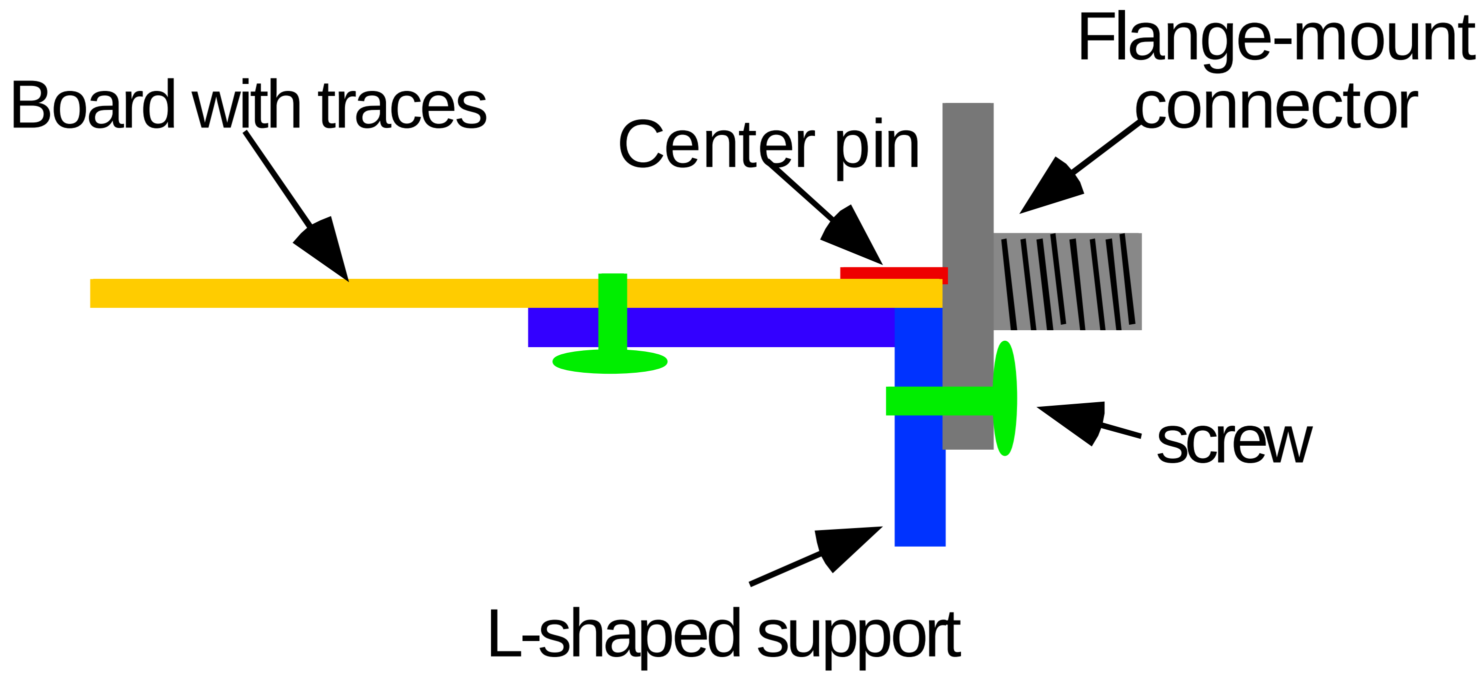
ECE 451
Automated Microwave Measurements

