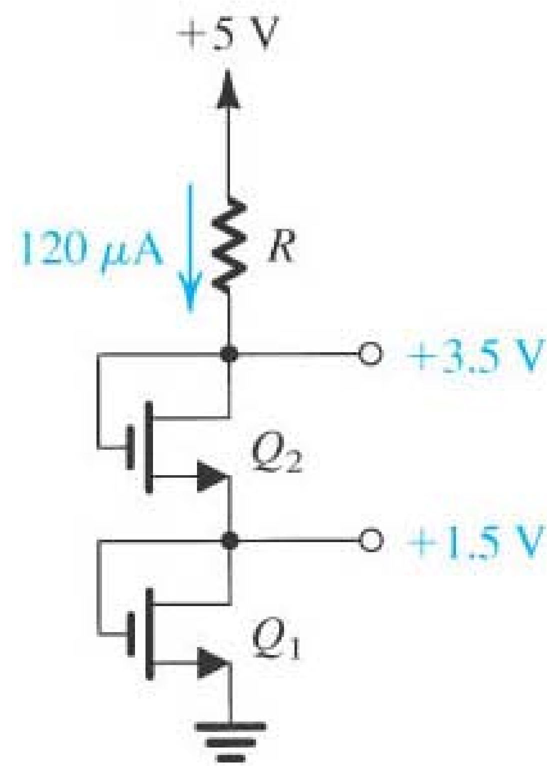

$$I_D = k_n' \frac{W}{L}\left[ (V_{GS} - V_t) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$1 = 100 \times 10^{-3} \frac{W}{L}\left[ (5-1) \times 0.05 - \frac{0.05^2}{2} \right]$$

$$\frac{W}{L} = 50$$

# MOSFET Circuit at DC – Problem 2

**The NMOS transistors in the circuit shown have $V_t = 1V$, $\mu_n C_{ox} = 120 \mu A/V^2$, $\lambda = 0$ and $L_1 = L_2 = 1 \mu m$. Find the required values of gate width for each of $Q_1$ and $Q_2$ and the value of $R$, to obtain the voltage and current values indicated.**



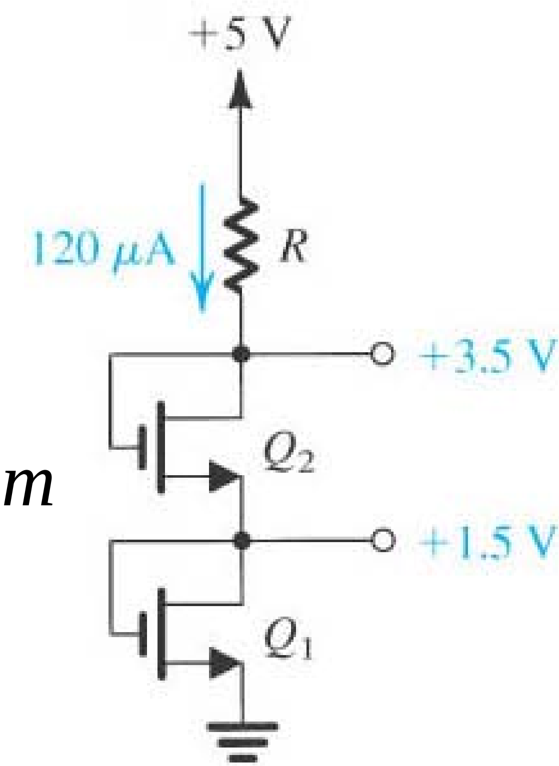

$$V_{GS1} = 1.5V$$

$$\text{Using } I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$$

$$120\ \mu A = \frac{1}{2} \times 120 \frac{W_1}{1} (1.5 - 1)^2 \Rightarrow W_1 = 8 \mu m$$

# MOSFET Circuit at DC – Problem 2

Using $I_D = \frac{1}{2} k_n^{'} \frac{W}{L} \left(V_{GS} - V_t\right)^2$




$$V_{GS2} = 3.5 - 1.5 = 2V$$

$$120\ \mu A = \frac{1}{2} \times 120 \frac{W_2}{1} \left(2-1\right)^2 \Rightarrow W_2 = 2\,\mu m$$

$$R = \frac{5-3.5}{0.120} = 12.5\ k\Omega$$