8-Term Error Model Derivation

The measuring system considered here is shown below. The entire system is represented using signal flow graph notation with the unknown two-port inserted between the two measuring ports. (See Figure 1.) The signals shown without parenthesis apply to the case where the stimulus signal is introduced from the left port, while the signals in parenthesis apply when the stimulus is introduced from the right ports.

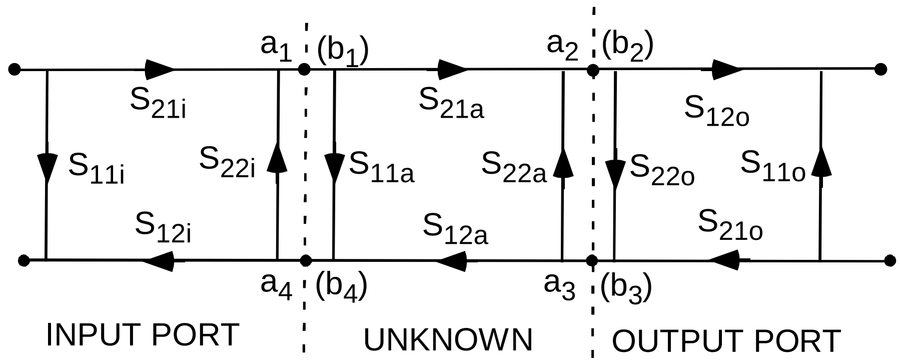

Figure 1. Signal Flow Graph of S-Parameters in Two-Port Device

Calibration of the test set consists of measuring the reflected signals at both ports with open, short, and matched terminations and measuring the transmitted signal in both directions with the ports connected together. These measurements are then used to determine the S-terms either explicitly or in combinations. Open terminations are modeled with a small fringing reactance. Its reflection coefficient is equal to:

$$\Gamma = 1e^{-j\beta} = \cos\beta - j\sin\beta \qquad (1)$$

$$\beta = 2\tan^{-1}\left(\frac{\pi fCZ_o}{5\times10^5}\right)$$

with frequency (*f*) in MHz and capacitance (*C*) in picofarads, and system characteristic impedance (Z_o) in ohms. Considering the input port, the open (*op*), short (*sh*) and matched load *(ld)* terminations result in the following measured values respectively:

$$S_{11m}^{(op)} = S_{11i} + \frac{S_{21i}S_{12i}}{e^{j\beta} - S_{22i}}$$

$$S_{11m}^{(sh)} = S_{11i} - \frac{S_{21i}S_{12i}}{1 + S_{22i}} \qquad (2)$$

$$S_{11m}^{(ld)} = S_{11i}$$

Solving (2) simultaneously, the following results are obtained:

$$S_{11i} = S_{11m}^{(ld)}$$

$$S_{22i} = \frac{e^{j\beta}\left[S_{11m}^{(op)} - S_{11m}^{(ld)}\right] - \left[S_{11m}^{(ld)} - S_{11m}^{(sh)}\right]}{S_{11m}^{(op)} - S_{11m}^{(sh)}} \qquad (3a)$$

$$S_{12i}S_{21i} = \frac{\left(1+e^{j\beta}\right)\left[S_{11m}^{(op)} - S_{11m}^{(ld)}\right]\left[S_{11m}^{(sh)} - S_{11m}^{(ld)}\right]}{S_{11m}^{(sh)} - S_{11m}^{(op)}}$$

Using the same technique, the corresponding relationships for the output port can be obtained.

$$S_{11o} = S_{22m}^{(ld)}$$

$$S_{22o} = \frac{e^{j\beta}\left[S_{22m}^{(op)} - S_{22m}^{(ld)}\right] - \left[S_{22m}^{(ld)} - S_{22m}^{(sh)}\right]}{S_{22m}^{(op)} - S_{22m}^{(sh)}} \qquad (3b)$$

$$S_{12o}S_{21o} = \frac{\left(1+e^{j\beta}\right)\left[S_{22m}^{(op)} - S_{22m}^{(ld)}\right]\left[S_{22m}^{(sh)} - S_{22m}^{(ld)}\right]}{S_{22m}^{(sh)} - S_{22m}^{(op)}}$$

With the measuring ports connected together, the transmitted signals are:

$$S_{21m}^{(thr)} = \frac{S_{21i}S_{12o}}{1 - S_{22i}S_{22o}} \qquad (4)$$

$$S_{12m}^{(thr)} = \frac{S_{12i}S_{21o}}{1 - S_{22i}S_{22o}} \qquad (5)$$

Using these relationships and the values of the S_{22}'s obtained in (3), the products $S_{21i}S_{12o}$ and $S_{12i}S_{21o}$ can be calculated. In these derivations, it is assumed that sufficient isolation is present between the test ports and switches of the network analyzer test set so that the S-terms remain constant for the entire measuring procedure.

