ECE 451
Black Box Modeling

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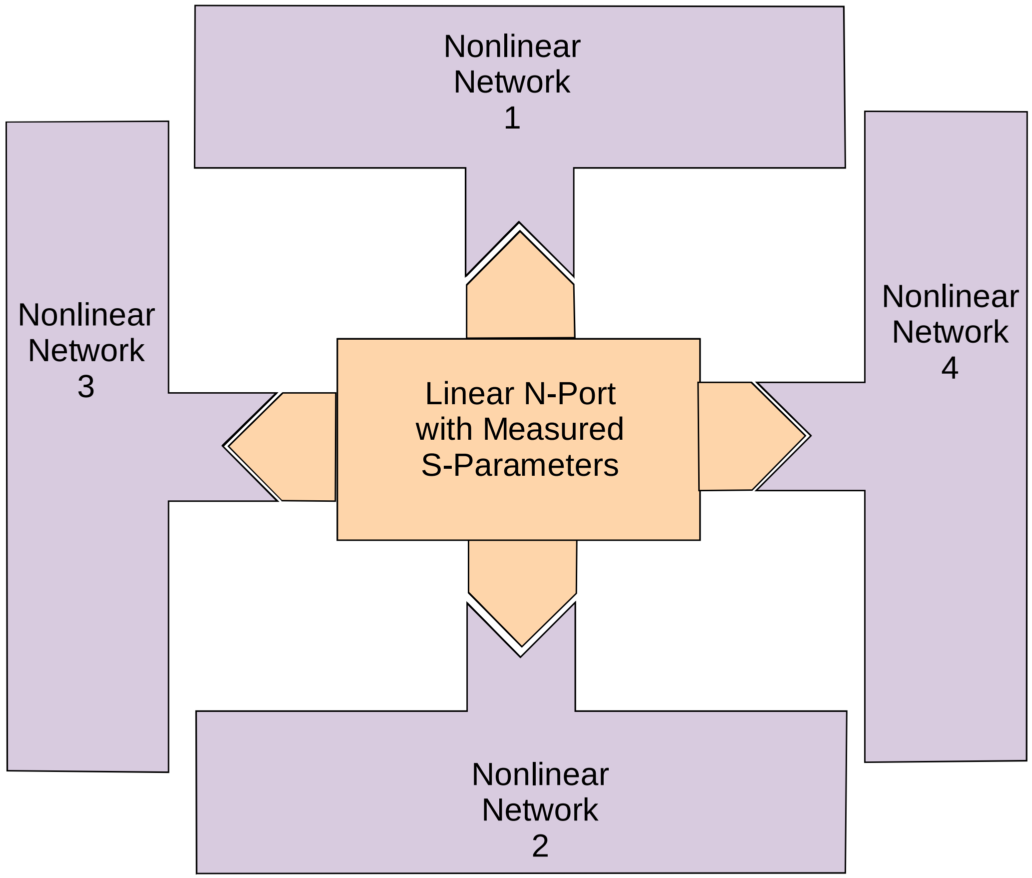
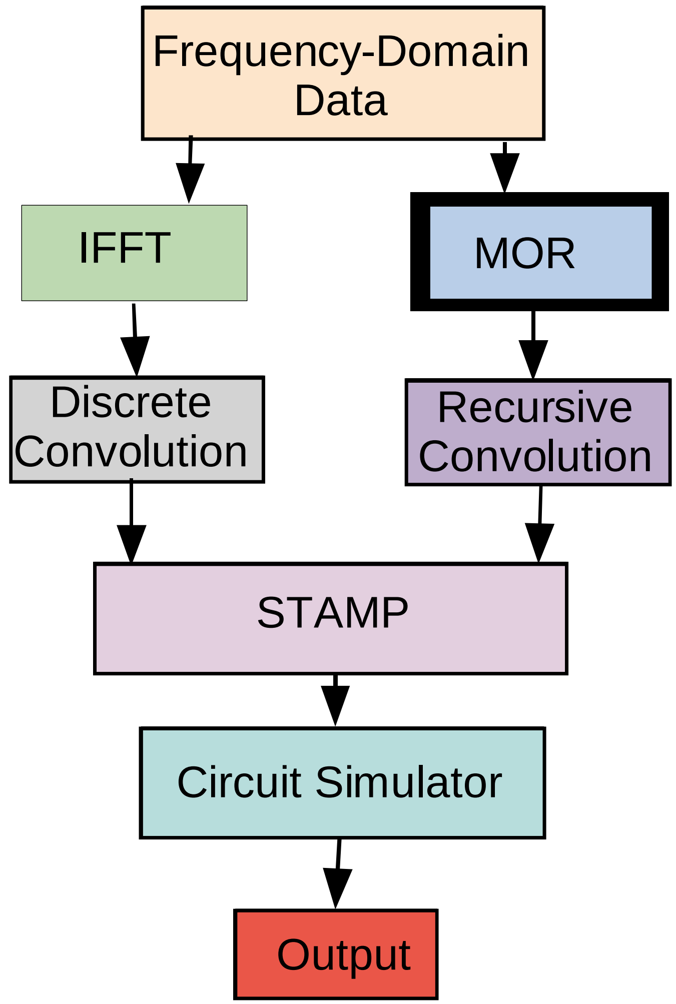
Simulation for Digital Design



Objective: **Perform time-domain simulation of composite network to determine timing waveforms, noise response or eye diagrams**

Macromodel Implementation



Black Box Synthesis

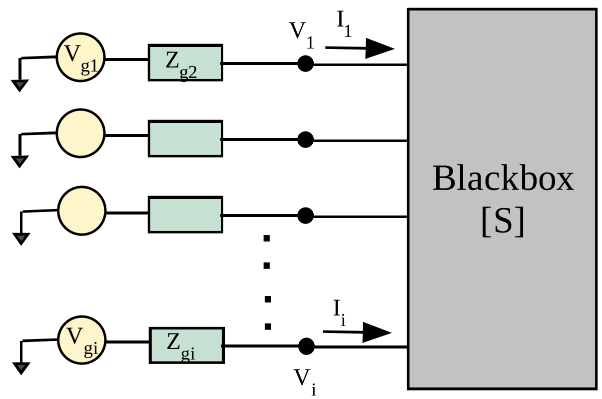
Motivations

- Only measurement data is available
- Actual circuit model is too complex

Methods

- Inverse-Transform & Convolution
  - IFFT from frequency domain data
  - Convolution in time domain
- Macromodel Approach
  - Curve fitting
  - Recursive convolution