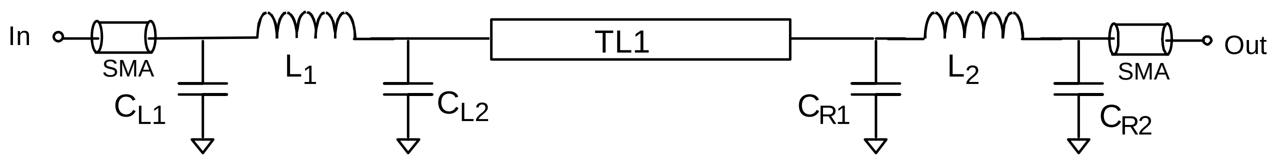
TRL Calibration

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jose@emlab.uiuc.edu

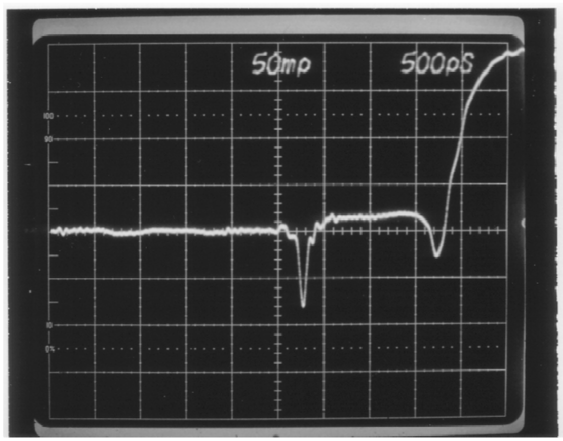
Coaxial-Microstrip Transition



Coaxial-Microstrip Transition



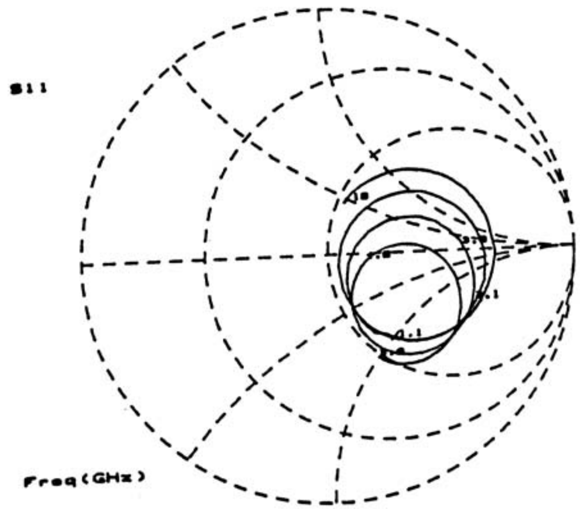
Equivalent Circuit

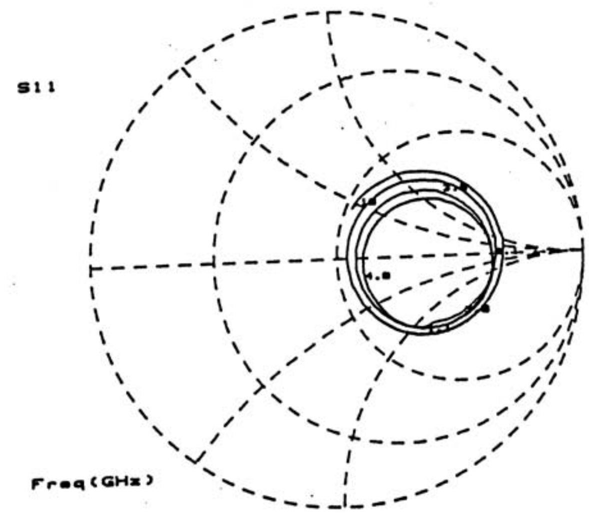


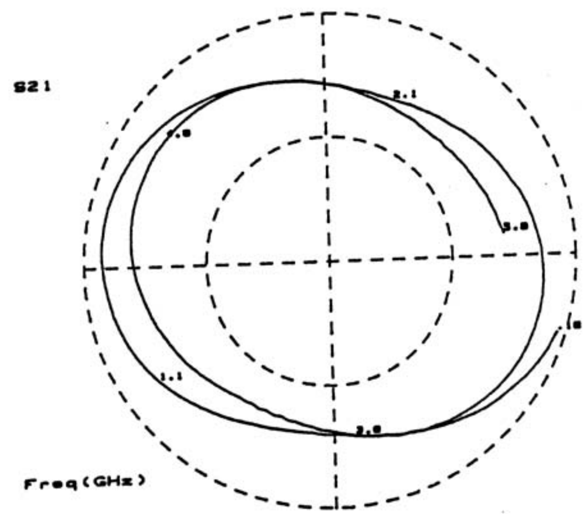
TDR Plot

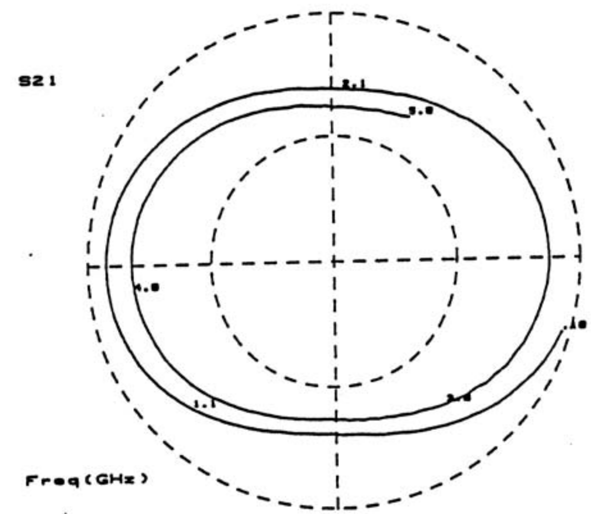
With parasitics

No parasitics

