ECE 451
Planar Transmission Lines

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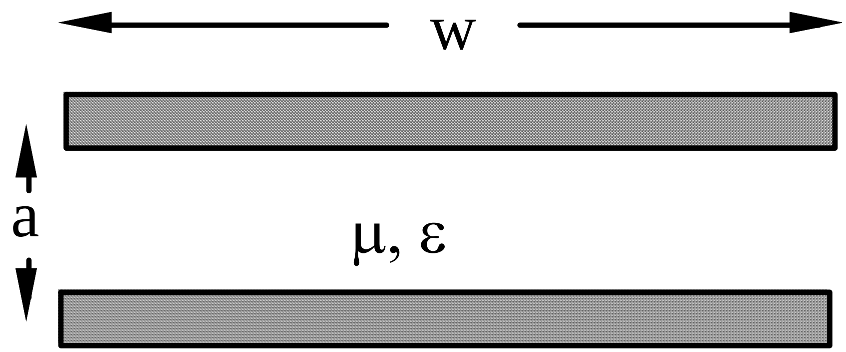
Parallel-plate Transmission Line



$$L = \frac{\mu a}{w}$$

$$C = \frac{\varepsilon w}{a}$$

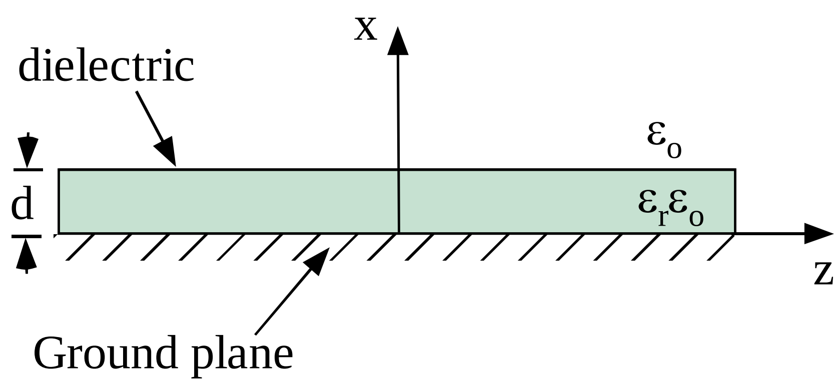
Surface Waves – TM Modes

Assume the form:

$$E_z(x,y,z) = e_z(x,y)e^{-j\beta z}$$

$$\left(\frac{\partial^2}{\partial x^2} + \varepsilon_r k_o^2 - \beta^2\right)e_z(x,y) = 0 \qquad \text{Inside dielectric}$$

$$\left(\frac{\partial^2}{\partial x^2} + k_o^2 - \beta^2\right)e_z(x,y) = 0 \qquad \text{Outside dielectric}$$



Dispersion relations

$$k_c^2 = \varepsilon_r k_o^2 - \beta^2$$

$$h^2 = \beta^2 - k_o^2$$

Surface Waves – TM Modes: Solutions

Solutions are:

$$e_z(x,y) = A\sin k_c x + B\cos k_c x$$ Inside dielectric

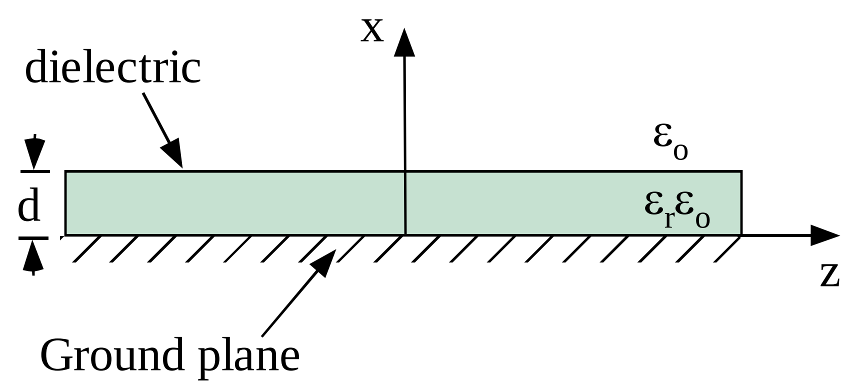
$$e_z(x,y) = Ce^{hx} + De^{-hx}$$ Outside dielectric