IFFT/Convolution Approach to Macromodels



In frequency domain $B = SA$

In time domain $b(t) = s(t)*a(t)$

Convolution: $$s(t)*a(t) = \int_{-\infty}^{\infty} s(t-\tau)a(\tau)d\tau$$

Discrete Convolution

When time is discretized the convolution becomes

$$s(t)*a(t)=\sum_{\tau=1}^{t} s(t-\tau)a(\tau)\Delta\tau$$

Isolating *a(t)*

$$s(t)*a(t)=s(0)a(t)\Delta\tau+\sum_{\tau=1}^{t-1} s(t-\tau)a(\tau)\Delta\tau$$

Since $a(\tau)$ is known for $\tau < t$, we have:

$$H(t)=\sum_{\tau=1}^{t-1} s(t-\tau)a(\tau)\Delta\tau \quad : \quad History$$

Termination Conditions

Defining $s'(0) = s(0)\Delta\tau$, we finally obtain

$$b(t) = s'(0)a(t) + H(t)$$

$$a(t) = \Gamma(t)b(t) + T(t)g(t)$$

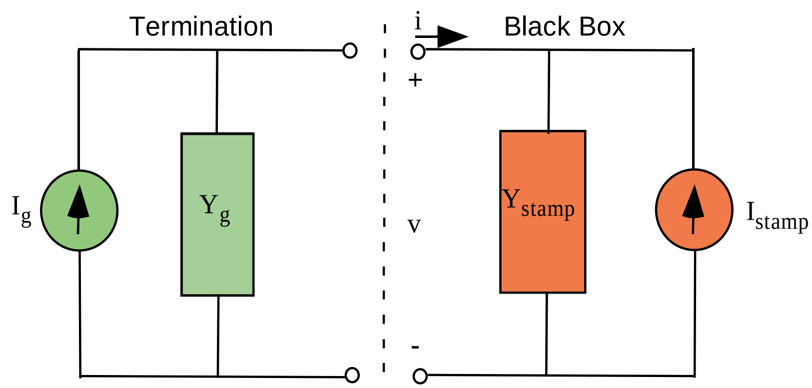
By combining these equations, the stamp can be derived

Stamp Generation from Convolution

$$i(t) = Y_{stamp} v(t) - I_{stamp}$$

$$Y_{stamp} = Z_o^{-1}\left[1 + s'(0)\right]^{-1}\left[1 - s'(0)\right]$$

$$I_{stamp} = 2Z_o^{-1}\left[1 + s'(0)\right]^{-1} H(t)$$



$$(Y_g + Y_{stamp})v(t) = I_g + I_{stamp}$$

Inverse FFT Procedure

1. The sampled data for the blackbox will come with the following set of parameters: number of points M, start frequency f_1, stop frequency f_2 (and frequency step Δf).

2. We wish to process the data. For this purpose we will define our own set of parameters which may or may not be the same as those of the original data. Number of points N, start frequency f_{start}, stop frequency f_{stop}. From the discussion above we learned that the start frequency is best set to zero; therefore, all of the parameters will be set by the choice of two quantities: N and f_{stop}.

3. Perform extrapolation and interpolation of raw data and map the M points from the raw data into the N points of the processed data

4. Determine the frequency step, $f_{step}=f_{stop}/N$. From this, the time step ($t_{step}=1/(2Nf_{step})$), the total duration of the simulation ($t_{stop}=Nt_{step}$) are determined; the frequency-domain data points can now be be arranged as follows

a	*b*	*c*	*d*	*e*	*f*

5. Set the imaginary parts of the first and last points to zero: $Imag\{a\}=0$, and $Imag\{g\}=0$

Inverse FFT Procedure

6. Fold the data with respect to its conjugate mirror image in order to insure that the time-domain response will be pure real; this looks as follows (* indicates complex conjugate):

g	*f**	*e**	*d**	*c**	*b**	*a*	*b*	*c*	*d*	*e*	*f*

where g and a are now real quantities. The total number of points is $N_2=2N$

7. Feed those N_2 points to IFFT routine to obtain inverse FFT. After the IFFT call, scale all the points in the returned array by dividing them by $(N_2 t_{step})$. The data now looks like

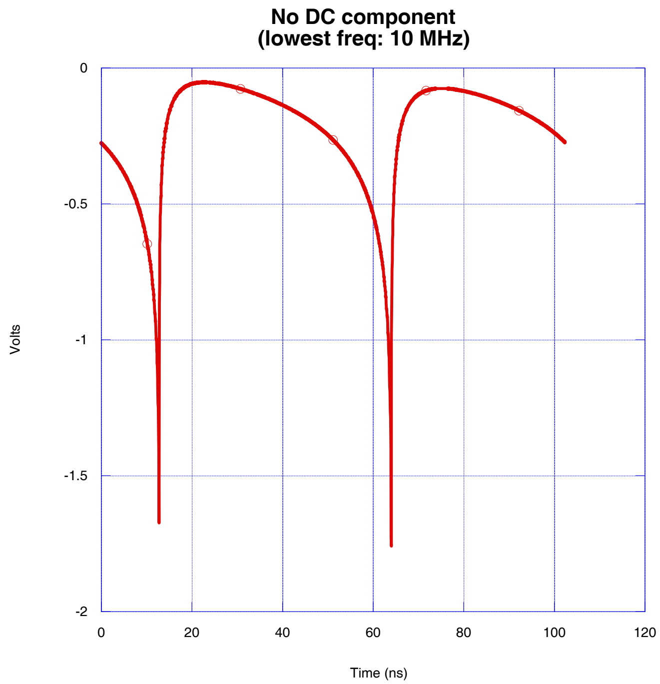
o	*p*	*q*	*r*	*s*	*t*	*u*	*v*	*w*	*x*	*y*	*z*

