

Experiment 7 - Physics-based Model Extraction from S-parameters

1 Introduction

With the increase in both speed and complexity of modern circuits, modeling interconnects becomes crucial for signal integrity analysis and design. This lab will introduce to you a simple method to model transmission line by extracting some of its parameters such as its characteristic impedance, phase velocity, attenuation etc.

2 Background

Because a transmission line is a linear, time-invariant (LTI) device, its behavior can be completely described by its response to complex exponentials at every possible frequency. When we operate in the real-world, we must deal with real-world signals; therefore, we measure our LTI device's response to sinusoids of varying frequency. In addition, we obviously cannot measure the response of every possible frequency due to bandwidth limitations of our measurement devices and sheer practicality. Thus, we determine an appropriate bandwidth of interest based on how our LTI device will be used in our application and a number of samples based on a compromise between measurement time and desired accuracy. So, keep in mind that the extraction method introduced here is not a universal ingredient for every cases. You will need to build up for yourself enough experience and confidence for knowing exactly when the method would not hold valid.

Returning to our goal of fully characterizing a transmission line, we now discuss how to model its frequency dependence. The parameters of interest are the resistance R , inductance L , conductance G , and capacitance C per unit length. These parameters define the propagation constant γ and the characteristic impedance Z_c as

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (1)$$

From network analyzer measurements at a frequency f , with $\omega = 2\pi f$, the scattering parameters are obtained. They can be related to the transmission line parameters from above using the relations

$$\Gamma = \frac{Z_c - Z_o}{Z_c + Z_o} = Q \pm \sqrt{Q^2 - 1} \text{ and } X = e^{-\gamma l} = \frac{(S_{11} + S_{21}) - \Gamma}{1 - (S_{11} + S_{21})\Gamma} \quad (2)$$

where

$$Q = \frac{(S_{11}^2 - S_{21}^2) + 1}{2S_{11}} \quad (3)$$

Substituting (3) in Γ of (2),

$$\Gamma \approx \frac{S_{11}}{S_{11}^2 - S_{21}^2 + 1} \quad (4)$$

Γ is the reflection coefficient, X is the propagation function, S_{11} and S_{21} are the measured reflection and transmission scattering parameters, respectively, l is the length of the transmission line and Z_o is the reference impedance of the measurement system.

X can be rewritten as

$$X = e^{-\gamma l} = e^{-j\beta l} e^{-\alpha l} \quad (5)$$

in which the propagation constant β and the attenuation constant α are separated. In most practical cases involving cables, dielectric losses are very small so that the conductance per unit length, G can be ignored; also, when the ratio $\frac{R}{\omega L}$ is very small, an approximation for the complex propagation constant gives

$$\gamma \cong \frac{R}{2} \sqrt{\frac{C}{L}} + j\omega\sqrt{LC} = \frac{R}{2Z_c} + j\frac{\omega}{v_p} = \alpha + j\beta \quad (6)$$

where v_p is the propagation velocity in the cable. The first term of γ is associated with the attenuation along the direction of propagation. The second term is associated with the phase shift and velocity. They can be related to other parameters by

$$\alpha \cong \frac{R}{2Z_c} \text{ and } \beta \cong \frac{\omega}{v_p}. \quad (7)$$

At each frequency, Γ and X are obtained from the measured scattering parameters using (2) and (4). Next, (5) is used to find γ ; (6) is used to approximate α and β ; and (7) to find Z_c , R , v_p . The results can then be combined with (1) to obtain R , L , and C .

Below is an example of transmission line parameter extraction.