$$E_z(x,y,z) = 0 \text{ at x=0}$$

$$E_z(x,y,z) < \infty, \quad for\ x \to \infty$$

$$E_z(x,y,z) \text{ continuous at boundary}$$

$$H_y(x,y,z) \text{ continuous at boundary}$$



$$A\sin k_c d = De^{-hd}$$

$$k_c \tan k_c d = \varepsilon_r h$$

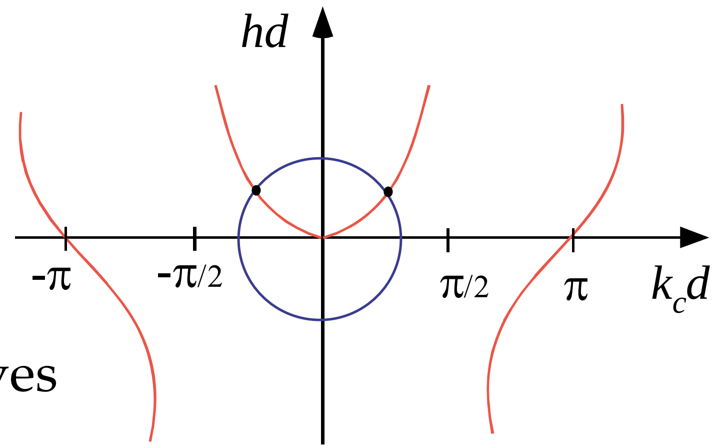
$$\frac{\varepsilon_r A}{k_c}\cos k_c d = \frac{D}{h}e^{-hd}$$

$$k_c^2 + h^2 = \left(\varepsilon_r - 1\right)k_o^2$$

Surface Waves – TM Modes: Solutions

$$(k_c d)^2 + (hd)^2 = (\varepsilon_r - 1)(k_o d)^2$$

$$k_c d \tan k_c d = \varepsilon_r hd$$



Solution is found at intersection of curves

First TM mode is TM_o mode

Cutoff frequencies for TM modes are given by:

$$f_c = \frac{nc}{2d\sqrt{\varepsilon_r - 1}}, \qquad n = 0, 1, 2, \ldots$$

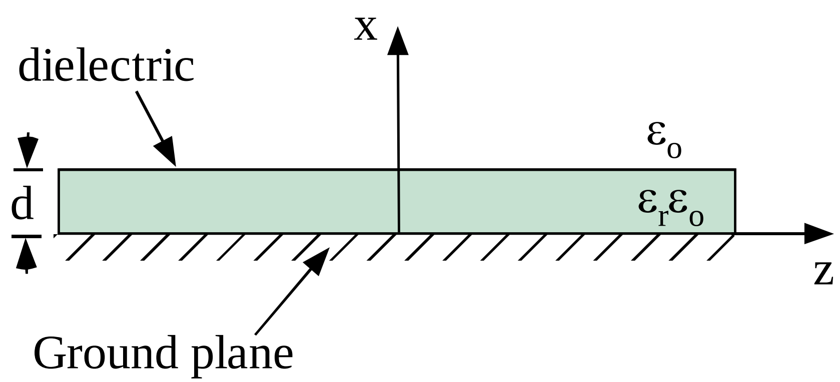
Surface Waves – TE Modes

Assume the form:

$$H_z(x,y,z) = h_z(x,y)e^{-j\beta z}$$

$$\left(\frac{\partial^2}{\partial x^2} + \varepsilon_r k_o^2 - \beta^2\right)h_z(x,y) = 0$$ Inside dielectric

$$\left(\frac{\partial^2}{\partial x^2} + k_o^2 - \beta^2\right)h_z(x,y) = 0$$ Outside dielectric