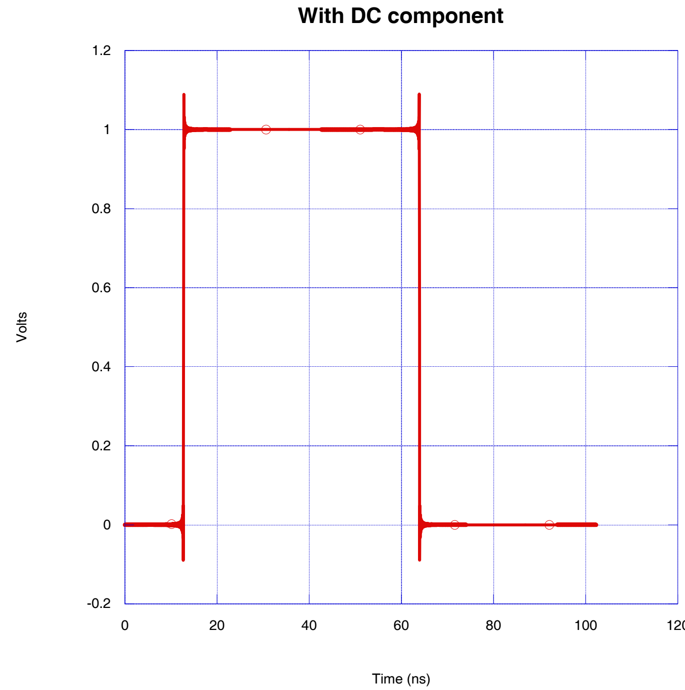
8. These points should all be real. They are the time-domain impulse response.

9. If everything is fine, proceed by keeping only the first N points of the sequence (*o* to *t*). They represent the impulse response of the data and will be used for the time-domain convolution

Importance of Low-Frequency

Calculating inverse Fourier Transform of: $$V(f)=\frac{2\sin(2\pi ft)}{2\pi ft}$$





Left: IFFT of a sinc pulse sampled from 10 MHz to 10 GHz. Right: IFFT of the same sinc pulse with frequency data ranging from 0-10 GHz. In both cases 1000 points are used

Recursive Convolution

Limitations

- Convolution is slow
- Linear network block can be large

Motivation for Recursive Convolution

- Faster
- Requires transfer function to have the form:

$$H(\omega) = \left[A_1 + \sum_{i=1}^{L} \frac{a_{1i}}{1 + j\omega/\omega_{c1i}} \right]$$

Time Domain Solution Using Recursive Convolution

Given the frequency-domain relation:

$$Y(\omega) = H(\omega) X(\omega)$$

If the transfer function is written as

$$\tilde{H}(\omega) = \left[A + \sum_{i=1}^{L} \frac{a_i}{1 + j\omega / \omega_{ci}} \right]$$

Then, the time domain relationship (for step invariant model) is:

$$y(nT) = Ax\left[(n-K)T\right] + \sum_{i=1}^{L} y_{pi}(nT)$$

where

$$y_{pi}(nT) = a_i (1 - e^{-\omega_{ci}T}) x\left[(n-K-1)T\right] + e^{-\omega_{ci}T} y_{pi}\left[(n-1)T\right]$$

This is also called *recursive convolution*

Macromodel - Approximation

Objective: Approximate frequency-domain transfer function to take the form:

$$H(\omega)=\left[A_1+\sum_{i=1}^{L}\frac{a_{1i}}{1+j\omega/\omega_{c1i}}\right]$$

Methods

- **AWE – Pade**
- **Pade via Lanczos (Krylov methods)**
- **Rational Function**
- **Chebyshev-Rational function**
- **Vector Fitting Method**

Attributes for Macromodel

- Numerical Robustness
  - Accuracy
  - Passivity
  - Causality
- User Flexibility
  - Automatic selection of starting poles
  - Automatic determination of order
- Compensation Features
  - Frequency truncation
  - No DC information
  - Measurement noise

Causality - Hilbert Transform Analysis

Enforce causality

$$h(t) = 0, \quad t < 0$$

Every function can be considered as the sum of an even function and an odd function

$$h(t) = h_e(t) + h_o(t)$$

$$h_e(t) = \frac{1}{2}\left[h(t) + h(-t)\right]$$ **Even function**

$$h_o(t) = \frac{1}{2}\left[h(t) - h(-t)\right]$$ **Odd function**

$$h_o(t) = \begin{cases} h_e(t), & t > 0 \\ -h_e(t), & t < 0 \end{cases}$$

$$h_o(t) = \mathrm{sgn}(t) h_e(t)$$

Causality - Hilbert Transform Analysis

$$h(t) = h_e(t) + \operatorname{sgn}(t) h_e(t)$$

In frequency domain this becomes

$$H(f) = H_e(f) + \frac{1}{j\pi f} * H_e(f)$$

$$H(f) = H_e(f) - j\hat{H}_e(f)$$

Imaginary part of transfer function is related to the real part through the Hilbert transform