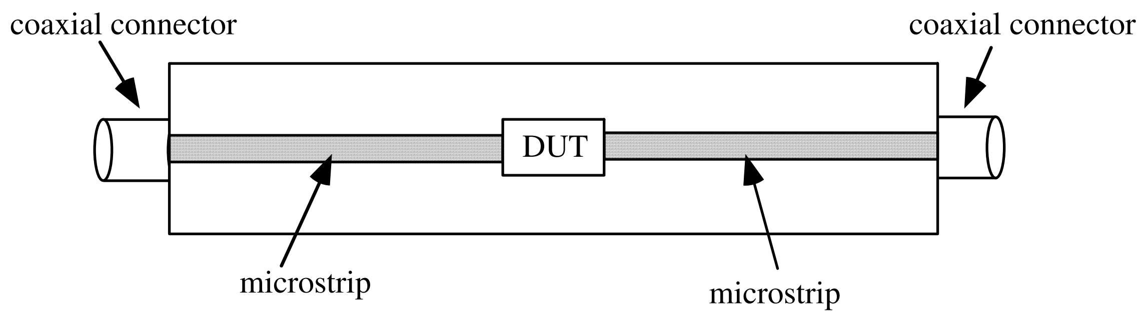








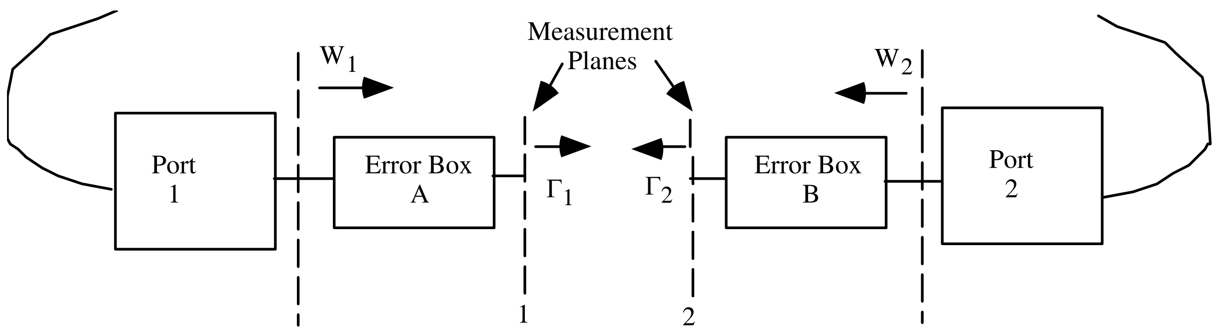
TRL CALIBRATION SCHEME



Want to measure DUT only and need to remove the effect of coax-to-microstrip transitions. Use TRL calibration

TRL Error Box Modeling

A model for the different error boxes can be implemented

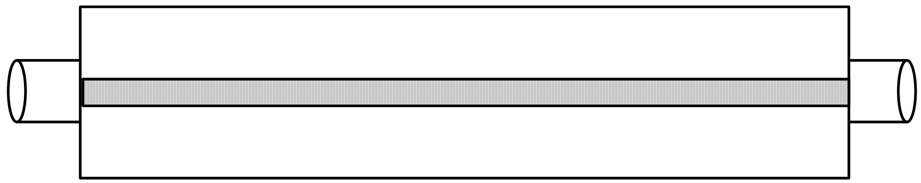


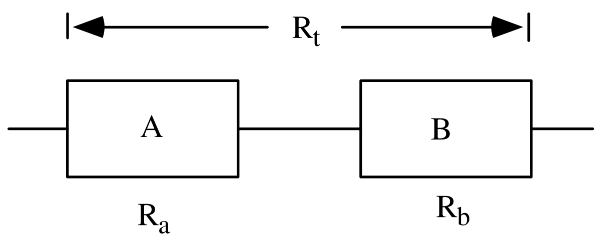
Error boxes A and B account for the transition parasitics and the electrical lengths of the microstrip.

