"Thru-Reflect-Line": An Improved Technique for Calibrating the Dual Six-Port Automatic Network Analyzer

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Abstract—**In an earlier paper, the use of a "thru-short-delay" (TSD) technique for calibrating the dual six-port automatic network analyzer was described. Another scheme required only a length of precision transmission line and a "calibration circuit." The better features of these two somewhat different approaches have now been combined and the requirement for either a known short, or a "calibration circuit" eliminated. This paper will develop the theory for this new procedure.**

I. Introduction

THE APPLICATION of digital technology to the field of microwave measurement is perhaps best illustrated by the automatic network analyzer (ANA). In addition to the time-saving features, however, a major shift in measurement strategy has been introduced. In particular, the requirement for an ideal item of test equipment (e.g., reflectometer) has been replaced by a more complete theory in which deviations from the ideal are explicitly recognized, evaluated, and, in theory, eliminated from the final measurement result. The determination of these "deviations from the ideal" is generally referred to as the ANA "calibration."

In the case of the conventional ANA, which is based upon the four-port reflectometer, it is convenient [1] to visualize the calibration as shown in Fig. 1. Here the nonideal reflectometer has been modeled by an ideal one in cascade with a two-port "error box."

The properties of the ideal reflectometer can be chosen in such a way that its sidearm wave amplitudes b_3, b_4 are, respectively, equal to the emergent and incident waves at a fictitious "detector plane" which is the input to the two-port "error box." The parameters of the "error box" are provided by the ANA calibration. After this has been done, these parameters, in combination with the reading of the complex ratio detector, permit an "exact" determination of the signals at the measurement plane.

In order to make two-port measurements, it is convenient to introduce a second (nonideal) reflectometer and complex ratio detector which is then modeled in the same way as the first one. This is shown in Fig. 2.

The calibration requirement now calls for obtaining the parameters of error boxes A and B. One procedure for doing this is via the "thru-short-delay" (TSD) technique [2], [3]. In this technique, the error boxes A, B become the key components in three additional (fictitious) two-ports. The first of these is the cascade combination of A and B and is formed (Fig. 2) merely by connecting measurement planes 1 and 2 together (thru). The second two-port is a degenerate one and results from terminating the measurement planes with shorts (short). Finally, the substitution of an (unknown) length of (nonreflecting) line between the measurement planes 1, 2 yields the third two-port which is thus comprised of A, the length of line, and B in cascade (delay). Given the scattering parameters of these three two-ports, it is possible to solve for the individual scattering parameters of error two-ports A, B.

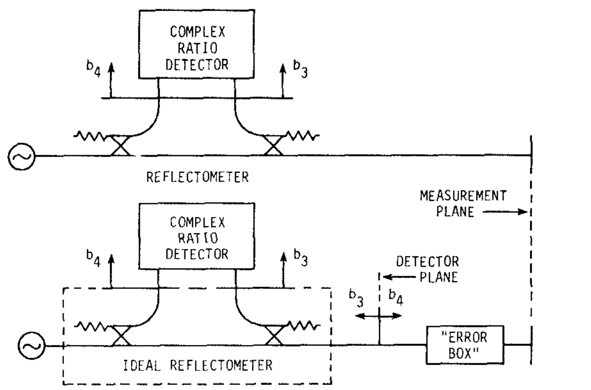

Fig. 1. It is convenient to model a reflectometer as an ideal one in cascade with a two-port "error box."

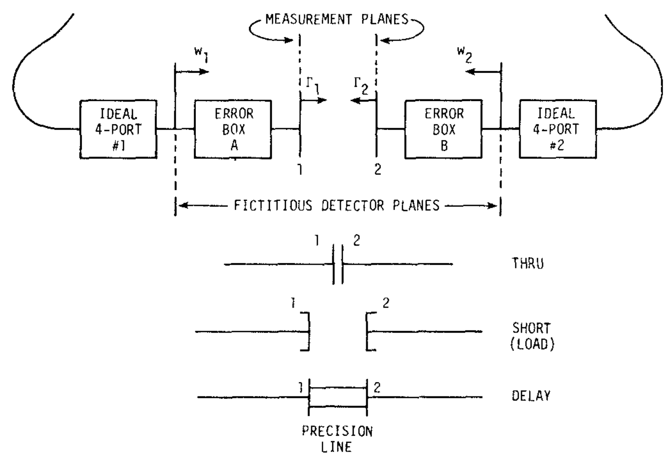

Fig. 2. For two-port measurements a second reflectometer is added and modeled as the first. Parameters of the two error boxes are obtained from measurements with the connections shown, where for TSD a known short is generally used, while for TRL any unknown highly reflecting termination may be used.