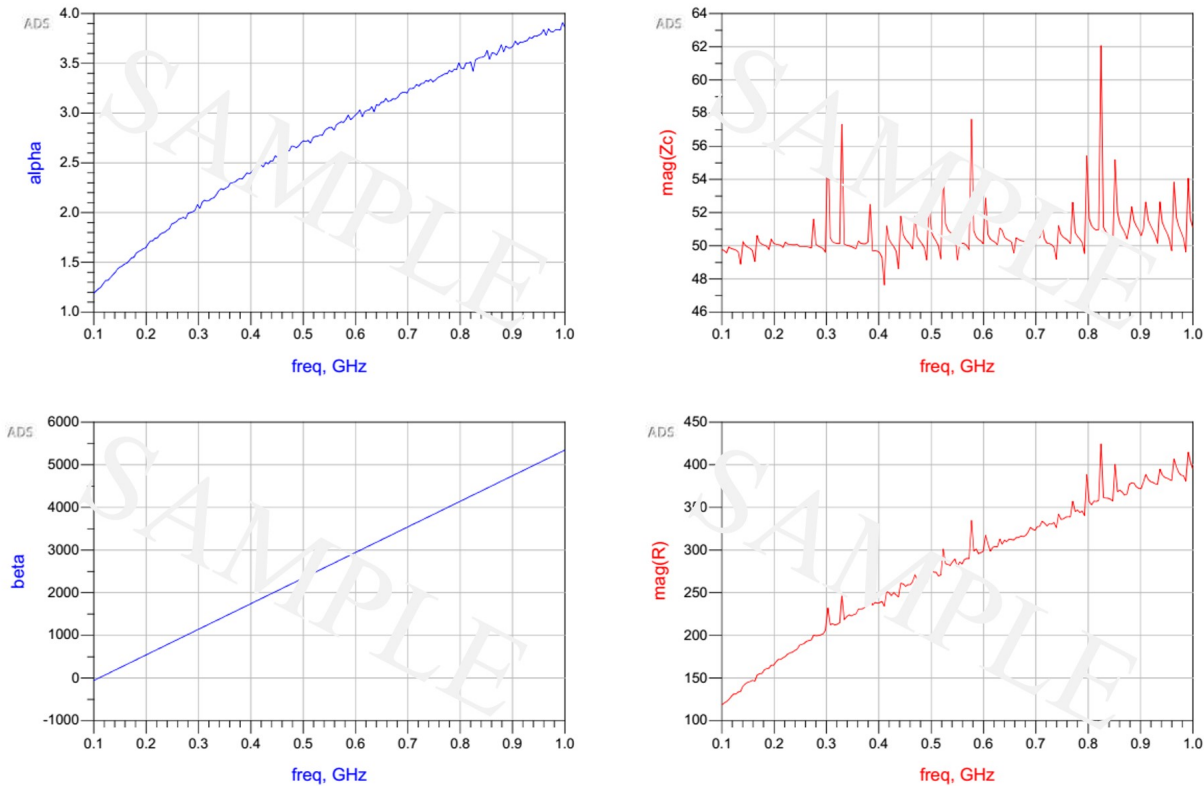


Figure 1: Transmission line extraction example: α , β , R , Z_c are shown

3 Prelab

1. At what conditions is Equation (7) accurate? State your answer in terms of ω , R , G , L , and C and then in terms of the physical meaning of those assumptions.
2. Explain the physical meaning of α and β in $X = e^{-\gamma l} = e^{-j\beta l} e^{-\alpha l}$.

- Derive $\Gamma \approx \frac{S_{11}}{S_{11}^2 - S_{21}^2 + 1}$ from Equations (2) and (3). Explicitly state what assumptions you made in your derivation? (Hint: For the + case, solve for Q in terms of first then make an approximation based on the magnitude of Γ . For the - case, use the first two terms of the Taylor Series expansion of the square root term. What assumption about Q is this truncated expansion making?)
- You are given the Y parameters for the three sub-circuits (Y_1 , Y_2 , Y_3) shown in Figure 2, calculate Y_{11} , Y_{21} of the 2 port system in terms of Y_1 , Y_2 , Y_3 .

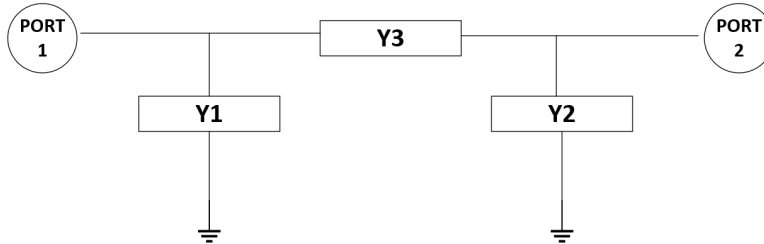


Figure 2: 2 port system for pre-lab

4 Equipment

- Advanced Design System (ADS)
- Jupyter notebook
- S-A-A-2 NanoVNA
- 3.5 mm calibration kit
- Student board: TRL set (operating frequency range 100MHz to 1GHz), DUT 6
- 50Ω coaxial cable (blue cable included within VNA box)

5 Procedure

Part I - Extraction of Transmission Line Parameters

Be advised that poor measurement data will lead to failure of the extraction method, even when it is in working range. Make sure you have as good measurement results as possible before you process the data or the result will not make sense and discourage the theory you were supposed to validate. Prepare yourselves to the circumstances where you will have to perform many attempts to get the good measurement result.

1. Calibrate the network analyzer with SOLT calibration over the frequency range 100MHz to 1GHz.
2. Measure the TRL set and DUT 6. Use your JupyterLab's TRL code to perform TRL calibration on DUT 6.
3. Import your post-processed DUT 6's s-parameter to ADS. Using the equations in the background section, calculate and plot the following (make sure the screenshots you take are legible):
 - $|S_{11}|$ in dB
 - $|S_{21}|$ in dB
 - Phase of S_{21}
 - Magnitude of Γ
 - Phase of Γ
 - Magnitude of X

- Phase of X
 - α , use this equation in ADS: $\alpha = -\text{real}(\ln(X))/l$
 - β , use this equation in ADS: $\beta = -\text{unwrap}(\text{phaserad}(X), \pi)/l$
 - Z_c
 - R
 - v_p
4. Calibrate and measure the 50Ω coaxial cable with SOLT (NOT TRL) from 100MHz to 1GHz. Calculate and plot the items listed in the previous step. Overlay your results with the ones from DUT 6. In the report, explain **in detail** why or why not the two results matches with each other.