Make three standards: Thru, Line and Reflect

Step 1 - THRU Calibration

connect thru

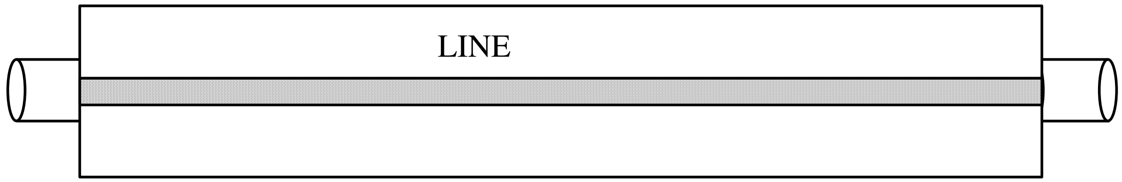


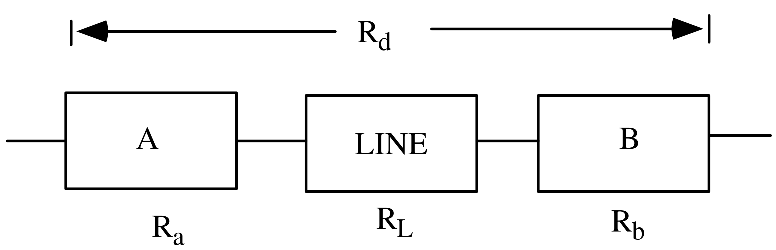


$$R_t = R_a R_b$$

Step 2 - LINE Calibration

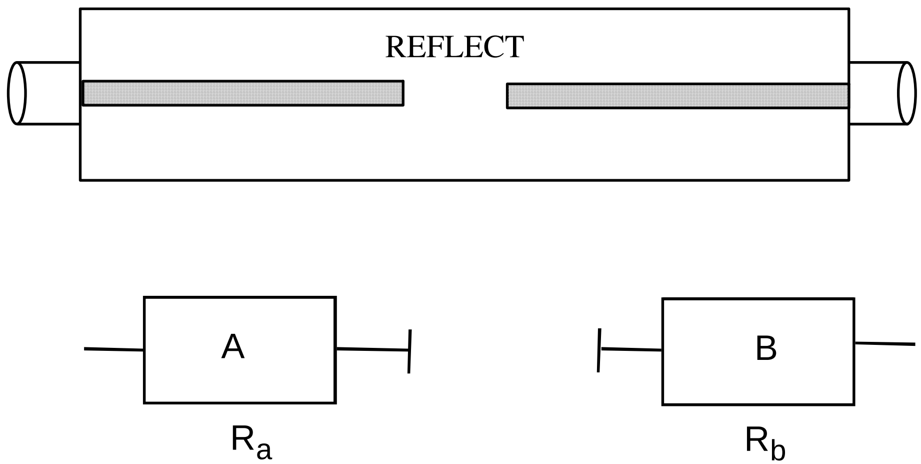
connect line (Note: difference in length between thru and line)



