# Experiment 06 - Extraction of Transmission Line Parameters

## 1 Introduction

With the increase in both speed and complexity of modern circuits, modeling interconnects becomes crucial for signal integrity analysis and design. This lab will introduce you to a simple method to model transmission line by extracting some of its parameters such as its characteristic impedance, phase velocity, attenuation etc.

You will have the very first hands-on experience with TRL calibration.

## 2 Background

Because a transmission line is a linear, time-invariant (LTI) device, its behavior can be completely described by its response to complex exponentials at every possible frequency. When we operate in the real-world, we must deal with real-world signals; therefore, we measure our LTI device’s response to sinusoids of varying frequency. In addition, we obviously cannot measure the response of every possible frequency due to bandwidth limitations of our measurement devices and sheer practicality. Thus, we determine an appropriate bandwidth of interest based on how our LTI device will be used in our application and a number of samples based on a compromise between measurement time and desired accuracy. So, keep in mind that the extraction method introduced here is not a universal ingredient for every case. You will need to build up for yourself enough experience and confidence for knowing exactly when the method would not hold valid.

Returning to our goal of fully characterizing a transmission line, we now discuss how to model its frequency dependence. The parameters of interest are the resistance $R$, inductance $L$, conductance $G$, and capacitance $C$ per unit length. These parameters define the propagation constant $\gamma$ and the characteristic impedance $Z_c$ as

\[
\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{1}
\]

From network analyzer measurements at a frequency $f$, with $\omega = 2\pi f$, the scattering parameters are obtained. They can be related to the transmission line parameters from above using the relations

\[
\Gamma = \frac{Z_c - Z_o}{Z_c + Z_o} = Q \pm \sqrt{Q^2 - 1} \text{ and } X = e^{-\gamma l} = \frac{(S_{11} + S_{21}) - \Gamma}{1 - (S_{11} + S_{21})\Gamma} \tag{2}
\]

where

\[
Q = \frac{(S_{11}^2 - S_{21}^2) + 1}{2S_{11}} \tag{3}
\]

Substituting (3) in $\Gamma$ of (2),

\[
\Gamma \approx \frac{S_{11}}{S_{11}^2 - S_{21}^2 + 1} \tag{4}
\]

$\Gamma$ is the reflection coefficient, $X$ is the propagation function, $S_{11}$ and $S_{21}$ are the measured reflection and transmission scattering parameters, respectively, $l$ is the length of the transmission line and $Z_o$ is the reference impedance of the measurement system.

$X$ can be rewritten as

\[
X = e^{-\gamma l} = e^{-j\beta l}e^{-\alpha l} \tag{5}
\]
in which the propagation constant $\beta$ and the attenuation constant $\alpha$ are separated. In most practical cases involving cables, dielectric losses are very small so that the conductance per unit length, $G$ can be ignored; also, when the ratio $R/\omega L$ is very small, an approximation for the complex propagation constant gives

$$\gamma \approx \frac{R}{2} \sqrt{\frac{C}{L}} + j \omega \sqrt{LC} = \frac{R}{2Z_c} + j \frac{\omega}{v_p} = \alpha + j \beta$$

where $v_p$ is the propagation velocity in the cable. The first term of $\gamma$ is associated with the attenuation along the direction of propagation. The second term is associated with the phase shift and velocity. They can be related to other parameters by

$$\alpha \approx \frac{R}{2Z_c} \text{ and } \beta \approx \frac{\omega}{v_p}.$$

At each frequency, $\Gamma$ and $X$ are obtained from the measured scattering parameters using (2) and (4). Next, (5) is used to find $\gamma$; (6) is used to approximate $\alpha$ and $\beta$; and (7) to find $Z_c$, $R$, $v_p$. The results can then be combined with (1) to obtain $R$, $L$, and $C$.

Below is an example of transmission line parameter extraction.

