ECE 342
Electronic Circuits

Lecture 22
Transistor Capacitances

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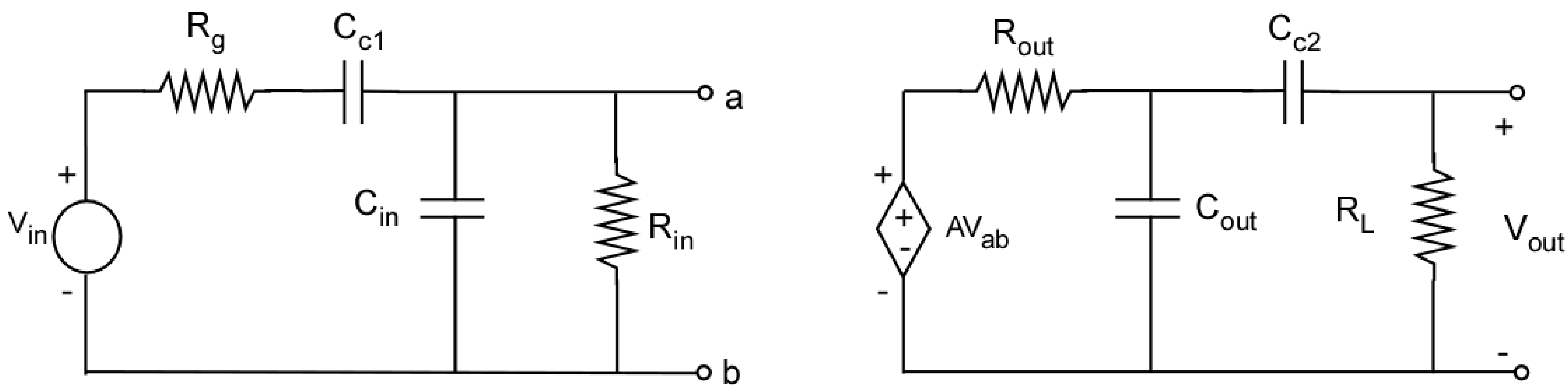
Model for general Amplifying Element

C_{c1} and C_{c2} are coupling capacitors (large) ➔ μF

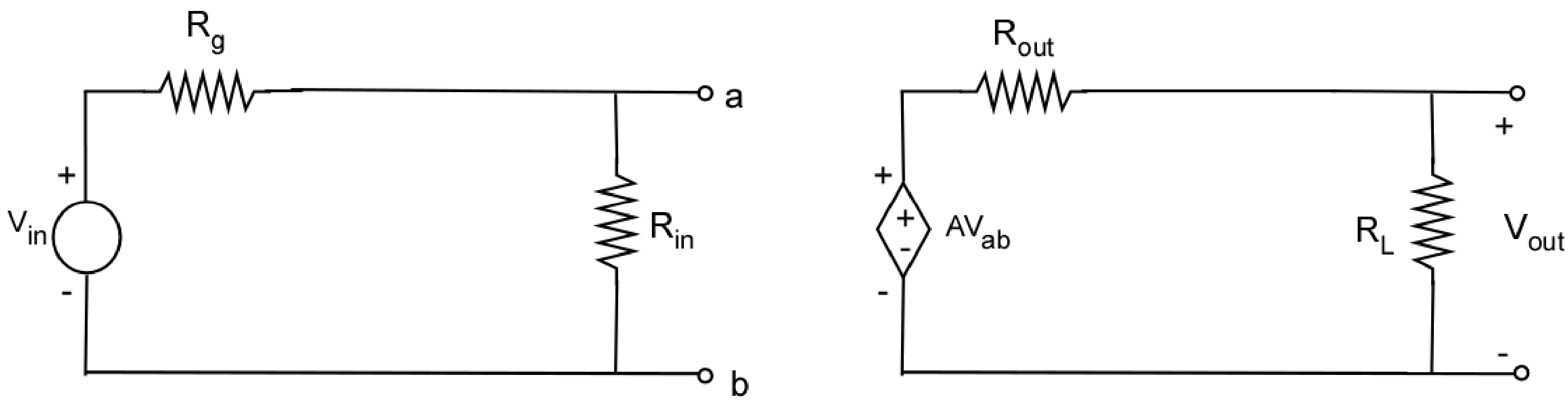
C_{in} and C_{out} are parasitic capacitors (small) ➔ pF