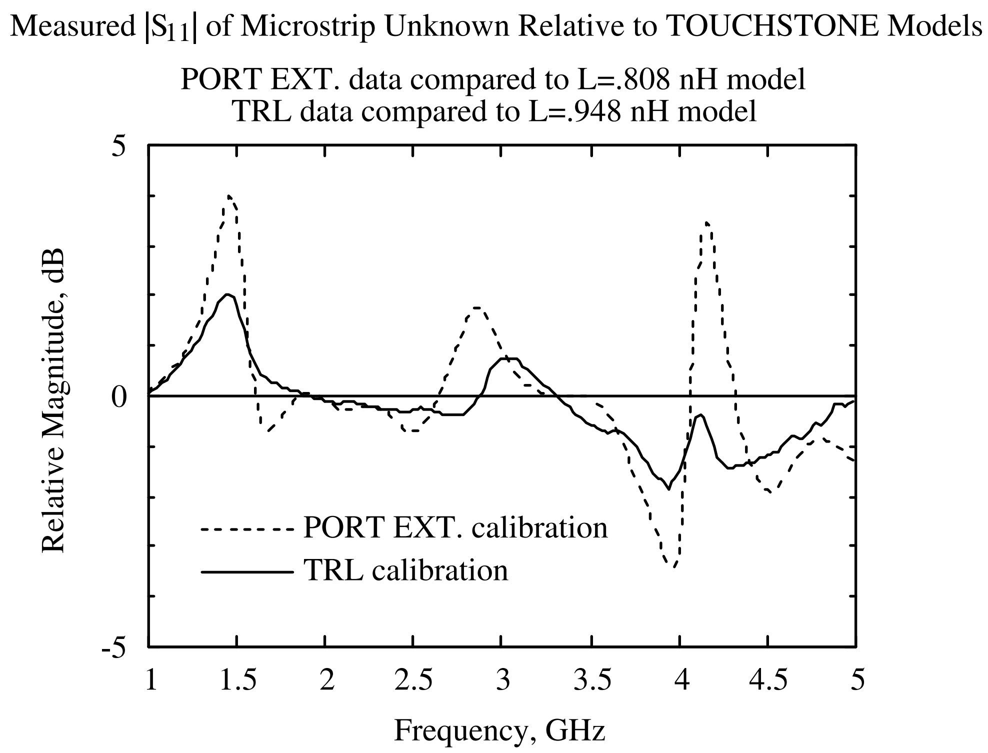


Step 3 - REFLECT Calibration

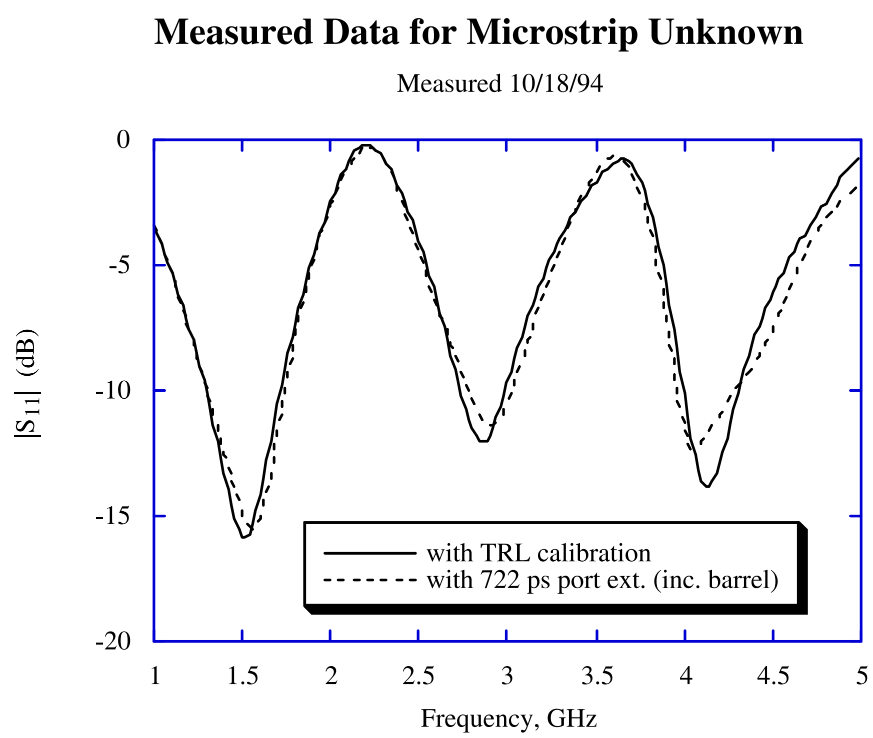
connect reflect



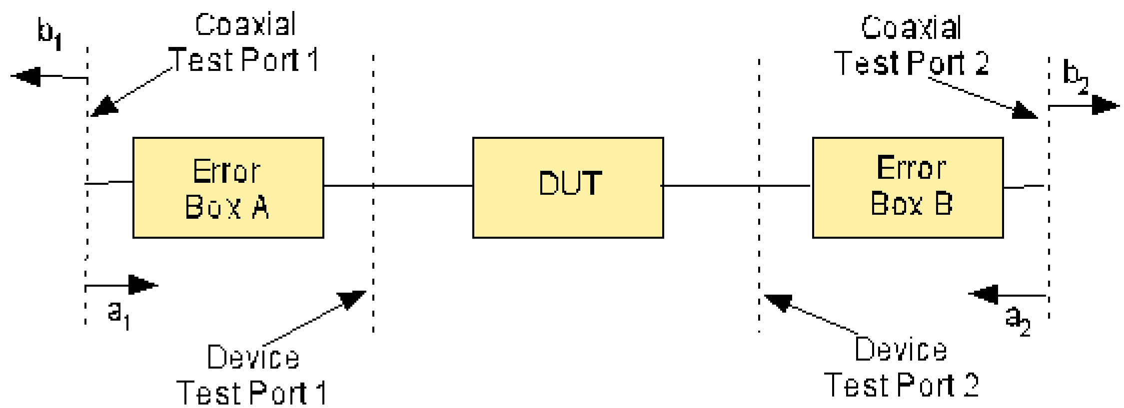
TRL – Measurement Comparison



TRL – Measurement Comparison



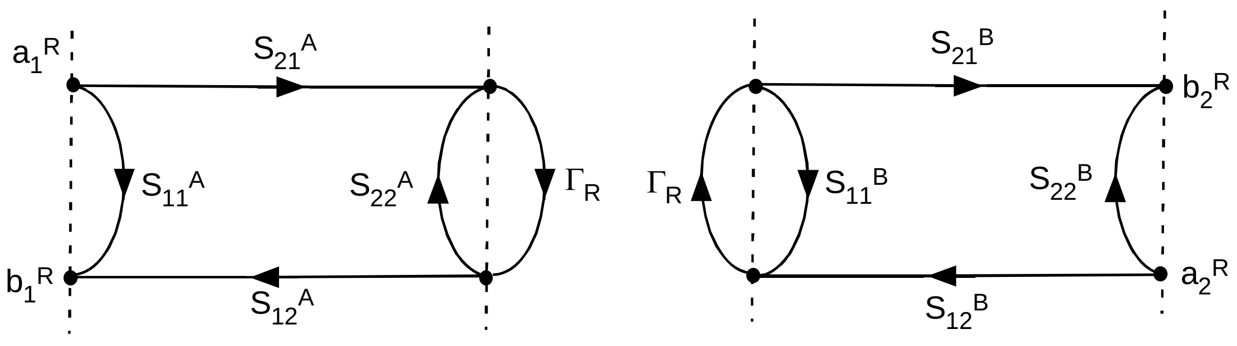
TRL Derivation



TRL Objectives

- **Obtain network parameters of error boxes A and B**
- **Remove their effects in subsequent measurements**