With the unknown two-port inserted as shown in Figure 1 and a unit reference signal introduced from the input the following relationships apply:

$$a_1 = S_{21i} + S_{22i}a_4$$

$$a_3 = S_{22o}a_2 \qquad (6)$$

$$S_{11m} = S_{11i} + S_{12i}a_4$$

$$S_{21m} = S_{12o}a_2$$

$$a_4 = S_{11a}a_1 + S_{12a}a_3 \tag{7}$$

$$a_2 = S_{21a}a_1 + S_{22a}a_3$$

Equations (6) can be solved simultaneously for the *a*'s. The results are:

$$a_1 = S_{21i} + S_{22i}\left[\frac{S_{11m} - S_{11i}}{S_{21i}}\right]$$

$$a_2 = \frac{S_{21m}}{S_{12o}}$$

$$a_3 = \frac{S_{22o}S_{21m}}{S_{12o}} \tag{8}$$

$$a_4 = \frac{S_{11m} - S_{11i}}{S_{12i}}$$

Following the same procedure, with the unit reference signal introduced from the right, the following relationships are obtained:

$$b_1 = S_{22i}b_4$$

$$b_3 = S_{22o}b_2 + S_{12o} \tag{9}$$

$$S_{22m} = S_{11o} + S_{12o}b_2$$

$$S_{12m} = S_{12i}b_4$$

$$b_4 = S_{11a}b_1 + S_{12a}b_3$$

$$b_2 = S_{21a}b_1 + S_{22a}b_3$$

where

$$b_1 = \frac{S_{22i}S_{12m}}{S_{12i}} \tag{10}$$

$$b_2 = \frac{S_{22m} - S_{11o}}{S_{12o}}$$

$$b_3 = S_{22o}\left[\frac{S_{22m} - S_{11o}}{S_{12o}}\right] + S_{21o}$$

$$b_4 = \frac{S_{21m}}{S_{12i}}$$

Equations (7) and (9) constitute four equations with four unknowns, namely the desired two-port scattering parameters. These four equations break into two pairs with two unknowns in each and consequently can be easily solved for the unknown scattering parameters. By substituting the values of the *a*'s and *b*'s, the following results are obtained:

$$S_{11a} = \frac{\frac{S_{11m} - S_{11i}}{S_{21i}S_{12i}}\left[1 + \frac{S_{22o}\left(S_{22m} - S_{11o}\right)}{S_{12o}S_{21o}}\right] - \frac{S_{22o}S_{21m}}{S_{21i}S_{12o}} \times \frac{S_{12m}}{S_{21o}S_{12i}}}{D} \qquad (11)$$

$$S_{12a} = \frac{\frac{S_{12m}}{S_{21o}S_{12i}}}{D}$$

$$S_{21a} = \frac{\frac{S_{21m}}{S_{21i}S_{12o}}}{D}$$

$$S_{22a} = \frac{\frac{S_{22m} - S_{11o}}{S_{21o}S_{12o}}\left[1 + \frac{S_{22i}\left(S_{11m} - S_{11i}\right)}{S_{21i}S_{12i}}\right] - \frac{S_{22i}S_{21m}}{S_{21i}S_{12o}} \times \frac{S_{12m}}{S_{21o}S_{12i}}}{D}$$

where

$$D = \left[1 + \frac{S_{22i}\left(S_{11m} - S_{11i}\right)}{S_{21i}S_{12i}}\right]\left[1 + \frac{S_{22o}\left(S_{22m} - S_{11o}\right)}{S_{12o}S_{21o}}\right] - S_{22i}S_{22o}\frac{S_{21m}}{S_{21i}S_{12o}}\frac{S_{12m}}{S_{21o}S_{12i}}$$

Measurement Equation Solved for One-Port devices:

$$S_{11a} = \frac{S_{11m} - S_{11i}}{S_{22i}\left(S_{11m} - S_{11i}\right) + S_{12i}S_{21i}}$$

[1]. W. Kruppa and K. F. Sodomsky, “An Explicit Solution for the Scattering Parameters of a Linear Two-Port Measured with an Imperfect Test Set,**”** IEEE Transactions on Microwave Theory and Techniques, vol. 19, pp. 122-123, January 1971.