Dispersion relations

$$k_c^2 = \varepsilon_r k_o^2 - \beta^2$$

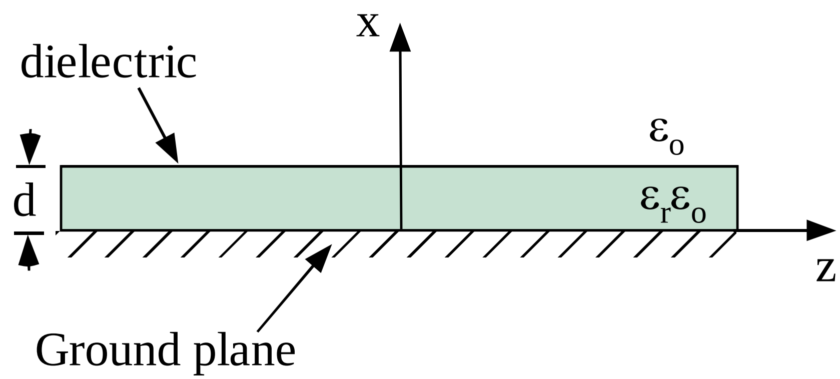
$$h^2 = \beta^2 - k_o^2$$

Surface Waves – TE Modes: Solutions

Solutions are:

$$h_z(x,y) = A\sin k_c x + B\cos k_c x$$ Inside dielectric

$$h_z(x,y) = Ce^{hx} + De^{-hx}$$ Outside dielectric



After matching the boundary conditions

$$\frac{-B}{k_c}\sin k_c d = \frac{D}{h}e^{-hd}$$

$$-k_c \cot k_c d = h$$

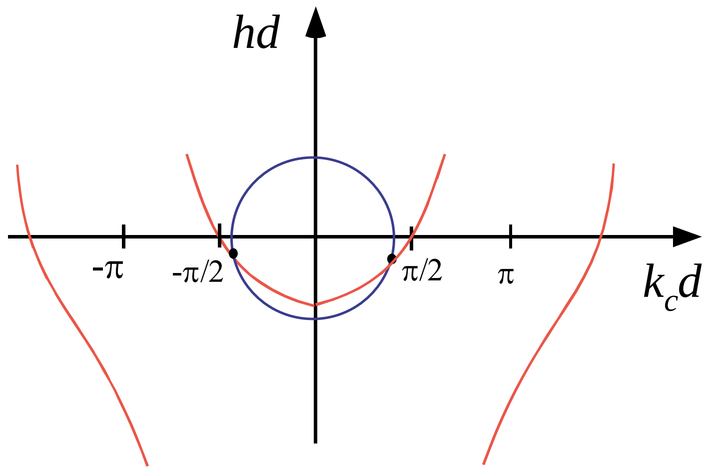
$$B\cos k_c d = De^{-hd}$$

$$k_c^2 + h^2 = (\varepsilon_r - 1)k_o^2$$

Surface Waves – TE Modes: Solutions

$$\left(k_c d\right)^2 + \left(hd\right)^2 = \left(\varepsilon_r - 1\right)\left(k_o d\right)^2$$

$$-k_c d \cot k_c d = hd$$



Solution is found at intersection of curves

Negative values of *h* must be excluded

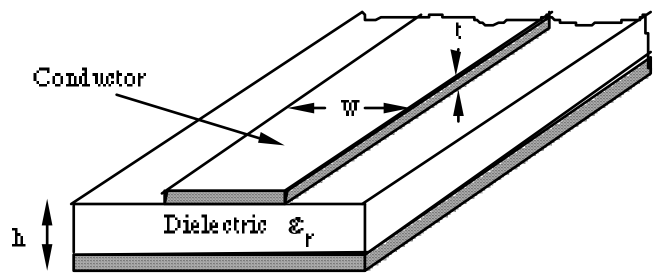
Cutoff frequencies for TE modes are given by:

$$f_c = \frac{(2n-1)c}{4d\sqrt{\varepsilon_r - 1}}, \qquad n = 1, 2, 3, \ldots$$

Modes will occur in the following order:

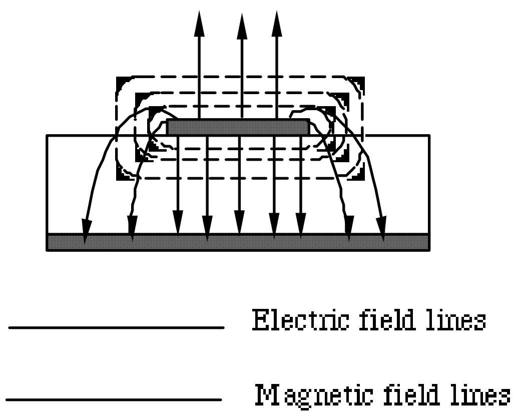
TM_0, TE_1, TM_1, TE_2, TM_2

Microstrip



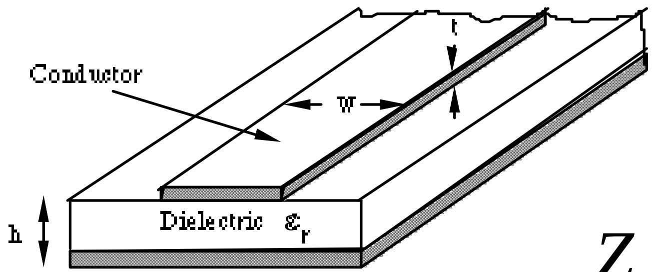
(a)

Need to determine characteristic impedance Z_o and effective permittivity ε_e



(b)

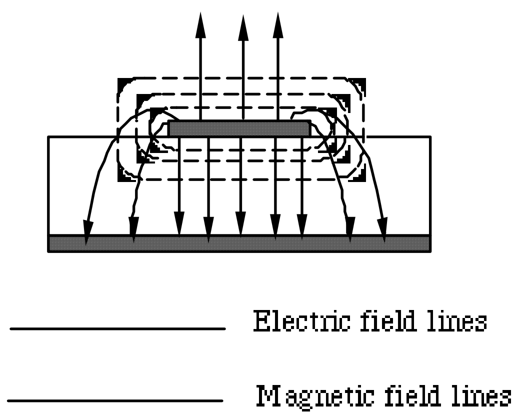
Microstrip – Analysis Equations



(a)

$w/h < 3.3$

$$Z_o = \frac{119.9}{\sqrt{2(\varepsilon_r+1)}} \ln\left[4\frac{h}{w} + \sqrt{16\left(\frac{h}{w}\right)^2 + 2}\right]$$



(b)

$w/h > 3.3$

$$Z_o = \frac{119.9\pi}{2\sqrt{\varepsilon_r}}\left\{\frac{w}{2h} + \frac{\ln(4)}{\pi} + \frac{\ln(e\pi^2/16)}{2\pi}\left(\frac{\varepsilon_r - 1}{\varepsilon_r^2}\right) + \frac{\varepsilon_r+1}{2\pi\varepsilon_r}\left[\ln\frac{\pi e}{2} + \ln\left(\frac{w}{2h} + 0.94\right)\right]\right\}$$