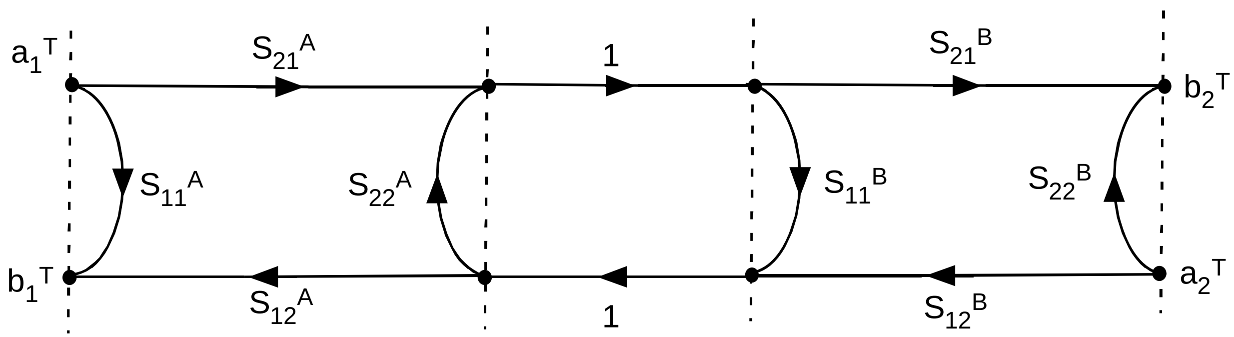
Model for Reflect



$$\left.\frac{b_1^R}{a_1^R}\right|_{a_2^R=0} \qquad \left.\frac{b_2^R}{a_2^R}\right|_{a_2^R=0}$$

2 Measurements

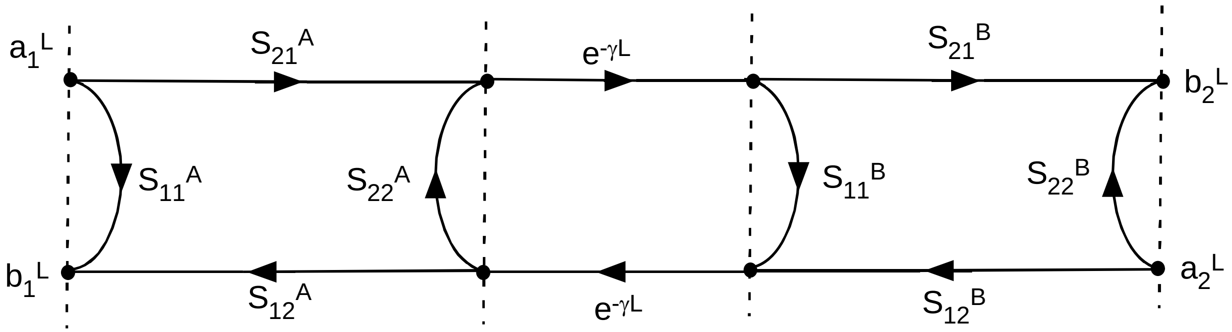
Model for Thru



$$\left.\frac{b_1^T}{a_1^T}\right|_{a_2^T=0} \qquad \left.\frac{b_2^T}{a_1^T}\right|_{a_2^T=0} \qquad \left.\frac{b_2^T}{a_2^T}\right|_{a_1^T=0} \qquad \left.\frac{b_1^T}{a_2^T}\right|_{a_1^T=0}$$

4 Measurements

Model for Line



$$\left.\frac{b_1^L}{a_1^R}\right|_{a_2^L=0} \qquad \left.\frac{b_2^L}{a_1^L}\right|_{a_2^L=0} \qquad \left.\frac{b_2^L}{a_1^L}\right|_{a_1^L=0} \qquad \left.\frac{b_1^L}{a_2^L}\right|_{a_1^L=0}$$

4 Measurements

Use R (or T) Parameters

Using T parameters (transfer parameters), we can show that if

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \frac{1}{S_{21}} \begin{pmatrix} -\Delta & S_{11} \\ -S_{22} & 1 \end{pmatrix} \begin{pmatrix} b_2 \\ a_2 \end{pmatrix}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

$$R = \frac{1}{S_{21}} \begin{pmatrix} -\Delta & S_{11} \\ -S_{22} & 1 \end{pmatrix}$$

TRL Derivation

The measurement matrix R_M is just the product of the matrices of the error boxes and the unknown DUT

$$R_M = R_A R R_B$$

or

$$R = R_A^{-1} R_M R_B^{-1}$$

Let R_A be written as

$$R_A = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = r_{22} \begin{bmatrix} a & b \\ c & 1 \end{bmatrix}$$

R_B is similarly written as

$$R_B = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} = \rho_{22} \begin{bmatrix} \alpha & \beta \\ \gamma & 1 \end{bmatrix}$$

The inverse of R_A is

$$R_A^{-1} = \frac{1}{r_{22}} \frac{1}{a - bc} \begin{bmatrix} 1 & -b \\ -c & a \end{bmatrix}$$

TRL Derivation

And the inverse of R_B is

$$R_B^{-1} = \frac{1}{\rho_{22}} \frac{1}{\alpha - \beta\gamma} \begin{bmatrix} 1 & -\beta \\ -\gamma & \alpha \end{bmatrix}$$

The matrix of the DUT is then found from

$$R = \frac{1}{r_{22}\rho_{22}} \frac{1}{a\alpha} \frac{1}{1 - b\frac{c}{a}} \frac{1}{1 - \gamma\frac{\beta}{\alpha}} \begin{bmatrix} 1 & -b \\ -c & a \end{bmatrix} R_M \begin{bmatrix} 1 & -\beta \\ -\gamma & \alpha \end{bmatrix}$$

Note that although there are eight terms in the error boxes, only seven quantities are needed to find R. They are $a, b, c, \alpha, \beta, \gamma$, and $r_{22}\rho_{22}$

From the measurement of the through and of the line, seven quantities will be found. They are b, c/a, β/α, γ, $r_{22}\rho_{22}$, αa and $e^{2\gamma l}$

In addition to the seven quantities, if a were found, the solution would be complete. Let us first find the above seven quantities.