Midband Frequencies

- Coupling capacitors are short circuits

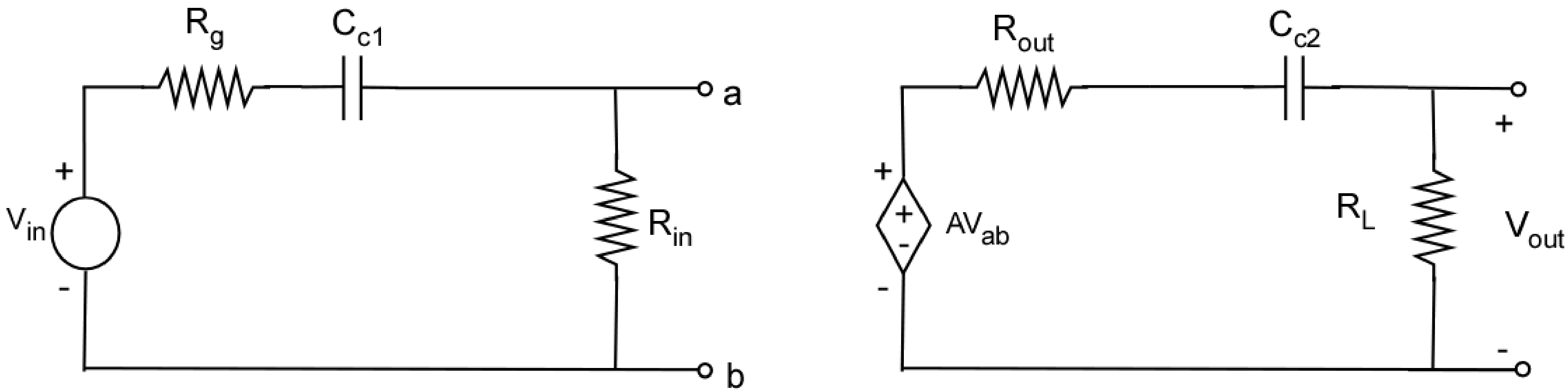
- Parasitic capacitors are open circuits



$$A_{MB} = \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_g + R_{in}} A \frac{R_L}{R_{out} + R_L}$$

Low Frequency Model

- Coupling capacitors are present
- Parasitic capacitors are open circuits



$$v_{ab} = \frac{v_{in} R_{in}}{R_g + R_{in} + \dfrac{1}{j\omega C_{c1}}} = \frac{v_{in} j\omega C_{c1} R_{in}}{1 + j\omega C_{c1}(R_g + R_{in})}$$

$$v_{ab} = v_{in} \frac{R_{in}}{R_g + R_{in}} \cdot \frac{j\omega C_{c1}(R_g + R_{in})}{\left[1 + j\omega C_{c1}(R_g + R_{in})\right]}$$

Low Frequency Model

$$define \quad f_{l1} = \frac{1}{2\pi\left(R_g + R_{in}\right)C_{c1}} \quad and \quad f_{l2} = \frac{1}{2\pi\left(R_L + R_{out}\right)C_{c2}}$$

$$v_{ab} = v_{in}\frac{R_{in}}{R_g + R_{in}} \cdot \frac{jf / f_{l1}}{1 + jf / f_{l1}}$$

$$Similarly, \quad v_{out} = Av_{ab}\frac{R_L}{R_L + R_{out}} \cdot \frac{jf / f_{l2}}{1 + jf / f_{l2}}$$

Low Frequency Model

$$Overall\ gain = \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_g + R_{in}} \cdot A \cdot \frac{R_L}{R_L + R_{out}} \cdot \frac{jf / f_{l1}}{1 + jf / f_{l1}} \cdot \frac{jf / f_{l2}}{1 + jf / f_{l2}}$$

$$\frac{v_{out}}{v_{in}} = A_{MB} \cdot \frac{jf / f_{l1}}{1 + jf / f_{l1}} \cdot \frac{jf / f_{l2}}{1 + jf / f_{l2}}$$

Example

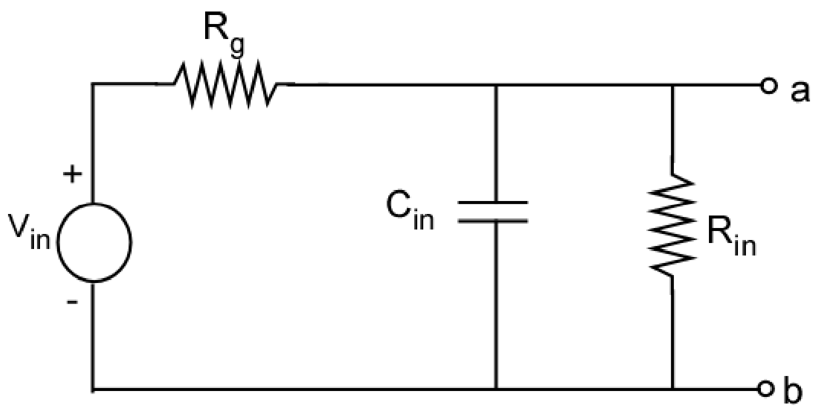
$R_{out} = 3$ kΩ, R_g=200 Ω, R_{in}=12 kΩ, R_L=10 kΩ
$C_{c1} = 5$ μF and $C_{c2} = 1$ μF