$\hat{H}_e(f)$ is the Hilbert transform of $H_e(f)$

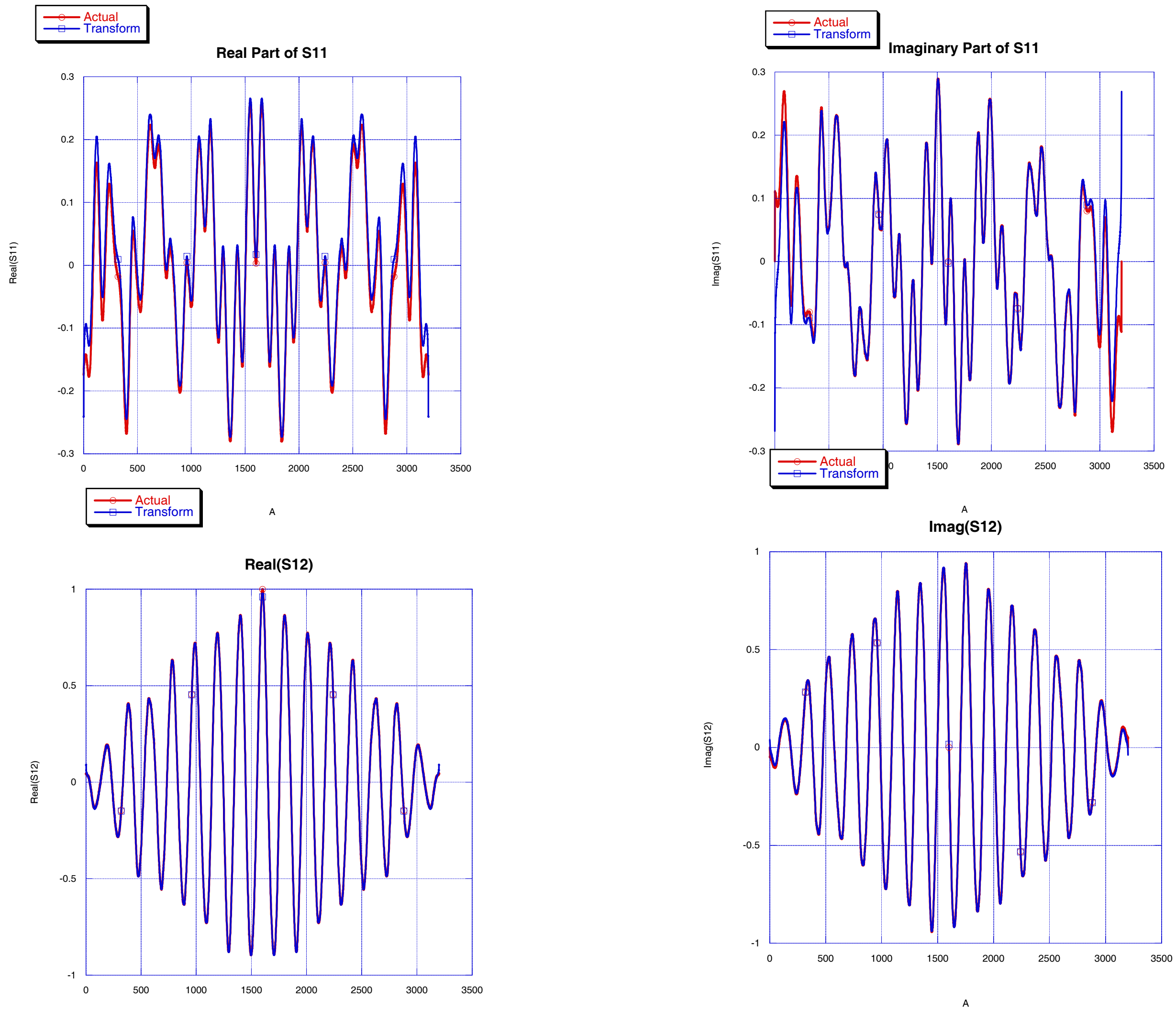
$$\hat{x}(t) = x(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau$$

Discrete Hilbert Transform

$$HT(h_n) = \hat{h}_k = \begin{cases} \dfrac{2}{\pi} \displaystyle\sum_{n \text{ odd}} \dfrac{h_n}{k-n}, & k \text{ even} \\ \dfrac{2}{\pi} \displaystyle\sum_{n \text{ even}} \dfrac{h_n}{k-n}, & k \text{ odd} \end{cases}$$