The ideal through has an R matrix which is the 2 x 2 unit matrix. The measured R matrix with the through connected will be denoted by R_T and is given by

$$R_T = R_A R_B$$

Where R_A and R_B are the R matrices of the error box A and B respectively. With the line connected, the measured R matrix will be denoted by R_D and is equal to

TRL Derivation

$$R_D = R_A R_L R_B$$

where R_L is the R matrix of the line

Now $R_B = R_A^{-1} R_T$

so that $R_D = R_A R_L R_A^{-1} R_T$

$$R_D R_T^{-1} R_A = R_A R_L$$

Define $T = R_D R_T^{-1}$ Which when substituted into the above equations results in

$$T R_A = R_A R_L$$

The matrix T is known from measurements and will be written as

$$T = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$$

$$R_L = \begin{bmatrix} e^{-\gamma l} & 0 \\ 0 & e^{+\gamma l} \end{bmatrix}$$, since the line is non-reflecting

TRL Derivation

R_A is unknown and was written as

$$R_A = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = r_{22} \begin{bmatrix} a & b \\ c & 1 \end{bmatrix}$$

R_B similarly was written as

$$R_B = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} = \rho_{22} \begin{bmatrix} \alpha & \beta \\ \gamma & 1 \end{bmatrix}$$

Recalling $TR_A = R_A R_L$ and writing the matrices results in

$$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} a & b \\ c & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & 1 \end{bmatrix} \begin{bmatrix} e^{-\gamma l} & 0 \\ 0 & e^{+\gamma l} \end{bmatrix}$$

Next, writing out the four equations gives:

TRL Derivation

$$t_{11}a + t_{12}c = ae^{-\gamma l}$$

$$t_{21}a + t_{22}c = ce^{-\gamma l}$$

$$t_{11}b + t_{12} = be^{+\gamma l}$$

$$t_{21}b + t_{22} = e^{+\gamma l}$$

Dividing the first of the above equation by the second results in

$$\frac{t_{11}a + t_{12}c}{t_{21}a + t_{22}c} = \frac{a}{c} = \frac{t_{11}\frac{a}{c} + t_{12}}{t_{21}\frac{a}{c} + t_{22}}$$

which gives a quadratic equation for *a/c*

$$t_{21}\left(\frac{a}{c}\right)^2 + \left(t_{22} - t_{11}\right)\frac{a}{c} - t_{12} = 0$$

Dividing the third equation in the group by the fourth results in

TRL Derivation

$$\frac{t_{11}b+t_{12}}{t_{21}b+t_{22}}=b$$ which gives the analogous quadratic equation for b as

$$t_{21}b^2+\left(t_{22}-t_{11}\right)b-t_{12}=0$$

Dividing the fourth equation in the group by the second results in

$$e^{2\gamma L}=c\frac{t_{21}b+t_{22}}{t_{21}a+t_{22}c}=\frac{t_{21}b+t_{22}}{t_{21}\frac{a}{c}+t_{22}}$$

Since $e^{2\gamma L}$ is not equal to 1, *b* and *c/a* are distinct roots of the quadratic equation. The following discussion will enable the choice of the root. Now $b=r_{12}/r_{22}=S_{11}$ and

$$\frac{a}{c}=\frac{r_{11}}{r_{21}}=S_{11}-\frac{S_{12}S_{21}}{S_{22}}$$

TRL Derivation

For a well designed transition between coax and the non-coax $|S_{22}|$, $|S_{11}| \ll 1$ which yields $|b| \ll 1$ and $|a/c| \gg 1$. Therefore,

$$|b| \ll \left|\frac{a}{c}\right|$$ which determines the choice of the root

Recalling $$TR_A = R_A R_L$$

$$(det\ T)(det\ R_A) = (det\ R_A)(det\ R_L)$$

or

$$(det\ T) = (det\ R_L) = 1$$

so that

$$t_{11}t_{22} - t_{12}t_{21} = 1$$

which implies that there are only three independent T_{ij}. Then there are only three independent results, e.g. b, a/c, and $e^{2\gamma L}$.