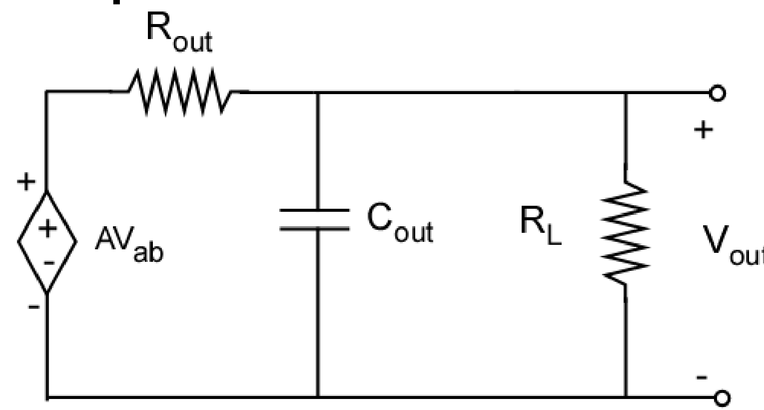
$$f_{l1} = \frac{1}{2\pi(12,200 \times 5 \times 10^{-6})} = 2.61\ Hz$$

$$f_{l2} = \frac{1}{2\pi(13,000 \times 10^{-6})} = 12.2\ Hz$$

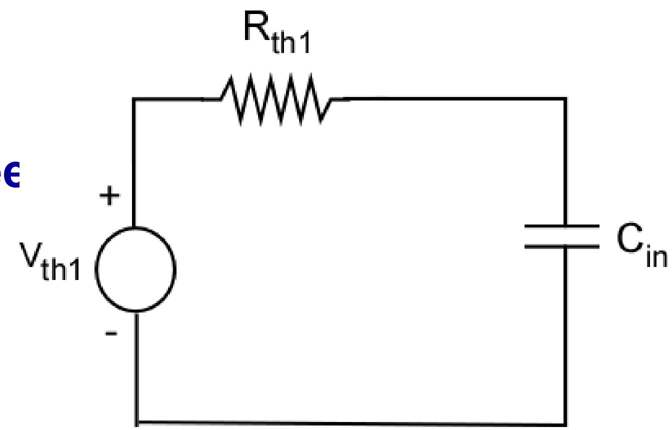
High Frequency Model

- Assume coupling capacitors are short
- Account for parasitic capacitors





Potential Thevenin equivalent for input as see by C_{in}



$$V_{th1} = \frac{v_{in} R_{in}}{R_g + R_{in}}$$

$$R_{th1} = R_g \, || \, R_{in}$$

High Frequency Model

$$v_{ab} = \frac{v_{in} R_{in}}{R_g + R_{in}} \cdot \frac{1}{1 + j\omega C_{in} R_{th1}}$$

$$v_{ab} = \frac{v_{in} R_{in}}{R_g + R_{in}} \cdot \frac{1}{1 + jf / f_{h1}} \quad where \quad f_{h1} = \frac{1}{2\pi R_{th1} C_{in}}$$

$$Likewise \; v_{out} = \frac{A v_{ab} R_L}{R_{out} + R_L} \cdot \frac{1}{1 + j\omega C_{out} R_{th2}}$$

$$with \quad R_{th2} = R_{out} \,||\, R_L$$

$$v_{out} = \frac{A v_{ab} R_L}{R_L + R_{out}} \cdot \frac{1}{1 + jf / f_{h2}} \quad where \quad f_{h2} = \frac{1}{2\pi R_{th2} C_{out}}$$

High Frequency

Overall gain is:

$$\frac{v_o}{v_i} = A \cdot \frac{R_{in}}{R_{in} + R_g} \cdot \frac{R_L}{R_L + R_{out}} \cdot \frac{1}{1 + jf / f_{h1}} \cdot \frac{1}{1 + jf / f_{h2}}$$

or

$$\frac{v_o}{v_i} = A_{MB} \cdot \frac{1}{1 + jf / f_{h1}} \cdot \frac{1}{1 + jf / f_{h2}}$$

Example

Example: R_{out} = 3 kΩ, R_g=200 Ω, R_{in}=12 kΩ, R_L=10 kΩ
C_{in} = 200 pF and C_{out}= 40 pF

$$f_{h1} = \frac{1}{2\pi \times 2 \times 10^{-10} \times (12,200 \,||\, 200)} = 4.05\ MHz$$

$$f_{h2} = \frac{1}{2\pi \times 40 \times 10^{-12} \times (10,000 \,||\, 3,000)} = 1.72\ MHz$$