HT for Coupled Lines: 300 KHz – 6 GHz

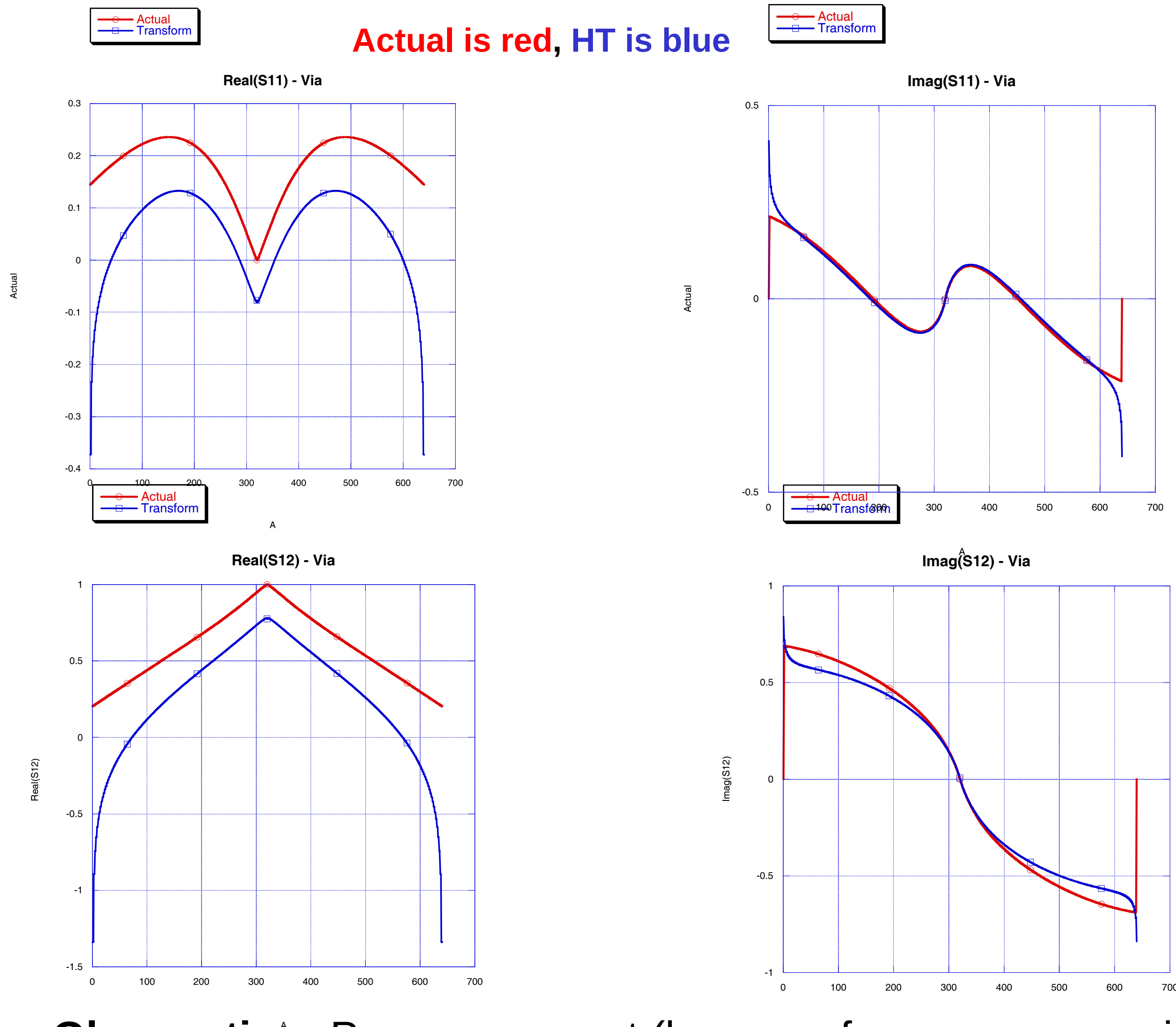
Actual is red, HT is blue



Observation: Good agreement

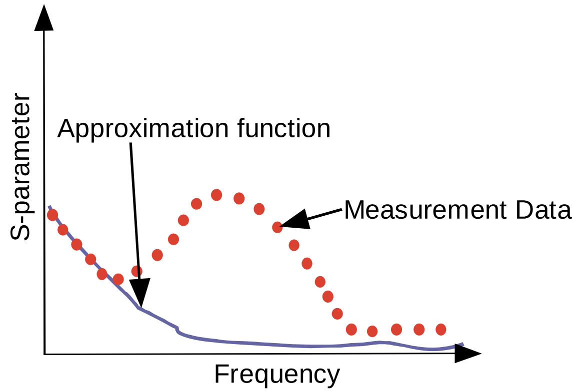
HT for Via: 1 MHz – 20 GHz

Actual is red, HT is blue

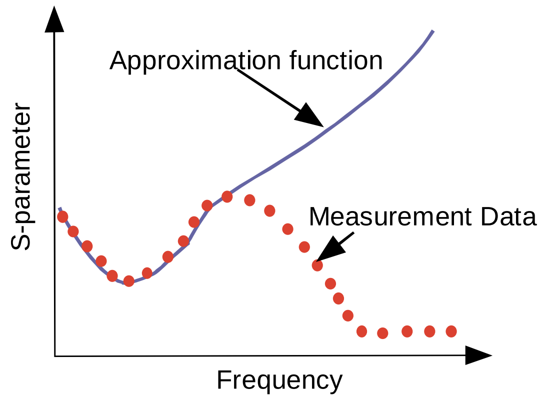


Observation: Poor agreement (because frequency range is limited)

