Modeling I/O Links with X Parameters

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Abstract
In this work, the polyharmonic distortion (PHD) model is used to analyze the behavior of high-speed nonlinear links. The model assumes the validity of the harmonic superposition principle for high-speed I/O links. From the PHD formalism, a frequency-domain X-parameter matrix formulation can be derived and shown to predict signal propagation in high-speed links. The formulation accommodates both port and harmonic dependence of the signals. The X-parameter formalism is reviewed as well as its application to circuit simulation. Advantages of the X-parameter representation are discussed and various types of simulations are performed and compared with standard simulation techniques.

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**Introduction**

The design of a multi-gigabit SerDes can be challenging because of the high-speed, mixed-signal circuitry involved and stringent electrical specifications. Robust design of these systems requires access to reliable modeling and simulation tools that can help predict the performance and optimize the design. One major challenge in the prediction of waveforms propagating in high-speed links is the nonlinear behavior of the active building blocks. These blocks may include I/O buffer as well as equalizer systems that aim at improving signal integrity. The behaviors of these high-speed equalizer circuitries are often not linear. In order to handle nonlinearities, most simulation techniques have been performed in the time domain using convolution-based techniques. Behavioral and macro-modeling techniques have been among the most popular methods used to predict the performance of these systems. In order to receive widespread acceptance in the communications industry, these simulation techniques and design environments must satisfy certain criteria.

**Vendor IP protection.** SPICE and SPICE-like simulators require internal device and process parameters as part of their input in order to generate accurate simulation results. Unfortunately, most IC vendors are reluctant to release such information in order to maintain a competitive advantage. This has forced the community of high-speed link designers to adopt a “blackbox” format for both the analytical treatment of data and the exchange of information. In the blackbox format, look-up tables or network parameters that offer a behavioral description of the systems from the terminals or ports are used as the vehicle for simulation and design.

**Efficiency and accuracy.** The network complexity found in serial links makes traditional SPICE simulations intractable. If in addition, statistical information is to be obtained to evaluate the impact of process and environmental variations, simulations can run for days if not weeks before meaningful data can be retrieved. When all the transistors in a network must be described in detail, the system matrix size describing the system becomes very large which makes the actual simulation very inefficient.

Behavioral models have been instrumental in meeting the simulation needs for high-speed applications because of their efficiency and blackbox nature. In particular, scattering or S parameters have been regarded as the most successful of behavioral models.
Because of their black box nature, S parameters protect intellectual property (IP) associated with a design. As network parameters, that describe interactions between terminals, they are computationally efficient. Frequency dependence can be included in their description which allows them to be treated with signal processing techniques for macromodeling. S parameters possess in addition the useful property that they can be concatenated. Thus in a serial link, several S-parameter blocks can be combined to generate the S-parameter representation of the composite system. This offers the potential for different vendors to combine different models into a design using the S-parameter description.

Finally, S parameters can be conveniently measured using vector network analyzers (VNA). These instruments have found widespread use not only in RF/microwave applications but also in signal integrity analysis. Accurate frequency-dependent representations of two- and four-port networks can be captured over wide frequency ranges and processed for modeling and simulation.

The main limitation with S parameters is that their use is restricted to linear time-invariant (LTI) systems. In a typical I/O channel, several of the sub-blocks behave as LTI networks; other blocks, however operate in the nonlinear regime and cannot be described accurately using the S-parameter representation. To circumvent these limitations, time-domain techniques must be used and combined with signal processing techniques to handle these nonlinear blocks. Unfortunately, a compact and universal standard for describing nonlinear behavioral model has been lacking preventing any significant breakthrough in nonlinear circuit analysis.

The invention of X parameters by Verspecht and Root [1], [2] and the introduction of nonlinear vector network analyzers (NVNA) by Agilent in 2008 [3] are two important milestones that have created new possibilities in the arena of nonlinear electronic computer-aided design. Using the polyharmonic distortion (PHD) method, a mathematically robust framework can be implemented and used to construct accurate representations for electronic systems that include nonlinearities. Based on the harmonic superposition principle, the X parameter formalism builds on scattering parameter theory and for which it is a superset. Relationship between incident and scattered waves are described using not only port-to-port but also harmonic-to-harmonic interactions. In addition, using an NVNA, these parameters can be measured for various power levels and frequency ranges. The PHD formalism presents unprecedented properties for the modeling analysis of high-speed links.
At first glance, the X-parameter formalism offers several advantages that warrant its use for modeling and simulation of high-speed links.