Summary: low-frequency < 12.2 Hz, High frequency > 1.72 MHz

$$\log\left(\frac{4.05 \times 10^6}{12.2}\right) = 5.52 \simeq 5\ decades$$

MOSFET - Gate Capacitance Effect

Triode region: $$C_{gs} = C_{gd} = \frac{1}{2} WLC_{ox}$$

Saturation region: $$C_{gs} = \frac{2}{3} WLC_{ox} \qquad C_{gd} = 0$$

Cutoff: $$C_{gd} = C_{gs} = 0$$

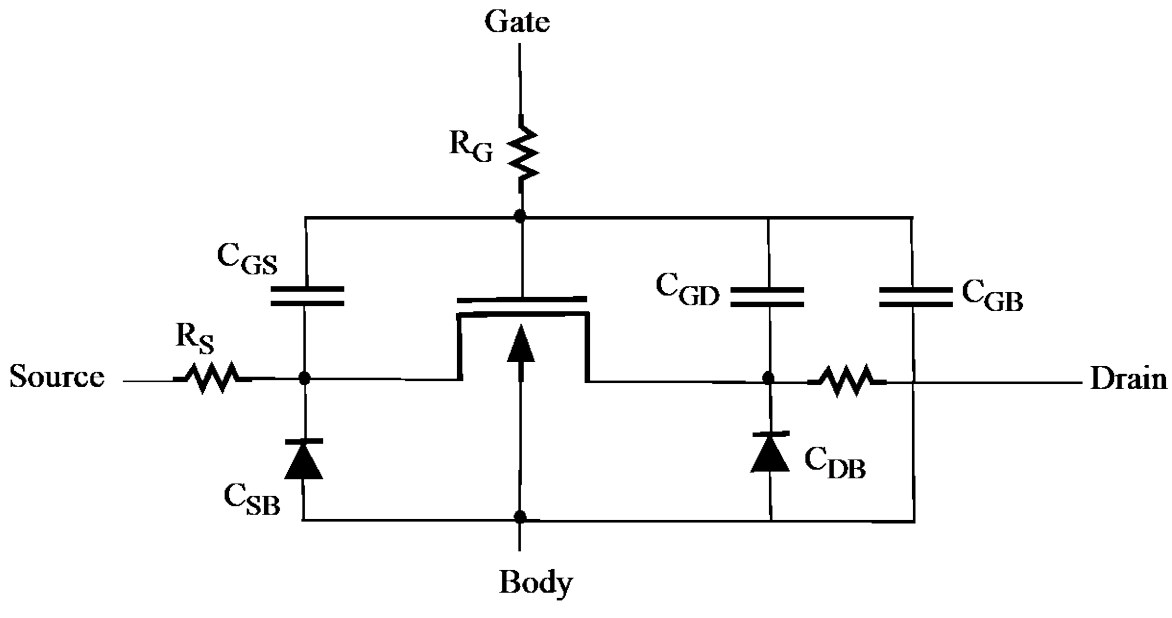
$$C_{gb} = WLC_{ox}$$

MOSFET – Junction Capacitances

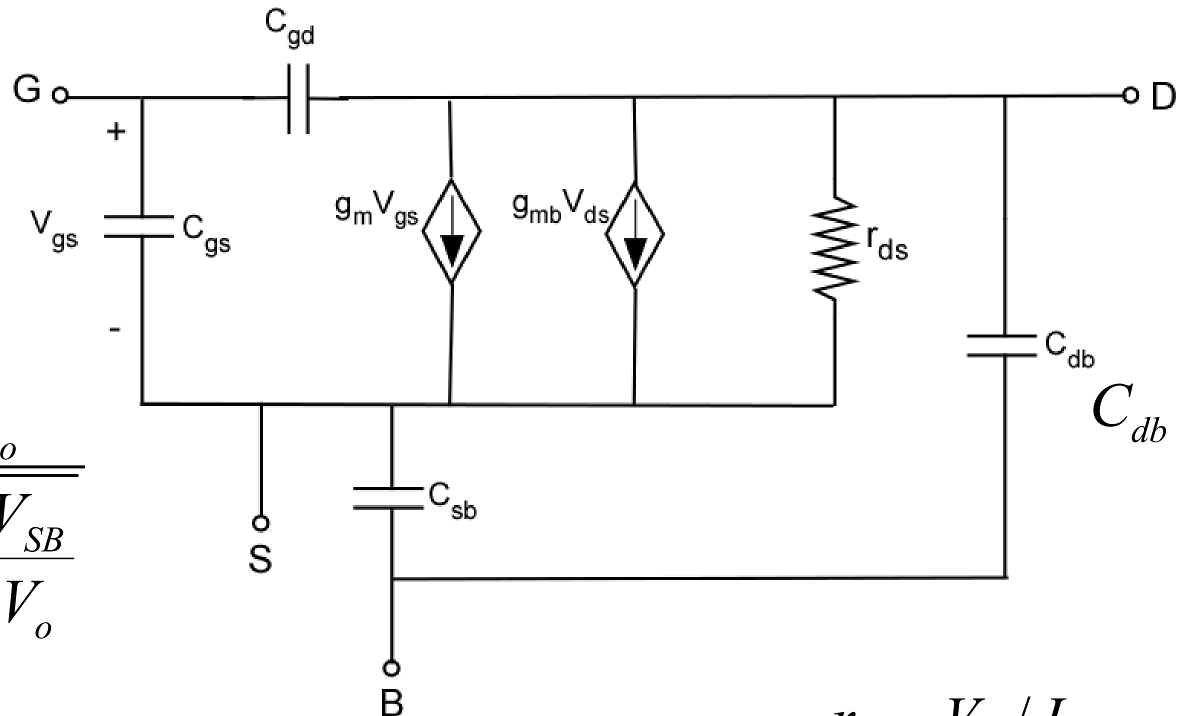
Overlap capacitance (gate-to-source): $C_{ov} = WL_{ov}C_{ox}$