![Graphs showing transmission line parameter extraction](image)

Figure 1: Transmission line extraction example: $\alpha$, $\beta$, $R$, $Z_c$ are shown

## 3 Prelab

1. At what conditions is (7) accurate? State your answer in terms of $\omega$, $R$, $G$, $L$, and $C$ and then in terms of the physical meaning of those assumptions.
2. Explain the physical meaning of $\alpha$ and $\beta$ in $X = e^{-\gamma l} = e^{-j\beta l}e^{-\alpha l}$.

3. Derive $\Gamma \approx \frac{S_{11}}{S_{11}^2 - S_{21}^2 + 1}$ from Equations (2) and (3). Explicitly state what assumptions you made in your derivation? (Hint: For the + case, solve for $Q$ in terms of first then make an approximation based on the magnitude of $\Gamma$. For the − case, use the first two terms of the Taylor Series expansion of the square root term. What assumption about $Q$ is this truncated expansion making?)

4. What should the return loss (|$S_{11}$| in dB), insertion loss (|$S_{21}$| in dB), and phase($S_{21}$) of a lossless transmission line with no dispersion look like? (Hint: Think about the physical meaning of these parameters and what they ideally would be for a good transmission line.)

5. To illustrate how tiresome it could be just to get the data correctly, especially when TRL calibration is involved, observe the following 4 measurements. They are all measurements of a simple transmission line using TRL calibration. Compare them and indicate which one can be considered good measurement result and why?

![Figure 2: Transmission line measurement attempts](image)

4 Equipment

- CMT S5085 VNA.
- N-type Calibration Kit 85032F.
- 3.5mm Calibration Kit 85052D.
- In-house TRL Calibration set.
- 15-foot coaxial cable.
- Typical microstrip line.
- Keysight ADS

5 Procedure

Be advised that poor measurement data will lead to failure of the extraction method, even when it is in working range. Make sure you have as good measurement results as possible before you process the data or the result will not make sense and discourage the theory you were supposed to validate. Prepare yourselves to the circumstances where you will have to perform many attempts to get the good measurement result.

1. Calibrate the network analyzer with SOLT calibration method for N-type connector for the frequency range from 300 MHz to 1 GHz with 401 frequency points. Connect your 15-foot coaxial cable and measure S parameters.

2. Calibrate the network analyzer with 3.5mm SOLT calibration. Measure the the LINE of the TRL standard.

3. Calibrate the network analyzer with TRL calibration set that the TA provides over the frequency range 300kHz to 2GHz. Connect the LINE of the TRL standard and measure S parameters. **Explain why it is said that you are collecting S-parameter of a microstrip line of length 0.75in while the LINE standard is 3.75in long?**

4. Import all of the S parameters into ADS. You have now collected S parameters for three (03) different devices:
   - 15-foot N-type coaxial cable
   - A typical microstrip line \((l = 3.75\text{in})\)
   - Another typical microstrip line \((l = 0.75\text{in})\)

5. Using the equations in the background section, calculate and plot the following for each device (the plots for each device should all be arranged neatly on one page each):
   - \(|S_{11}|\) in dB
   - \(|S_{21}|\) in dB
   - Phase of \(S_{21}\)
   - Magnitude of \(\Gamma\)
   - Phase of \(\Gamma\)
   - Magnitude of \(X\)
   - Phase of \(X\)
   - \(\alpha\), use this equation in ADS: \(\alpha = -\text{real(ln}(X)/l\)
   - \(\beta\), use this equation in ADS: \(\beta = -\text{unwrap(phaserad}(X),pi)/l\)
   - \(Z_c\)
   - \(R\)
   - \(v_p\)
6 Conclusion

1. What is the phenomenon of dispersion? How does it affect phase and group velocities? What would you conclude about the significance of dispersion from your measurement results?

2. What is skin effect? How would it affect your interconnects in your circuit? Do you observe it from your measurement in this Experiment?

3. Compare the loss per unit length of the 15-foot cable extracted in this lab with that from Experiment 3. Comment on discrepancies if any.

4. Do your extraction results match with your expectation? Do all approximation assumptions helping us to derive the extraction equations hold true according to your measurement? Support your answer by citing appropriate measurement plots.