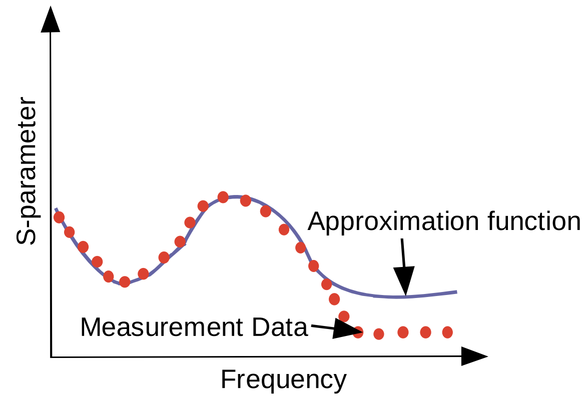
Orders of Approximation



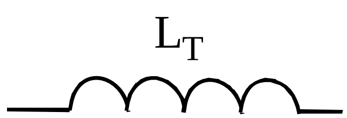
Low order

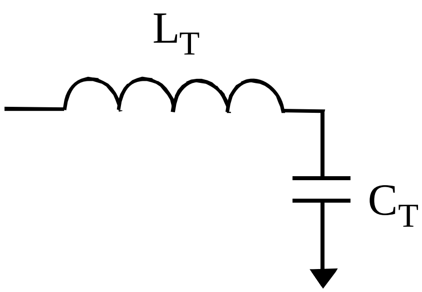


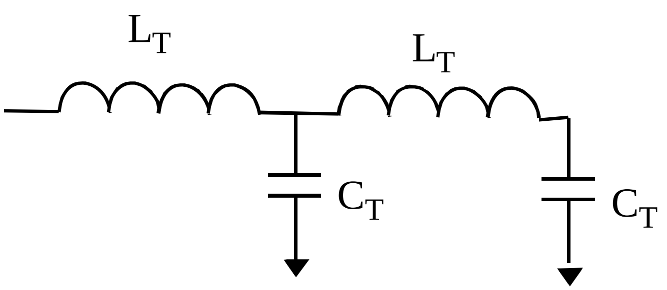
Medium order



Higher order





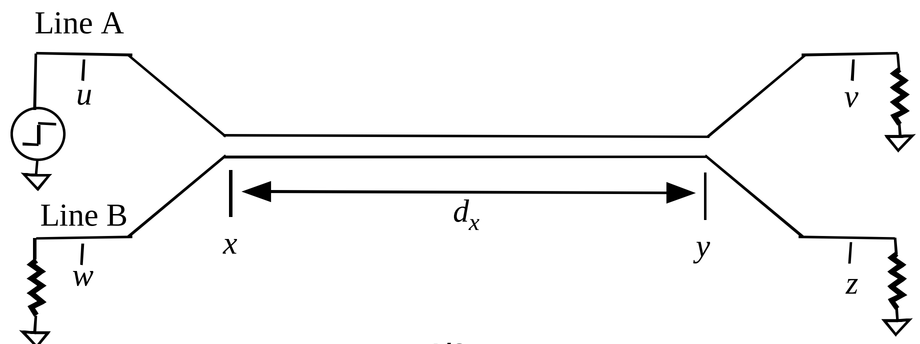


Measurement Data

1.- DISC: Transmission line with discontinuities

Length = 7 inches

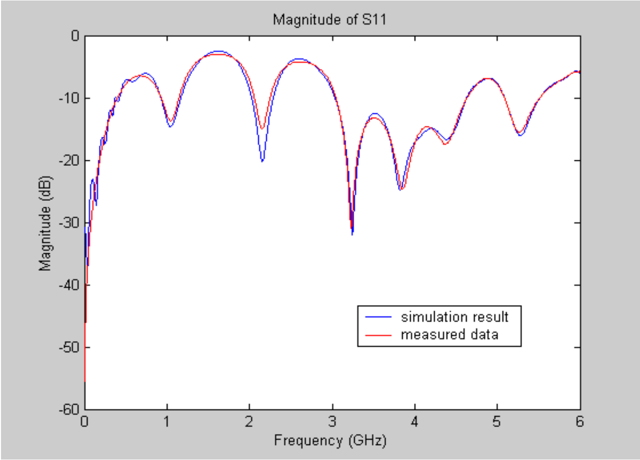
2.- COUP: Coupled transmission line2

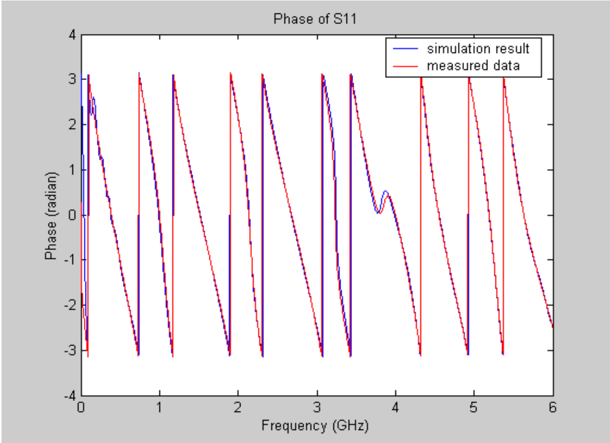


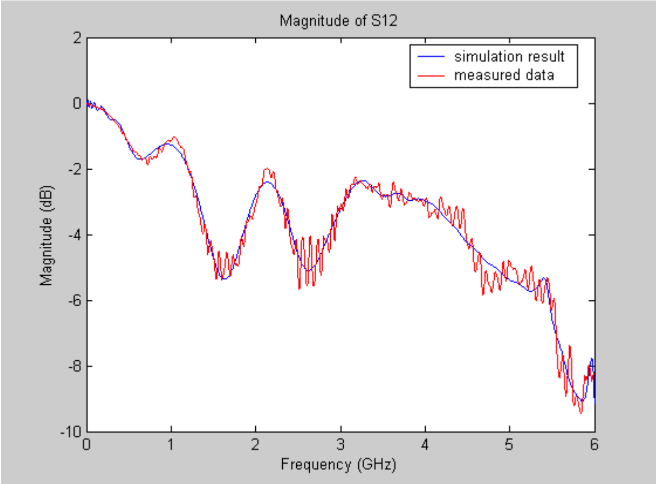
d_x **= 5**$^{1/2}$ **inches**

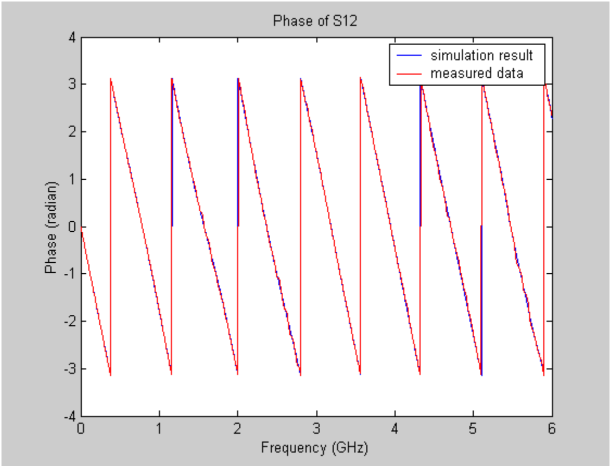
Frequency sweep: 300 KHz – 6 GHz

DISC: Approximation Results



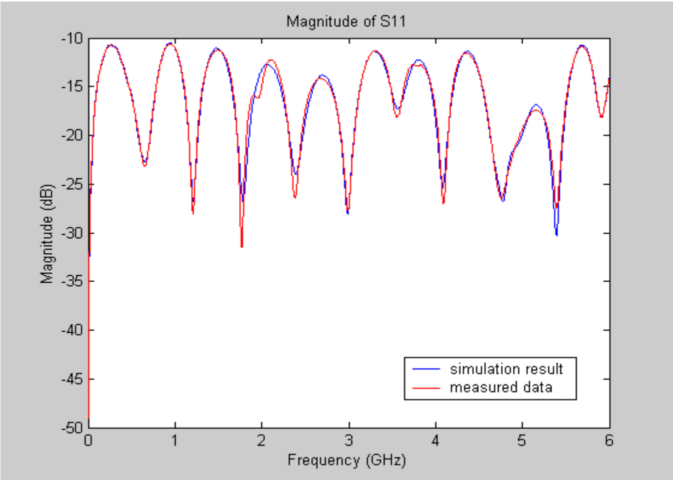