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The six-port reflectometer [4] differs from the four-port version in that it is based upon a six- rather than a four-port junction, and where the requirement for a complex ratio detector has been eliminated in exchange for four power meters or other detectors which yield amplitude information only. It has been shown ([5], [6]), however, that the six-port may be modeled by an equivalent (nonideal) four-port and complex ratio detector. Following this the TSD procedure may be applied as outlined above [7]. This is illustrated in Fig. 3. An alternative procedure for applying TSD to the dual six-port is found in [8].

Another calibration procedure [9] was based upon the use of a length of line as the only impedance standard. This method also required, however, the use of a "calibration circuit" which provided for terminating the two six-ports with equal impedances and at the same power levels.

The better features of these methods have now been combined in a technique known as "thru-reflect-line" (TRL). Here the "calibration circuit" is no longer required. As compared with TSD, the need for a short of known reflection has been eliminated. In its place a termination of unknown reflection (but different from zero!) is used to terminate, in turn, each of the six-ports. While a nominal short continues to be one of the more convenient choices for the unknown termination, the key point is that a value for its reflection is no longer required. Instead this is obtained as a by-product of the procedure. In addition, the word "line" has been substituted for "delay" as more descriptive of the technique. In common with TSD, the line length is arbitrary and unknown (other than being different from $\lambda/2$!). Moreover, it need not be free of dissipation.

There are three distinct parts to the associated theory: 1) the six-port to four-port reduction, 2) the determination of the scattering parameters of the two-ports which result from the thru, reflect, and line connections, and 3) the application of the TRL solution. These will be discussed in the order given.

II. The Six-Port to Four-Port Reduction

As described in an earlier paper [6], the four power measurements associated with the six-port represent an overdetermined set in that three of them determine the fourth to the extent of a choice between two possible values. It is convenient to express this result by means of the constraining relationship,

$$\begin{aligned}
&a(P_3/P_4)^2 + b\zeta^2(P_5/P_4)^2 + c\rho^2(P_6/P_4)^2 \\
&+(c-a-b)\zeta(P_3P_5/P_4^2) + (b-a-c)\rho(P_3P_6/P_4^2) \\
&+(a-b-c)\zeta\rho(P_5P_6/P_4^2) + a(a-b-c)(P_3/P_4) \\
&+b(b-a-c)\zeta(P_5/P_4) + c(c-a-b)\rho(P_6/P_4) + abc = 0
\end{aligned} \tag{1}$$

where a, b, c, ζ, and ρ are five real constants, whose values are intrinsic properties of the six-port. These five constants also characterize the reduction from a six-port to a four-port.

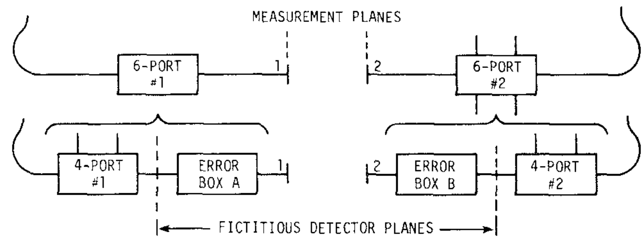

Fig. 3. The six-port reflectometer may also be reduced to an ideal four-port in cascade with an "error box."

Let P_3/P_4, P_5/P_4, and P_6/P_4 represent a point in a three-dimensional "P-space." Equation (1), which is of second degree, is thus represented by a quadric surface in P-space. However, the nine coefficients in (1) (i.e., a, $b\zeta^2$, $c\rho^2$, etc.), are functions of the five parameters $a \cdots \rho$. Moreover, it has been shown [6] that the quadric surface described by (1) is an elliptic paraboloid which is tangent to the planes $P_3/P_4=0$, $P_5/P_4=0$, $P_6/P_4=0$. The immediate task is to determine $a \cdots \rho$, which, in turn, permits the six-port to be reduced to a four-port.