$$C_{sb} = \frac{C_{sbo}}{\sqrt{1+\frac{V_{SB}}{V_o}}}$$

$$C_{db} = \frac{C_{dbo}}{\sqrt{1+\frac{V_{DB}}{V_o}}}$$



MOSFET High-Frequency Model



$$C_{sb} = \frac{C_{sbo}}{\sqrt{1 + \frac{V_{SB}}{V_o}}}$$

$$C_{db} = \frac{C_{dbo}}{\sqrt{1 + \frac{V_{DB}}{V_o}}}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{eff} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{V_{eff}}$$

$$g_{mb} = \chi g_m = \frac{\gamma}{2\sqrt{2\phi_F + V_{sb}}} g_m$$

$$r_{ds} = V_A / I_D = \frac{1}{\lambda I_D}$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov}C_{ox}$$

$$C_{gd} = WL_{ov}C_{ox}$$

BJT Capacitances

Base: Diffusion Capacitance: C_{de} (small signal)

$$C_{de} \equiv \frac{dQ_n}{dv_{BE}}$$

where Q_n is the minority carrier charge in base

$$C_{de} = \tau_F \frac{di_C}{dv_{BE}} = \tau_F g_m = \frac{\tau_F I_C}{V_T}$$

where τ_F is the forward transit time (time spent crossing base)

BJT Capacitances

Base-emitter junction capacitance:

$$C_{je} = \frac{C_{jeo}}{\left(1 - \frac{V_{BE}}{V_{oe}}\right)^m}$$

C_{jeo} is C_{je} at 0 V. V_{oe} is EBJ built in voltage ~ 0.9 V
m is the grading coefficient (typically, 0.2-0.5)

BJT Capacitances

In hybrid pi model, $C_{de}+C_{je}=C_{\pi}$

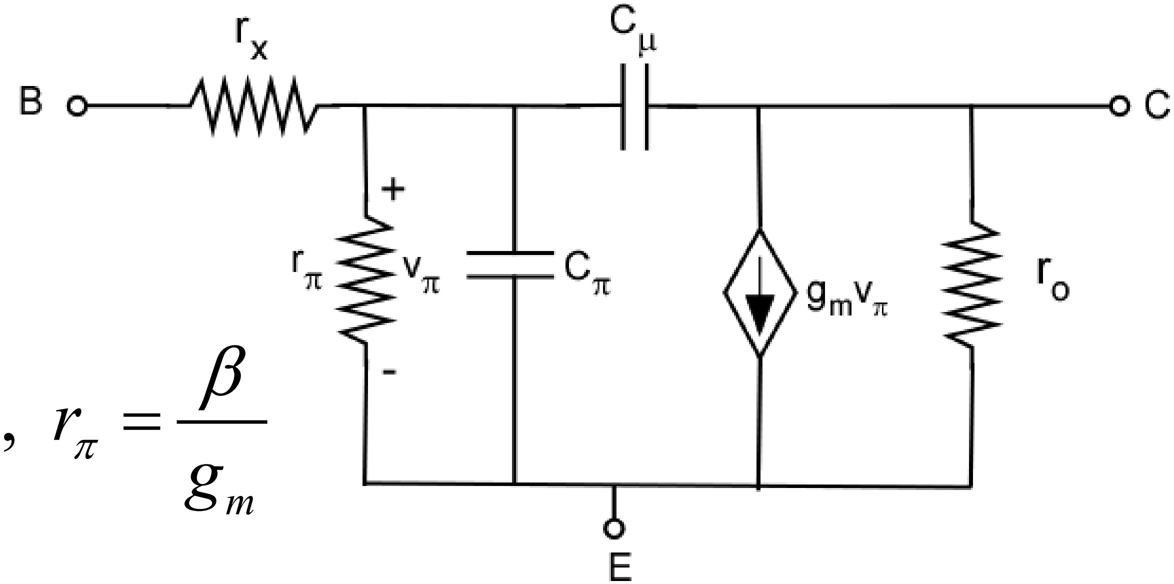
Collector-base junction capacitance

$$C_{\mu} = \frac{C_{\mu o}}{\left(1+\frac{V_{CB}}{V_{oc}}\right)^{m}}$$

$C_{\mu o}$ is C_{μ} at 0 V. V_{oc} is CBJ built in voltage ~ 0.9 V

C_{π} is around a few tens of pF
C_{μ} is around a few pF
m is the grading coefficient (typically, 0.2-0.5)