**Mathematical Robustness.** X parameters are based on the polyharmonic distortion (PHD) model which rests on the principle of harmonic superposition. From these assumptions, a robust construct can be defined to account for port-to-port and harmonic-to-harmonic interactions. As a result, X parameters are the mathematically correct superset of S parameters to which they reduce in the LTI limit.

**Vendor IP protection.** One major concern for SerDes vendors is the protection of their intellectual property. With X parameters, design and process information are completely invisible since only tabulated data is transferred from vendor to user. In this blackbox format, X parameters offer a complete behavioral description that captures static, dynamic, reactive and nonlinear characteristics of the systems.

**Cascadability.** Successive X-parameter blocks can be combined and concatenated to yield composite X-parameter descriptions. This is a powerful feature because it helps simplify the exchange of behavioral models between designers and vendors without compromising accuracy or efficiency.

**Ease of Standardization.** In using X parameters from different sources, no major standardization effort is needed since the PHD formalism already predefines the set of excitations needed to construct the X-parameter representation of a given device under test. There are several types of X parameters (S, T, B, Y and Z) that account for interactions between the different types of signals (DC, large, small, fundamental, harmonics) as a function of frequency and power level.

**Poly-Harmonic Distortion Modeling**

Power waves are quantities associated with voltage and current quantities at the terminals of a multiport network. By convention they are measured on ideal reference transmission lines at the port junctions. At an arbitrary port, the relationships between power waves and voltage and current variables are expressed by:

\[
V_p = \sqrt{2Z_o \left[ a_p + b_p \right]} \quad (1)
\]

\[
I_p = \sqrt{\frac{2}{Z_o} \left( a_p - b_p \right)} \quad (2)
\]

where \(V_p\) and \(I_p\) are the voltage and current measured in port \(p\), \(a_p\) is the incident power wave into the port and \(b_p\) is the corresponding reflected or scattered power wave at that
port. These waves are measured exactly at the junction plane between the reference transmission line and the port terminal. \( Z_o \) is the characteristic impedance of the reference line which is also the reference impedance for the system. For measurement systems \( Z_o \) is usually chosen to be 50 \( \Omega \). With this convention, scattering parameters can be defined that describe power wave interactions between the ports in a time-invariant manner.

In the PHD formalism, this formulation is expanded so as to include interactions between the harmonics of the excitation signal. The PHD formulation builds from the representation of independent and dependent power waves associated with a network in terms of its scattering parameters. For an \( n \)-port network; the relationship between the incident and scattered waves can be expressed as:

\[
b_p^{(k)} = \sum_{q,l} S_{pq}^{(kl)} a_{q}^{(l)} P^{k-l} + T_{pq}^{(kl)} a_{q}^{(l)*} P^{k+l}
\]

where \( a_{q}^{(l)} \) is the \( l^{th} \) harmonic of the incident wave into port \( q \) - subscripts indicate port interaction while superscripts within parentheses indicate harmonic interaction - \( b_p^{(k)} \) is the \( k^{th} \) harmonic of the scattered wave at port \( p \). \( S_{pq}^{(kl)} \) is a scattering parameter of type \( S \) that accounts for the contribution to the \( k^{th} \) harmonic at port \( p \) due to the \( l^{th} \) harmonic of the incident wave in port \( q \). \( T_{pq}^{(kl)} \) is a scattering parameter of type \( T \) that accounts for the contribution to the \( k^{th} \) harmonic at port \( p \) due to the \( l^{th} \) harmonic of the conjugate of the incident wave in port \( q \). The existence of a scattering parameter of type \( T \) is due to the nonanalyticity of the spectral mapping from the time domain to the frequency domain [2]; in essence, it emphasizes the fact that unlike in a linear system, the real and imaginary parts of the waves in a nonlinear system are treated differently through their transfer functions. \( P \) is a phase shift term which must be accounted for during the interactions between the various harmonics. By convention, \( P \) is chosen to be associated with the phase of \( a_{i}^{(1)} \)

\[
P = e^{j2\pi(\text{Arg}(a_{i}^{(1)}))}
\]