TRL Derivation

Now let us find four more quantities

$$r_{22}\rho_{22}\begin{bmatrix} a & b \\ c & 1 \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & 1 \end{bmatrix} = R_A R_B = R_T = g\begin{bmatrix} d & e \\ f & 1 \end{bmatrix}$$

Now

$$\begin{bmatrix} a & b \\ c & 1 \end{bmatrix}^{-1} = \frac{1}{a-bc}\begin{bmatrix} 1 & -b \\ -c & a \end{bmatrix}$$

So that

$$r_{22}\rho_{22}\begin{bmatrix} \alpha & \beta \\ \gamma & 1 \end{bmatrix} = \frac{g}{a-bc}\begin{bmatrix} 1 & -b \\ -c & a \end{bmatrix}\begin{bmatrix} d & e \\ f & 1 \end{bmatrix}$$

or

$$r_{22}\rho_{22}\begin{bmatrix} \alpha & \beta \\ \gamma & 1 \end{bmatrix} = \frac{g}{a-bc}\begin{bmatrix} d-bf & e-b \\ af-cd & a-ce \end{bmatrix}$$

TRL Derivation

from which we can extract

$$r_{22}\rho_{22} = g\frac{a-ce}{a-bc} = g\frac{1-e\frac{c}{a}}{1-b\frac{c}{a}}$$

We also have

$$\begin{bmatrix} \alpha & \beta \\ \gamma & 1 \end{bmatrix} = \frac{1}{a-ce}\begin{bmatrix} d-bf & e-b \\ af-cd & a-ce \end{bmatrix}$$

from which we obtain

$$\gamma = \frac{f-\frac{c}{a}d}{1-\frac{c}{a}e}$$

and

$$\frac{\beta}{\alpha} = \frac{e-b}{d-bf}$$

TRL Derivation

and

$$\alpha a = \frac{d - bf}{1 - \frac{c}{a}e}$$

The additional four quantities found are β/α, γ, $r_{22}\rho_{22}$ and αa. To complete the solution, one needs to find *a*. Let the reflection measurement through error box A be w_1. Then

$$w_1 = \frac{a\Gamma_R + b}{c\Gamma_R + 1}$$

which may be solved for *a* in terms of the known *b* and *a/c* as

$$a = \frac{w_1 - b}{\Gamma_R\left(1 - w_1\frac{c}{a}\right)}$$

We need a method to determine *a*. Use the measurement for the reflect from through the error box B. Let w_2 denote the measurement

$$w_2 = S_{22} + \frac{S_{12}S_{21}\Gamma_R}{1 - S_{11}\Gamma_R} = \frac{S_{22} - \Delta\Gamma_R}{1 - S_{11}\Gamma_R}$$

TRL Derivation

$$w_2 = \frac{-\frac{\rho_{21}}{\rho_{22}} + \frac{\rho_{11}}{\rho_{22}}\Gamma_R}{1 - \frac{\rho_{12}}{\rho_{22}}\Gamma_R}$$

or

$$w_2 = -\frac{\alpha\Gamma_R - \gamma}{\beta\Gamma_R - 1}$$

α may be found in terms of γ and β/α as

$$\alpha = \frac{w_2 + \gamma}{\Gamma_R\left(1 + w_2\frac{\beta}{\alpha}\right)}$$

Recall $$a = \frac{w_1 - b}{\Gamma_R\left(1 - w_1\frac{c}{a}\right)}$$

TRL Derivation

so that

$$\frac{a}{\alpha}=\frac{w_1-b}{w_2+\gamma}\frac{1+w_2\dfrac{\beta}{\alpha}}{1-w_1\dfrac{c}{a}}$$

From earlier $\alpha a=\dfrac{d-bf}{1-\dfrac{c}{a}e}$

so that

$$a^2=\frac{w_1-b}{w_2+\gamma}\frac{1+w_2\dfrac{\beta}{\alpha}}{1-w_1\dfrac{c}{a}}\frac{d-bf}{1-\dfrac{c}{a}e}$$

or

$$a=\pm\left(\frac{w_1-b}{w_2+\gamma}\frac{1+w_2\dfrac{\beta}{\alpha}}{1-w_1\dfrac{c}{a}}\frac{d-bf}{1-\dfrac{c}{a}e}\right)^{\frac{1}{2}}$$

which determines *a* to within a ± sign.

TRL Derivation

$$\Gamma_R = \frac{w_1 - b}{a\left(1 - w_1 \dfrac{c}{a}\right)}$$

So if Γ_R is known to within ± then *a* may be determined as well. Calibration is complete and we can now proceed to the measurement of the DUT.

From earlier, the matrix of the DUT is found from

$$R = \frac{1}{r_{22}\rho_{22}} \frac{1}{a\alpha} \frac{1}{1 - b\dfrac{c}{a}} \frac{1}{1 - \gamma\dfrac{\beta}{\alpha}} \begin{bmatrix} 1 & -b \\ -c & a \end{bmatrix} R_M \begin{bmatrix} 1 & -\beta \\ -\gamma & \alpha \end{bmatrix}$$

in which all the terms have now been determined.