High-Frequency Hybrid-π Model

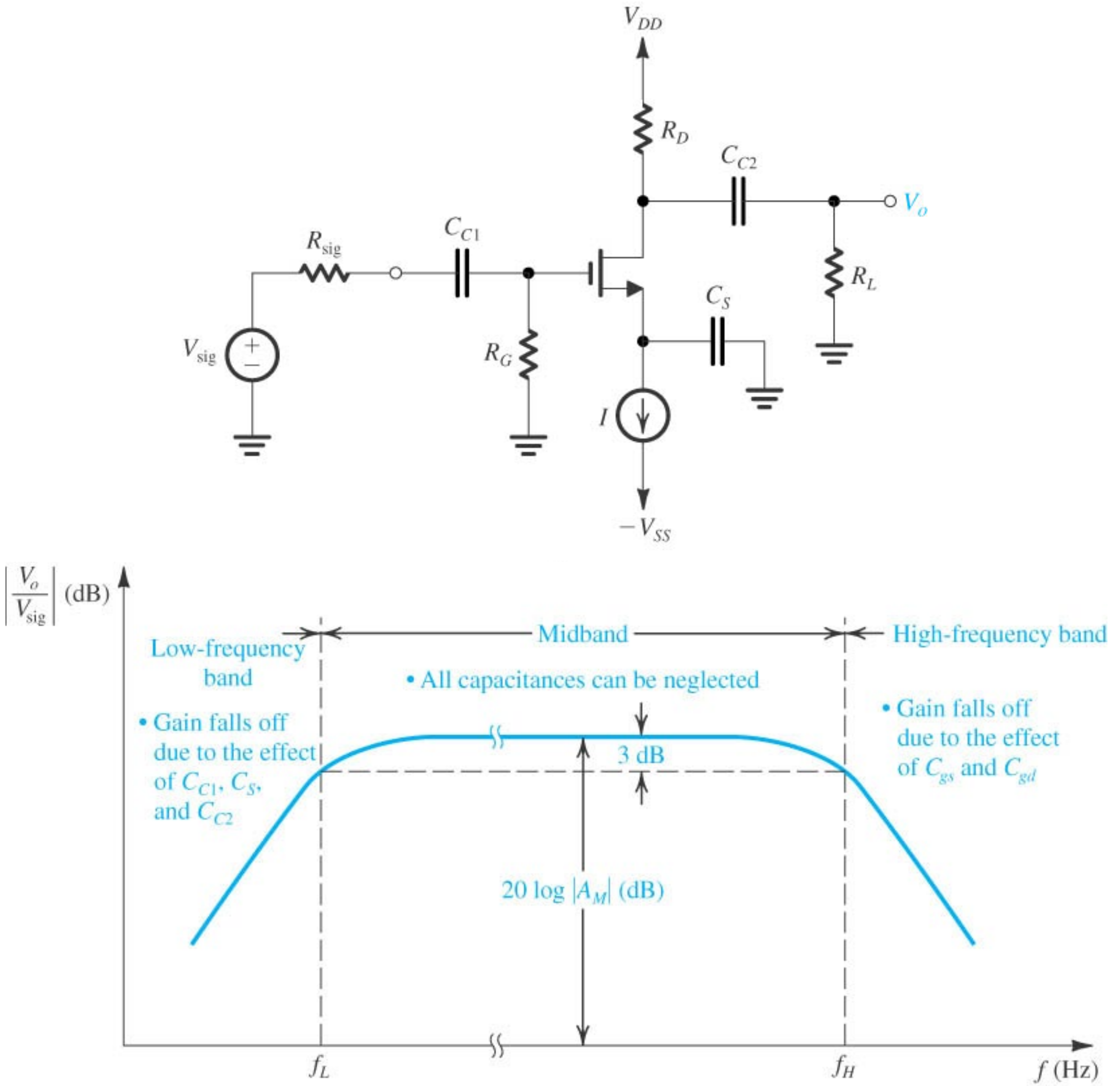


$$g_m = \frac{I_C}{V_T}, \quad r_o = \frac{|V_A|}{I_C}, \quad r_\pi = \frac{\beta}{g_m}$$

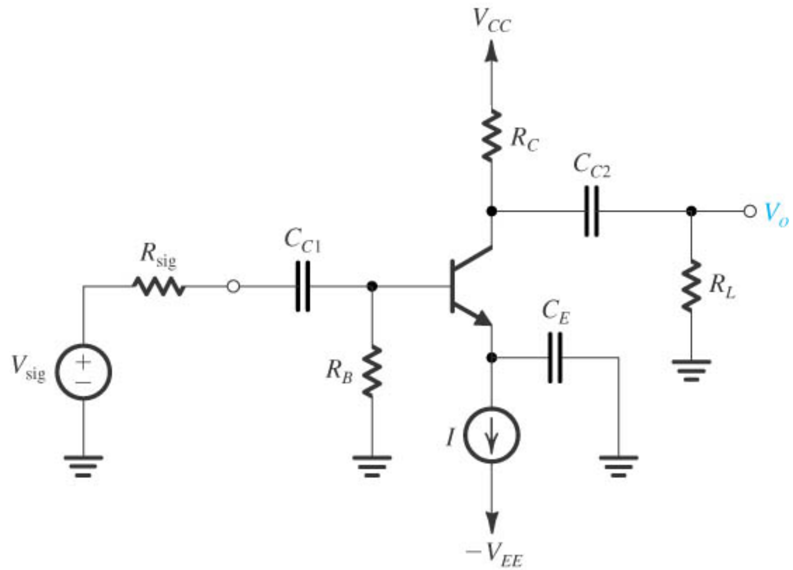
$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}, \quad C_\pi = C_{de} + C_{je}, \quad C_{de} = \tau_F g_m$$