The \( S \)’s and \( T \)’s represent a complete set of network parameters that fully characterize the reactive, nonlinear dynamics of the network under study. These parameters are obtained by first applying a large-signal stimulus at the fundamental frequency at the reference port; additional stimuli at the various harmonics are then superimposed with and without phase shift at all the ports to produce the corresponding responses from which parameters of the two types are extracted [2]. Extraction or generation of these parameters can be performed using a nonlinear vector network analyzer (NVNA) equipped with a narrowband modulation capability [3]. Alternatively, these parameters can be generated algorithmically through the use of a harmonic balance circuit simulator [4].
It is key to emphasize the importance of the large-signal operating point (LSOP) in the determination of the X parameters. It is always defined with one reference signal per fundamental frequency. A change in LSOP fundamental frequency or amplitude leads to a completely different set of X parameters for the same device under test (DUT). In addition, the phase of the LSOP is used as the reference phase and is usually set to be zero.

In addition to X parameters of the $S$ and $T$ types, other types are also defined. Since in a nonlinear system, higher powers of a continuous wave (CW) excitation will generate contributions to the DC (zero-frequency) component of the response, it is important to account for interactions between DC, fundamental and harmonics of the excitation signal. In the PHD formalism, these interactions are described through the use of $V$, $I$, $Y$ and $Z$ type X parameters.

The generated X parameters can be stored in a two dimensional format. The dimensions represent the fundamental frequency and the amplitude of the LSOP. The tabulated data can be interpolated for use during simulation. Figure 1 shows plots of various X parameters versus amplitude obtained for a simple CMOS inverter at two different frequencies. The variations of the X parameters with the amplitude are a clear indication of the non-LTI nature of the DUT.

One attractive feature of X parameters is that they can be cascaded. Figure 2 shows a block diagram of SerDes link consisting of passive, analog and nonlinear sub-blocks. While the passive blocks can be represented with S parameters the nonlinear and analog sub-blocks can be described with their X-parameter representation and combined with each other and the S-parameter blocks to define a composite X-parameter representation for the complete system.

**Matrix Representation**

In order to perform high-speed link time-domain simulations using the PHD formalism, a formulation must be implemented that incorporates termination conditions and combines them with the X-parameter representation of the complete channel of interest. For this effect, it is convenient to search for a matrix formulation that can be extended for an arbitrary number of ports and harmonics.

The presence of the scattering parameter of type $T$ multiplying the conjugate of the waves motivates the need for special manipulation of the scattering parameter coefficients. Without loss of generality, we can temporarily adopt a simpler notation in which subscripts and superscripts are omitted that reads
Figure 1. Plots of various types of X parameters as a function of LSOP amplitude ($|A_{11}|$).

Figure 2. A block diagram of a typical high-speed link.
\[ b = Sa + Ta^* \]  

(5)

in which \( a \) and \( b \) are the incident and reflected signals associated with arbitrary input and output ports and harmonics. \( S \) and \( T \) are the scattering parameters of type \( S \) and \( T \) respectively. We can split the wave variables and network coefficients into real and imaginary components to read

\[ b_r + jb_i = (S_r + jS_i)(a_r + ja_i) + (T_r + jT_i)(a_r - ja_i), \]  

(6)

where the subscripts \( r \) and \( i \) refer to real and imaginary parts respectively. After rearranging and separating the variables into their real and imaginary components we get:

\[ b_r = (S_r + T_r)a_r - (S_i - T_i)a_i \]  

(7)

\[ b_r = (S_r + T_i)a_r + (S_i - T_r)a_i \]  

(8)

This can be arranged in a matrix form as

\[
\begin{pmatrix}
  b_r \\
  b_i
\end{pmatrix} =
\begin{pmatrix}
  X_{rr} & X_{ri} \\
  X_{ir} & X_{ii}
\end{pmatrix}
\begin{pmatrix}
  a_r \\
  a_i
\end{pmatrix}
\]  

(9)

where

\[ X_{rr} = (S_r + T_r), \quad X_{ri} = -(S_i - T_i), \]  

(10)

\[ X_{ir} = (S_r + T_i), \quad X_{ii} = (S_i - T_r). \]  

(11)

Matrix equation (9) is an expression of the non-analytic nature of the spectral mapping function described by the X parameter formalism. It emphasizes that the real and imaginary parts of the waves are treated differently in a manner that cannot be described by a complex product. Consequently, an X-parameter matrix formulation must separate not only harmonics and ports but also real and imaginary components of the parameters and associated signals.