Microstrip Analysis & Synthesis Equations

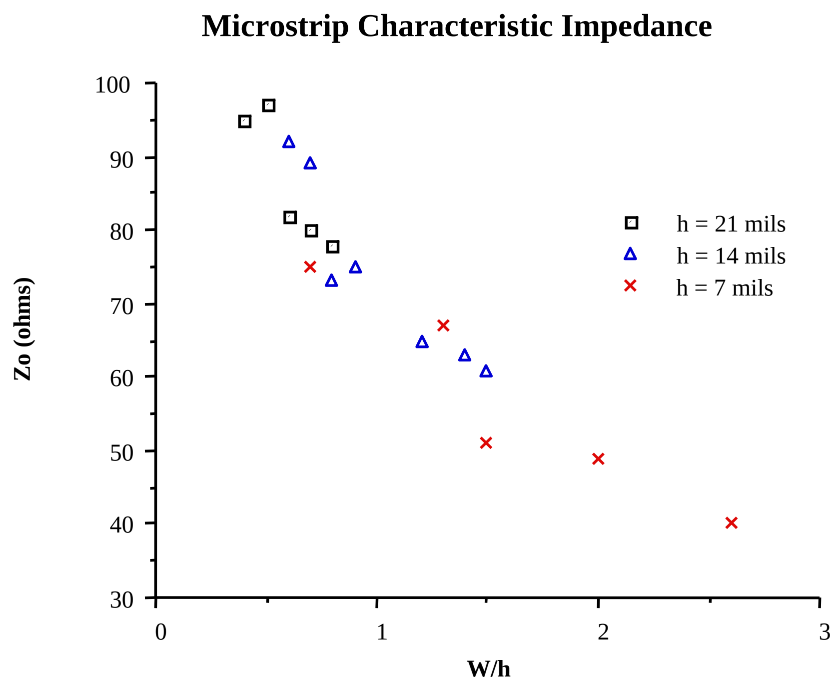
$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12d/W}}$$

$$Z_o = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_e}} \ln\left(\dfrac{8d}{W} + \dfrac{W}{4d}\right) & \text{for } W/d \le 1 \\ \dfrac{120\pi}{\sqrt{\varepsilon_e}\left[W/d + 1.393 + 0.667\ln(W/d + 1.444)\right]} & \text{for } W/d \ge 1 \end{cases}$$

$$\frac{W}{d} = \begin{cases} \dfrac{8e^A}{e^{2A} - 2} & \text{for } W/d < 2 \\ \dfrac{2}{\pi}\left[B - 1 - \ln(2B - 1) + \dfrac{\varepsilon_r - 1}{2\varepsilon_r}\left\{\ln(B - 1) + 0.39 - \dfrac{0.61}{\varepsilon_r}\right\}\right] & \text{for } W/d > 2 \end{cases}$$

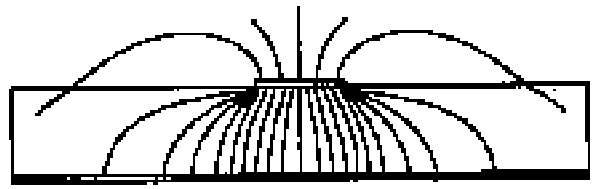
$$A = \frac{Z_o}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right) \qquad B = \frac{377\pi}{2Z_o\sqrt{\varepsilon_r}}$$

Microstrip

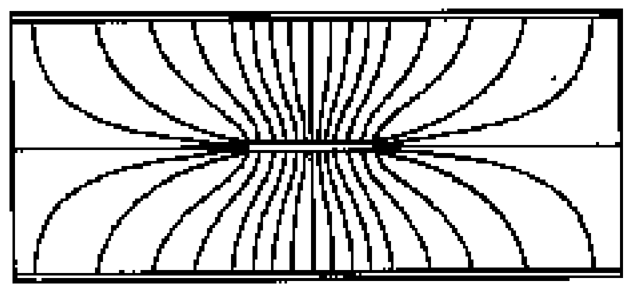


dielectric constant : 4.3.