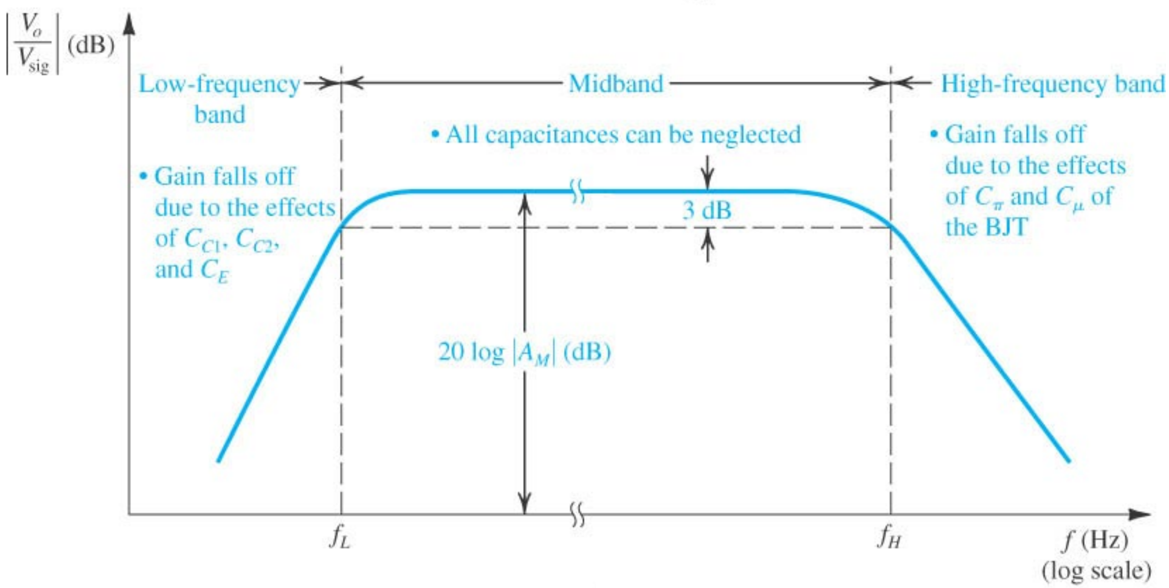
$$C_\mu = \frac{C_{jco}}{\left(1 + \frac{V_{CB}}{V_{oc}}\right)^m}, \quad m = 0.3 - 0.5 \qquad f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)}$$

CS - Three Frequency Bands



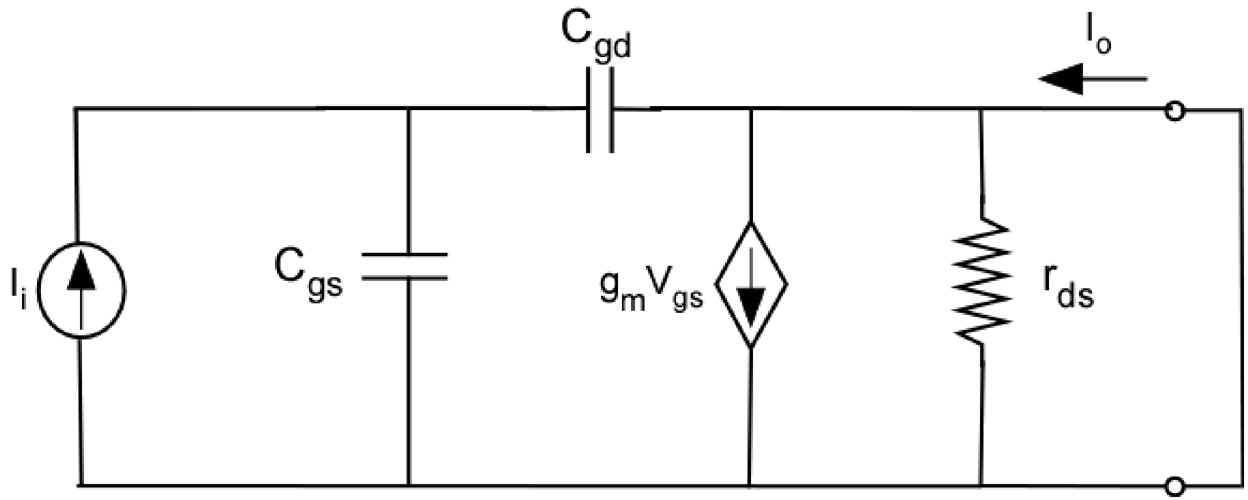
CE - Three Frequency Bands





Unity-Gain Frequency f_T

f_T is defined as the frequency at which the short-circuit current gain of the common source configuration becomes unity



(neglect $sC_{gd}V_{gs}$ since C_{gd} is small)