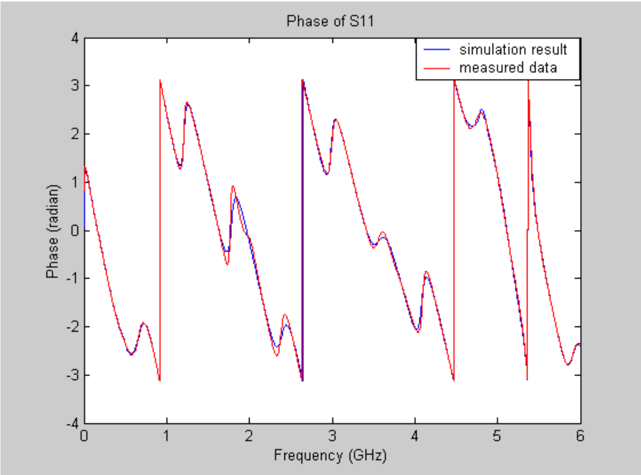


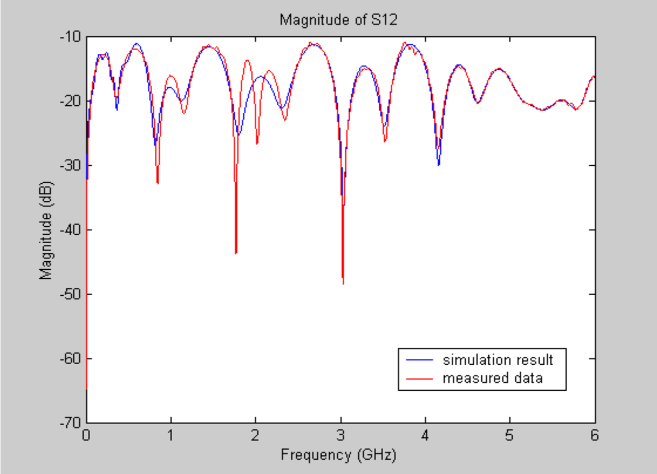


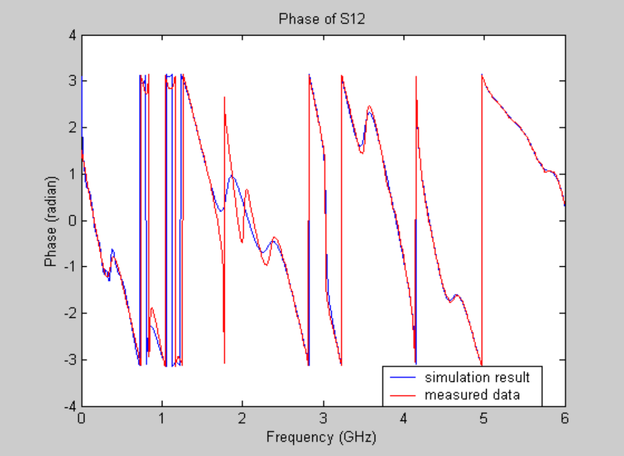
DISC: Approximation order 90

COUP: Approximation Results



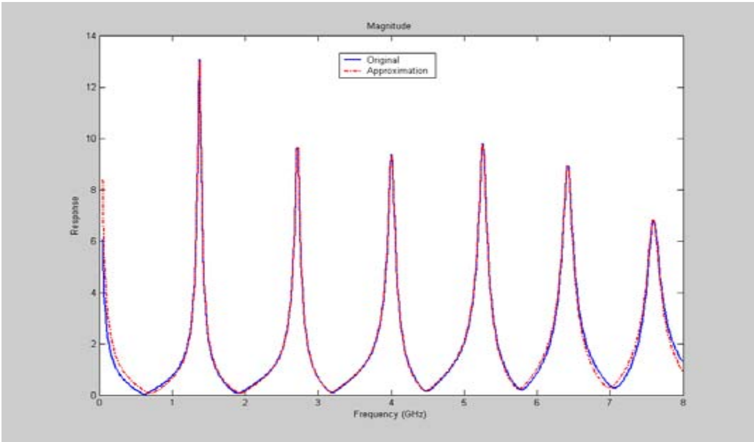


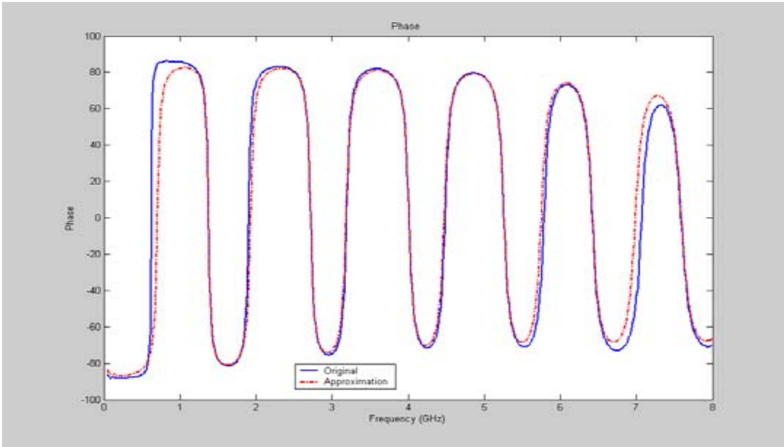




COUP: Approximation order 75 – Before Passivity Enforcement

Results After Passivity Check





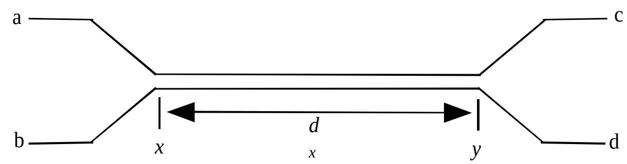
TL with capacitive discontinuity

Magnitude plot of Y_{11}, measured data and the 30-th order rational approximation **with passivity check**

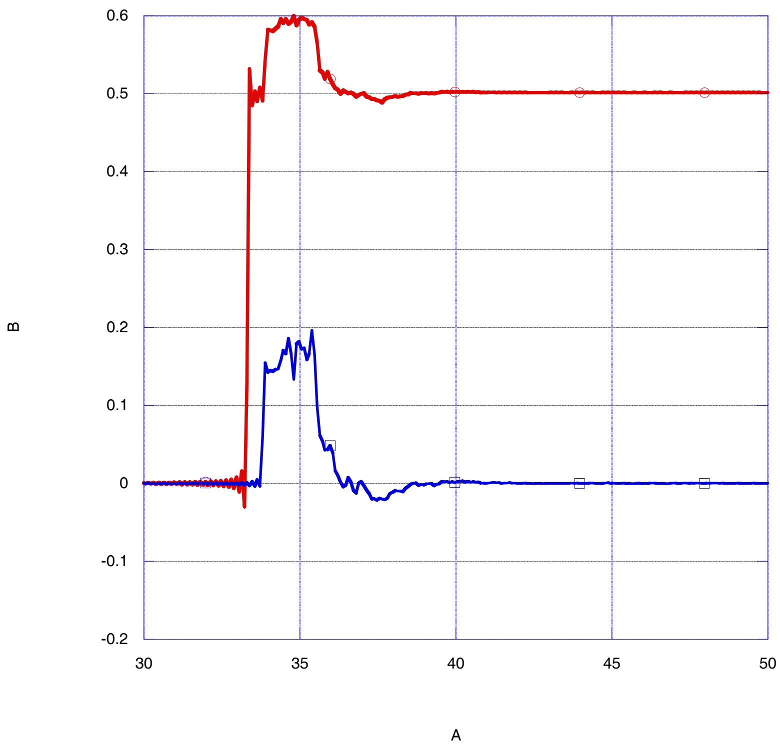
Phase plot of Y_{11}, measured data and the 30-th order rational approximation **with passivity check**