From (3) it is observed that the phase can be factored from the incident and reflected power wave variables. Consequently, (9) can be viewed as a relationship involving the phased variables; expressions involving phase-less variables are given by

\[
\begin{pmatrix}
  \cos \theta_b & -\sin \theta_b \\
  -\sin \theta_b & \cos \theta_b
\end{pmatrix}
\begin{pmatrix}
  b_r \\
  b_i
\end{pmatrix} =
\begin{pmatrix}
  X_{rr} & X_{ri} \\
  X_{ir} & X_{ii}
\end{pmatrix}
\begin{pmatrix}
  \cos \theta_a & -\sin \theta_a \\
  -\sin \theta_a & \cos \theta_a
\end{pmatrix}
\begin{pmatrix}
  a_r' \\
  a_i'
\end{pmatrix}
\]  

(12)

where
The equations are as follows:

\[
\begin{pmatrix}
\cos \theta_a & -\sin \theta_a \\
-\sin \theta_a & \cos \theta_a
\end{pmatrix}
\begin{pmatrix}
b_r \\
b_i
\end{pmatrix}
\]

and

\[
\begin{pmatrix}
\cos \theta_a & -\sin \theta_a \\
-\sin \theta_a & \cos \theta_a
\end{pmatrix}
\begin{pmatrix}
a_r \\
a_i
\end{pmatrix}
\]

In which \( \theta_a \) and \( \theta_b \) are the phases associated with harmonically weighted \( a \) and \( b \) waves, respectively. Consequently, generality is not lost in using a phase-less formulation since phased quantities can be recovered using the above transformations. Thus, for analysis purposes, the phase-normalized version (\( P=1 \)) of equation (3) is utilized for describing the wave interactions. Referring to the network diagram in Figure 3, for which we assume \( n \) ports and \( m \) harmonics, we can define incident and reflected wave vectors given by

\[
\begin{pmatrix}
a_1 \\
a_2 \\
\vdots \\
a_n
\end{pmatrix}
\]

and

\[
\begin{pmatrix}
b_1 \\
b_2 \\
\vdots \\
b_n
\end{pmatrix}
\]

where each of the \( a_p \)'s and \( b_p \)'s are subvector representing incident and reflected signals at port \( p \) respectively. More explicitly:

\[
a_p = \begin{pmatrix}
a^{(1)}_{pr} \\
a^{(1)}_{pi} \\
a^{(2)}_{pr} \\
a^{(2)}_{pi} \\
\vdots \\
a^{(m)}_{pr} \\
a^{(m)}_{pi}
\end{pmatrix}, \quad \text{and} \quad b_p = \begin{pmatrix}
b^{(1)}_{pr} \\
b^{(1)}_{pi} \\
b^{(2)}_{pr} \\
b^{(2)}_{pi} \\
\vdots \\
b^{(m)}_{pr} \\
b^{(m)}_{pi}
\end{pmatrix}
\]
where $a_{pr}^{(k)}$ and $b_{pr}^{(k)}$ are the real parts of the $k^{th}$ harmonic of the incident and reflected waves, respectively. From these relationships, it is clear that an X-parameter matrix can be defined to relate these waves in the form $b = Xa$ where

$$X = \begin{pmatrix}
X_{11} & X_{12} & \cdots & X_{1n} \\
X_{21} & X_{22} & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
X_{n1} & \cdots & \cdots & X_{nn}
\end{pmatrix} \tag{16}$$

in which each $X_{pq}$ is a submatrix (size $2m \times 2m$) given by

$$X_{pq} = \begin{pmatrix}
X_{pq}^{(11)} & X_{pq}^{(11)} & X_{pq}^{(12)} & \cdots & X_{pq}^{(1m)} \\
X_{pq}^{(11)} & X_{pq}^{(11)} & X_{pq}^{(12)} & \cdots & \cdots \\
X_{pq}^{(12)} & X_{pq}^{(12)} & X_{pq}^{(22)} & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
X_{pq}^{(m1)} & X_{pq}^{(m1)} & \cdots & X_{pq}^{(nn)} & X_{pq}^{(nn)}
\end{pmatrix} \tag{17}$$

for which we made use of the relationship in (10)-(11). More explicitly,

$$X_{pq}^{(kl)} = \text{Re}\left[ S_{pq}^{(kl)} \right] + \text{Re}\left[ T_{pq}^{(kl)} \right] \tag{18}$$

$$X_{pq}^{(kl)} = \text{Im}\left[ T_{pq}^{(kl)} \right] - \text{Im}\left[ S_{pq}^{(kl)} \right] \tag{19}$$

$$X_{pq}^{(kl)} = \text{Im}\left[ S_{pq}^{(kl)} \right] + \text{Im}\left[ T_{pq}^{(kl)} \right] \tag{20}$$

$$X_{pq}^{(kl)} = \text{Re}\left[ S_{pq}^{(kl)} \right] - \text{Re}\left[ T_{pq}^{(kl)} \right] \tag{21}$$