Define:

$$s = j\omega$$

$$I_o = g_m V_{gs} - sC_{gd}V_{gs}$$

$$I_o \simeq g_m V_{gs}$$

$$V_{gs} = \frac{I_i}{s\left(C_{gs} + C_{gd}\right)}$$

$$\frac{I_o}{I_i} = \frac{g_m}{s\left(C_{gs} + C_{gd}\right)}$$

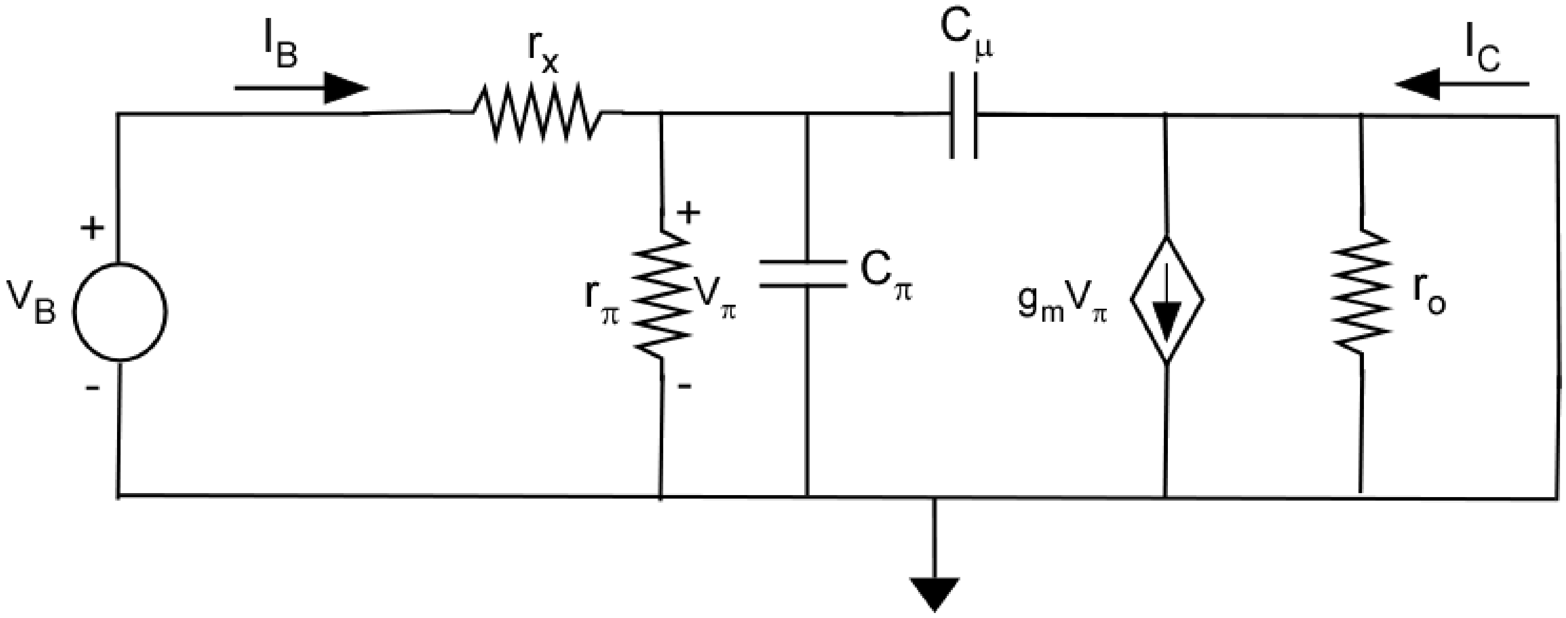
Calculating f_T

For $s=j\omega$, magnitude of current gain becomes unity at

$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}} \Rightarrow f_T = \frac{g_m}{2\pi\left(C_{gs} + C_{gd}\right)}$$

f_T ~ 100 MHz for 5-μm CMOS, f_T ~ several GHz for 0.13μm CMOS

BJT - Short-Circuit Current Gain



$$I_C = \left(g_m - sC_\mu\right) v_\pi$$

$$v_\pi = \frac{I_B}{\frac{1}{r_\pi} + sC_\mu + sC_\pi}$$

BJT - Short-Circuit Current Gain

Define h_{fe} as short-circuit current gain

$$h_{fe} = \frac{I_C}{I_B} = \frac{g_m - sC_\mu}{\frac{1}{r_\pi} + s\left(C_\pi + C_\mu\right)}$$

$$g_m \gg sC_\mu \text{ at freq. of interest}$$

$$h_{fe} = \frac{I_C}{I_B} = \frac{g_m r_\pi}{1 + s\left(C_\pi + C_\mu\right) r_\pi}$$

BJT - Short-Circuit Current Gain

$$h_{fe} = \frac{\beta_o}{1 + s\left(C_\pi + C_\mu\right) r_\pi}$$

Define h_{fe} has a single pole (or STC) response.
Unity gain bandwidth is for:

$$h_{fe} = \frac{g_m r_\pi}{1 + s\left(C_\pi + C_\mu\right) r_\pi} = 1 \quad or \quad \frac{g_m}{2\pi f_T\left(C_\pi + C_\mu\right)} = 1$$

In some cases, if C_μ is known, then

BJT - Short-Circuit Current Gain

$$f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)}$$

From which we get

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{g_m}{\omega_T}$$

$$Thus, \;\; C_\pi + C_\mu = \frac{g_m}{\omega_T} \Rightarrow C_\pi = \frac{g_m}{\omega_T} - C_\mu$$