Coupled Lines – Measured on ANA

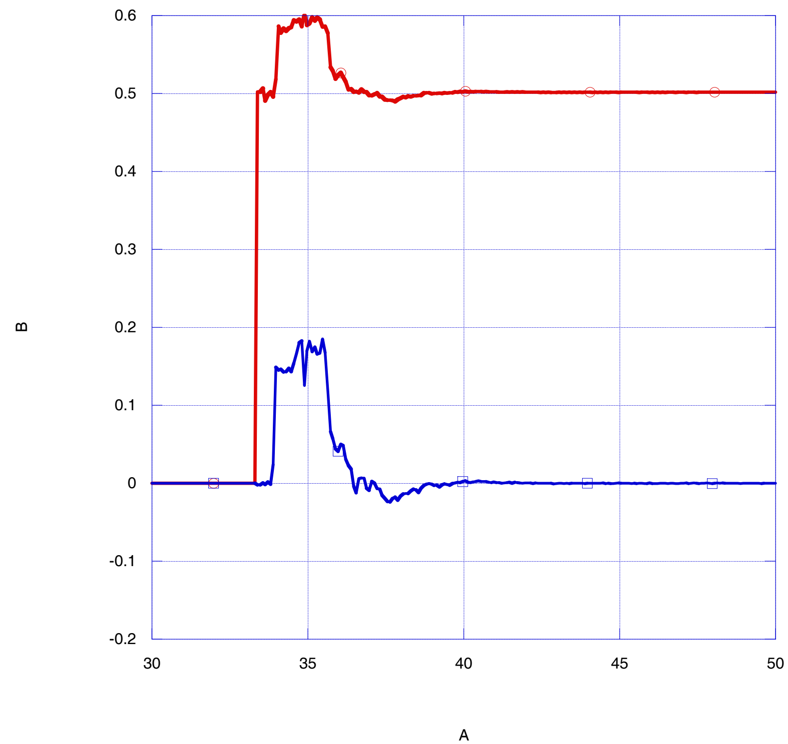
Port 1: a – Port 2: d
Data from 300 KHz to 6 GHz



Exact



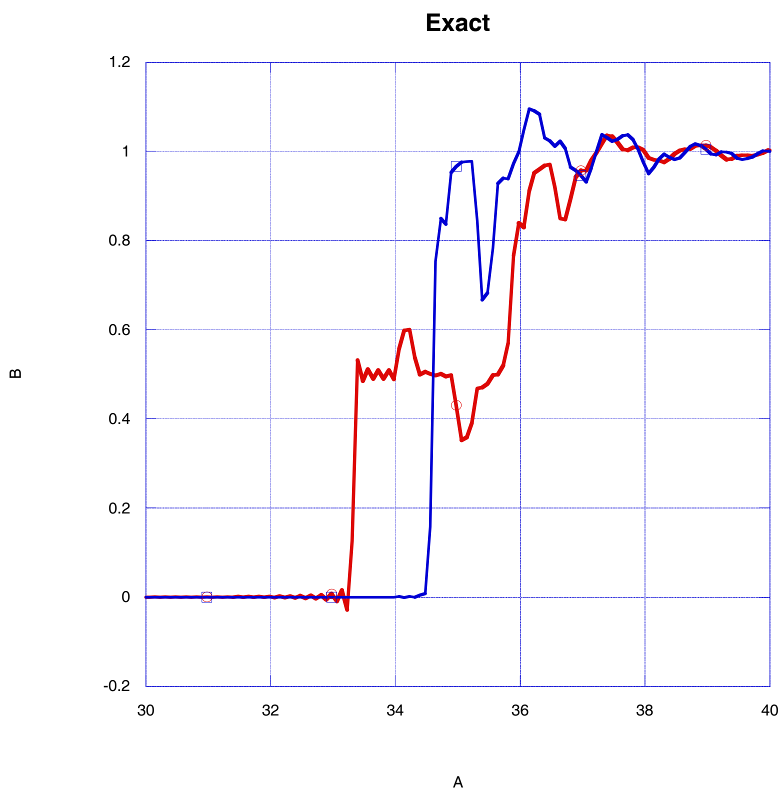
Convolution

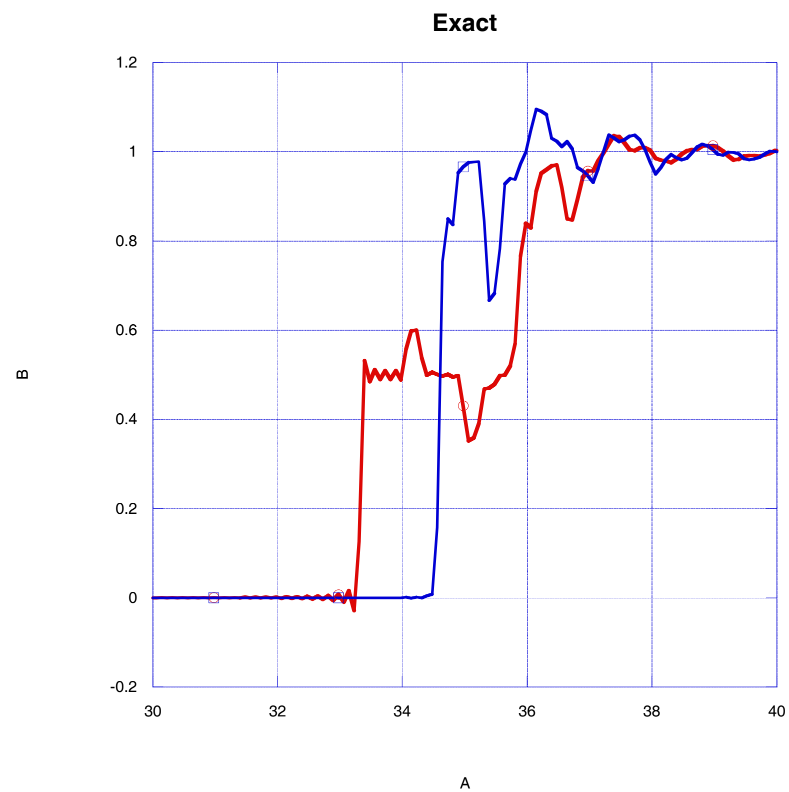


Observation: Good agreement

Microstrip with Steps – Measured on ANA

Microstrip line with discontinuities
Data from 300 KHz to 6 GHz





Observation: Good agreement