Part II - Extraction of Model: Inductor & Capacitor

Using measured data that was obtained from probe station measurements, students must build proper models and extract necessary parameters in the models. You will start with .s2p Touchstone files containing the measurement data and with a suggested model for the inductor and for the capacitor and then will find the appropriate parameters to simulate a response of that model that will match the measured data with high accuracy. This model could then be used to help with later designs or other simulations.

The DUTs (Device Under Test) are an inductor and a capacitor on a GaAs substrate. To accurately simulate the response of our inductor under test, we use the model shown in Figure 3. C_2 and C_3 represent the capacitances occurred at the ports, and C_1 represents the coupling capacitance between the spiral inductor lines. R is corresponding to the loss. Larger R means higher loss in the substrate. L_1 is, of course, the inductance of the inductor. It is easy to represent this model using “PI” network and $[Y]$ matrix, as shown in Figure 2 from Pre-lab. Using Y_1 and Y_2 , you can extract parameter C_2 and C_3 . Y_3 contains R , L_1 and C_1 's information. At low frequency, C_1 provide an open branch. If $Z_3 = 1/Y_3$, the real part of Z_3 at low frequency will be quite close to R while the imaginary part of Z_3 will provide us L_1 . From the curves of Y_3 , we can find a resonance frequency point that is corresponding to the resonance frequency of the paralleled LC circuits. Using that point and L_1 we got before, we can get C_1 .

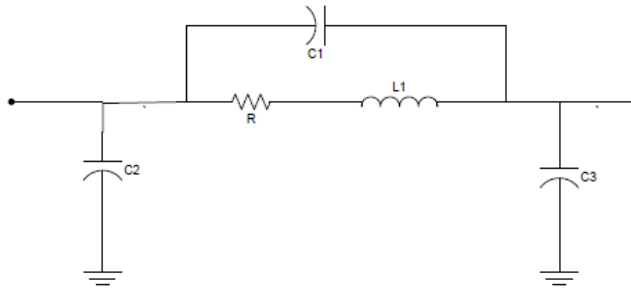


Figure 3: Suggested Inductor Model

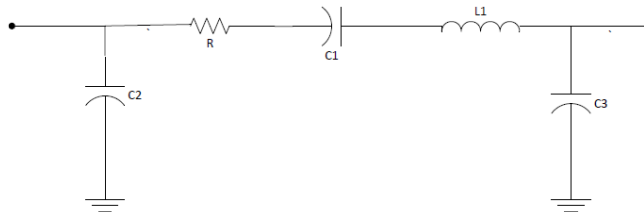


Figure 4: Suggested Capacitor Model

For a capacitor under test, a similar model (Figure 4) can be used. But the Y_3 branch now is a serial circuit. Using the same model extraction concepts, we can extract all the parameters needed for this “PI” network. Note

that we only can measure [S] parameters using the network analyzer. To get [Y], we have to change [S] to [Y] using standard transformation. These formulas are provided in Equation set (8).

$$\begin{aligned}
 Y_{Y11} &= 1/50 * ((1 - S_{11}) * (1 + S_{22}) + S_{12} * S_{21}) / ((1 + S_{11}) * (1 + S_{22}) - S_{12} * S_{21}) \\
 Y_{Y12} &= -1/50 * 2 * S_{12} / ((1 + S_{11}) * (1 + S_{22}) - S_{12} * S_{21}) \\
 Y_{Y21} &= -1/50 * 2 * S_{21} / ((1 + S_{11}) * (1 + S_{22}) - S_{12} * S_{21}) \\
 Y_{Y22} &= 1/50 * ((1 + S_{11}) * (1 - S_{22}) + S_{12} * S_{21}) / ((1 + S_{11}) * (1 + S_{22}) - S_{12} * S_{21})
 \end{aligned}
 \tag{8}$$

1. Using the given inductor s2p file (download link [HERE](#)), build the model and extract the parameters of a PI network. Please demonstrate all necessary steps in the plots and print them in one page. You might need to tune the calculated values in order to match with the original measurement data.
2. Using the given capacitor s2p file download link [HERE](#)) to build the model and extract the parameters of a PI network. Please demonstrate all necessary steps in the plots and print them in one page. Record your calculated values and tuned values for the report.
3. Required plots/tables for this part:
 - Plots of Real [Y₁], Imag [Y₁], Real [Y₂], Imag [Y₂], Real [Y₃], Imag [Y₃] vs. frequency with appropriate markers to show how you calculated C₁, C₂, C₃, L₁, and R.
 - Table of values of C₁, C₂, C₃, L₁, and R for both the inductor and the capacitor.
 - Smith Charts of S₁₁ and S₂₁ (overlay your tuned simulated result with our measurement data).

6 Conclusion

1. What is the phenomenon of dispersion? How does it affect phase and group velocities? What would you conclude about the significance of dispersion from your measurement results?
2. What is skin effect? How would it affect your interconnects in your circuit? Do you observe it from your measurement in this Experiment?
3. Do your extraction results match with your expectation? Do all approximation assumptions helping us to derive the extraction equations hold true according to your measurement? Support your answer by citing appropriate measurement plots.