As an example, $X_{pqri}^{(kl)}$ is the contribution to the real part of the $k^{th}$ harmonic of the wave scattered at the $p^{th}$ port due to the imaginary part of the $l^{th}$ harmonic of the incident wave at the $q^{th}$ port. The complete X matrix is of size $2mn \times 2mn$. For instance in the case of a two-port network and taking into account 2 harmonics (fundamental and second harmonic), the full vector and matrix description would be
Steady-State Simulations

In order to find a solution that incorporates the termination conditions, we start by stating a relationship between voltage waves and terminal variables. These variables satisfy the relationship

\[ v = a + b \]  \hspace{1cm} (24)

and

\[ i = Z_o^{-1} [a - b] \]  \hspace{1cm} (25)

where \( v \) and \( i \) are the total voltage and current vectors respectively. \( Z_o \) is the reference impedance matrix constructed from the system used to determine the X-parameters of the black box. We next state the scattering voltage wave relationship as

\[ b = Xa \]  \hspace{1cm} (26)

This must be combined with the termination conditions expressing the relationship between the voltage sources, terminal impedances, and voltage waves as

\[ a = Dv_s + \Gamma b \]  \hspace{1cm} (27)
\( \mathbf{D} \) is a voltage division matrix, \( \mathbf{\Gamma} \) is a reflection coefficient matrix and \( \mathbf{v}_s \) is the voltage source vector. These matrices and vector are constructed in the same manner as those in (22) and (23). By combining (26) and (27), we get

\[
\mathbf{a} = \left[ 1 - \mathbf{\Gamma X} \right]^\dagger \mathbf{Dv}_s
\]

which expresses the incident voltage wave vector in terms of the voltage source vector. The reflected voltage wave vector, \( \mathbf{b} \) can be obtained from \( \mathbf{a} \) using (26). Then, the voltage and current solutions can be determined using (24) and (25).

In order to test and validate the formulation, steady-state simulations were performed on a simple link consisting of a CMOS driver/receiver system connected via a 10-inch slightly lossy microstrip trace (Figure 4). First, the X-parameters for the complete circuit were generated using the ADS X-parameter generator [4]-[5]. A MATLAB program was next developed to handle the simulations as prescribed by equations (24)-28).

A 1-GHz sine wave was used to excite the system. The input and output voltages were constructed for various number of harmonics. Figure 5 shows plots of the response waveforms and their dependence of the number of harmonics. A maximum number of 12 harmonics was chosen for this case in order to obtain acceptable accuracy. Validation of the results was obtained by comparing with the standard simulator in ADS. Plot comparisons are shown in Figure 6 and indicate good correlation. As can be observed, the X-parameter simulation is able to capture the nonlinear effects such as the clipping of the negative excursions from the excitation.

**Transient Simulation**

In the PHD model, the X parameters of a device are obtained by exciting the reference port with a large signal at the fundamental frequency. Small-signal tones are then applied at each port one at a time and at each harmonic frequency of the fundamental. The responses at the ports due to the different combinations of excitation define the X parameters of the model.

For transient simulations, it is assumed that the primary effects of the nonlinearities in the system are manifested through the generation of harmonics. By applying the harmonic superposition principle, interactions between the harmonics allow to capture these nonlinear effects. It is then assumed that traditional signal processing techniques that rely on superposition can be applied as long as a sufficient number of harmonics are taken into account. We thus adopt the following these guidelines for transient simulation.
Figure 4. Schematic of CMOS driver/receiver circuit used for test

Figure 5. Plots showing input and output voltages for circuit of Figure 4 for various numbers of harmonics.
Figure 6. Circuit schematic (top), Matlab simulations using X-parameters (bottom-left) and ADS simulations (bottom-right). Simulations show input (dashed) and output (solid) waveforms.
1. Generate the X parameters of the system under study over a wide frequency range.