Electric Field Configuration



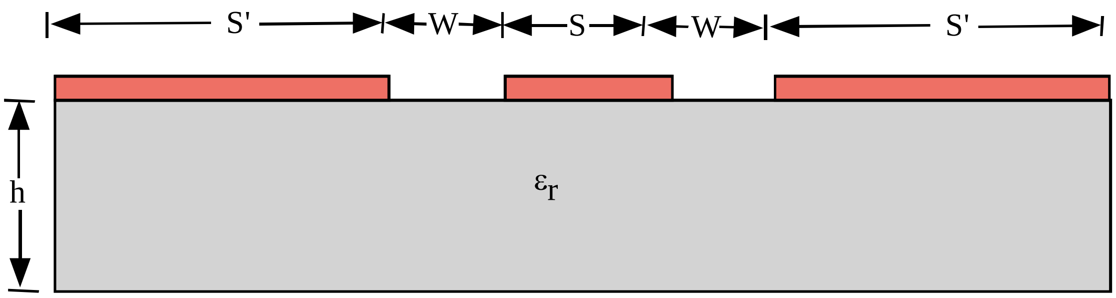
Microstrip



Stripline

<u>Consequence</u>: Wave propagation in stripline is closer to the TEM mode of propagation and the propagation of velocity is approximately $c/\sqrt{\varepsilon_r}$.

Coplanar Waveguide



$K(k)$: Complete Elliptic Integral of the first kind

$$k = \frac{S}{S+2W}$$

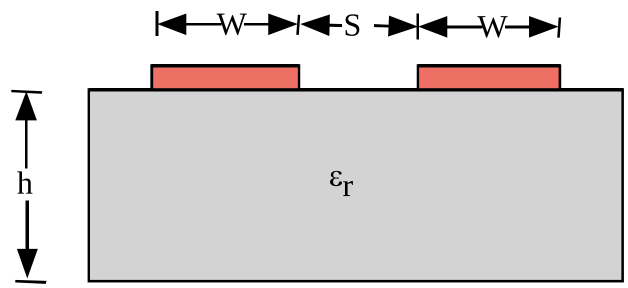
$$Z_{ocp} = \frac{30\pi}{\sqrt{\frac{\varepsilon_r + 1}{2}}} \frac{K'(k)}{K(k)} \text{ (ohm)}$$

$$K'(k) = K(k')$$

$$k' = (1-k^2)^{1/2}$$

$$v_{cp} = \left(\frac{2}{\varepsilon_r + 1}\right)^{1/2} c$$

Coplanar Strips



$$Z_{ocs} = \frac{120\pi}{\sqrt{\dfrac{\varepsilon_r + 1}{2}}} \frac{K'(k)}{K(k)} \text{ (ohm)}$$

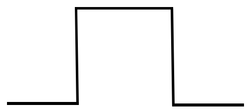
Qualitative Comparison

Characteristic	Microstrip	Coplanar Wguide	Coplanar strips
ε_{eff}*	~6.5	~5	~5
Power handling	High	Medium	Medium
Radiation loss	Low	Medium	Medium
Unloaded Q	High	Medium	Low or High
Dispersion	Small	Medium	Medium
Mounting (shunt)	Hard	Easy	Easy
Mounting (series)	Easy	Easy	Easy

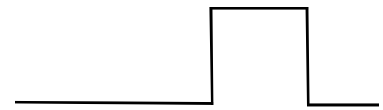
* Assuming ε_r=10 and h=0.025 inch

Dispersion & Velocities

$$\beta = 2\pi f\sqrt{LC}$$

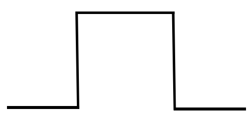


Ideal channel

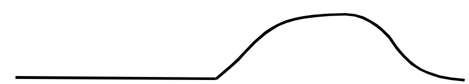


$$velocity = \frac{1}{\sqrt{LC}}$$

$$\beta = 2\pi f h(f)$$



Dispersive channel



$$velocity = g(f)$$

$$Phase\ velocity = \omega / \beta \qquad Group\ velocity = \left(\frac{d\beta}{d\omega}\right)^{-1}$$