2. Obtain the solution for the voltage vectors as a function of frequency as per equations (24)-(28).

3. For each harmonic obtain the time-domain solution via IFFT and using the frequency scaling relation: \( H(kf) \leftrightarrow \frac{1}{|k|} h\left(\frac{t}{k}\right) \) for each harmonic index \( k \).

4. The total time-domain solution is obtained by superposing contributions from all the harmonics.

These steps can be implemented in a computer program and used to simulate the transient response of a network characterized by its X parameters. A serial link with a simple equalizer was implemented to test the transient analysis procedure. The equalizer consists of a 1-tap FIR filter implemented in a modified single-ended push-pull configuration using a typical 2.5V CMOS process. Near-end signal driving the channel is obtained by voltage division, and the tap delay is implemented using a simple 4-inverter chain. Resistor sizing sets the tap coefficient and signal levels, chosen in such a way as to cancel out most of the ISI contributed by the linear channel, assuming NRZ signaling with a rate of 5Gb/s. The channel is chosen to represent a 50\(\Omega\), 40-inch backplane copper trace on a FR4 board, terminated at far-end with the input resistance of the receiver represented by a matching 50\(\Omega\) resistor.

A computer program was written in C to read the X parameters of a two port and perform the transient analysis using the method described above. First, the X parameters of the complete system were generated at power levels of 12 dBm and 14 dBm. This was chosen so that the amplitude of the large signal excitation would correspond to the magnitude of the pulse to be used in the transient simulation. Swept frequency data was obtained from 100 kHz to 50 GHz. At each frequency point 4 harmonics were included in the excitation. Next, the X parameter data was read by the C program and utilized for the transient simulation. Results are shown in Figure 8 for the two different power levels in the unequalized and equalized cases. Plots of the corresponding simulations using ADS are shown for comparison. Although a small difference exists in the responses, it is clear that the X-parameter behavioral model captures the essential characteristics of the response.
Next, the program was modified to simulate the response of the system for a 5-Gb/s pseudo-random bit sequence (PRBS) excitation. The results were used to construct several eye diagrams for the signal at the receiver end. These are shown in Figure 9 for both unequalized and equalized states. Different power levels for the X parameters were used. The 10 dBm model corresponds to a state in which the driver is not fully on; consequently, as anticipated, this model produces a closed eye. X-parameter models corresponding to the higher power levels of 12 and 14 dBm are associated with the devices being turned on and consequently lead to improved response diagrams especially in the equalized mode.

It is important to stress that these results were obtained by applying LTI techniques to the PHD concept. From a rigorous standpoint, the PHD method and the X parameter models apply to systems that are non-LTI. In the PHD model, it is assumed that the harmonics of the response of the system under test obey the principle of superposition. Our analysis of high-speed links extended the harmonic superposition principle to also include the large signal component of the response of the system. The validity of this approach rests on the assumption that there exist a range of mildly nonlinear systems for which this assumption works. High-speed links with their driver and receiver networks belong to that category.

Figure 7: Block diagram of the serial link (top), circuit schematic of the transmitter (bottom).
Figure 8. Plots of the transient responses for the serial link of Figure 7 for unequalized (left) and equalized (right) modes. Top: simulation using 12 dBm X-parameter model, middle: simulation using 14 dBm X-parameter model, bottom: Simulation using standard ADS transient simulator. Pulse characteristics: width: 180 ps, rise and fall times: 20 ps. A total of 4 harmonics was used for the X-parameter simulations.
Figure 9. Eye diagram simulations for the serial link of Figure 7 for unequalized (left) and equalized (right) modes. Top: simulation using 10 dBm X-parameter model, middle: simulation using 12 dBm X-parameter model, bottom: simulation using 14 dBm X-parameter model. A 5-Gb/s PRBS sequence was used. Each bit was a pulse with the following characteristics: width: 180 ps, rise and fall times: 20 ps. A total of 4 harmonics was in the X-parameter models.
Conclusion
This work demonstrated the use of the X-parameter/PHD formalism for the modeling and simulation of high-speed links. It assumed that the network driving conditions are such that the harmonic superposition principle is valid. A matrix formulation was presented and used to predict signal transmission in the frequency and time domains. Results showed good correlation with conventional circuit simulators. Thus, it is anticipated that X-parameters will be an efficient standard for the exchange of nonlinear behavioral models.

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