In theory, it is only necessary to observe $P_3 \cdots P_6$ for five arbitrary and unknown terminations. These may then be inserted in (1) to obtain a set of simultaneous equations in $a \cdots \rho$. Unfortunately, however, these are of third degree, and unless a good initial estimate is in hand, the iterations required by standard numerical methods tend to be lengthy and may fail to yield the desired root. To obtain the initial estimate, it is useful to start with nine (or more) arbitrary terminations, which in practice are provided by the phase shifter in conjunction with the thru, reflect, and line connections, etc. The corresponding sets of values for $P_3 \cdots P_6$ are substituted in the equation

$$\begin{aligned}
A(P_3/P_4)^2 + B(P_5/P_4)^2 + C(P_6/P_4)^2 + D(P_3P_5/P_4^2) \\
+ E(P_3P_6/P_4^2) + F(P_5P_6/P_4^2) + G(P_3/P_4) \\
+ H(P_5/P_4) + I(P_6P_4) + 1 = 0
\end{aligned} \tag{2}$$

and the resulting set of linear equations solved for $A \cdots I$. In principle, (1) and (2) represent the same surface so that by equating the coefficients one has $A = a/abc$, $B = b\zeta^2/abc, \cdots$, $I = c(c-a-b)\rho/abc$. The system of equations, which results from equating the coefficients, is next solved for $a \cdots \rho$ as functions of $A \cdots I$. By substitution it can be confirmed that[1]

$$b = (2D - GH)/(2AH - DG) \tag{3}$$

$$c = (2E - GI)/(2AI - EG) \tag{4}$$

$$a = b + c + G/A \tag{5}$$

$$\zeta = +\sqrt{Bac} \tag{6}$$

$$\rho = +\sqrt{Cab} \tag{7}$$

[1] In addition, in the ideal case, there are constraining relations among the $A \cdots I$ such as $DE = AHI$, $CDH = BEI$, $DF = BGI$, $EF = CGH$, etc., so that a variety of alternative expressions for $a \cdots \rho$ is possible. However, since (3)–(7) are only used to obtain an initial starting point, this additional information, in this context at least, is of no obvious practical value.

where, by definition [6], ζ and ρ are positive. Because of measurement error, however, the paraboloid and tangency conditions will only be approximately satisfied by (2). In order to improve the accuracy, these values of $a \cdots \rho$ are used as the starting point for an iteratative solution using the multidimensional Newton method, to the nonlinear system of equations based on (1). This also provides a convenient method for using all of the observations, although the system is now overdetermined.

After these improved values $a \cdots \rho$ have been obtained, these together with the observed $P_3 \cdots P_6$ may be used to determine the complex ratio b_3/b_4 which, as already noted, is also the reflection coefficient at the input of the error box. Let $b_3/b_4 = w = u + jv$. It has been shown [6] that

$$u = (P_3 - \zeta P_5 + cP_4)/(2P_4\sqrt{c}) \tag{8}$$

$$v = (P_3 - \rho P_6 + (b - 2uu_2)P_4)/(2P_4 v_2) \tag{9}$$

where

$$u_2 = (b + c - a)/(2\sqrt{c}) \tag{10}$$

and

$$v_2 = \sqrt{b - u_2^2}\,. \tag{11}$$

(The signs of the radicals are to be taken as positive.) This completes the six-port to four-port reduction.

III. Determination of Scattering Parameters

The next task is to determine the scattering parameters of the three fictitious two-ports which are obtained from the thru, reflect, and line connections as indicated by the dashed box in Fig. 4. The emergent wave amplitudes b_1, b_2, (b_3, b_3') at terminals 1 and 2 are related to the incident waves a_1, a_2, (b_4, b_4') by the well-known scattering equations

$$b_1 = S_{11}a_1 + S_{12}a_2 \tag{12}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \tag{13}$$

where the S_{mn} are the scattering coefficients. Dividing the first of these by a_1, the second by a_2, and then eliminating the ratio a_1/a_2 between them yields

$$w_2 S_{11} + w_1 S_{22} - \Delta = w_1 w_2 \tag{14}$$

where

$$\Delta = S_{11}S_{22} - S_{12}S_{21} \tag{15}$$

and

$$w_1 = b_1/a_1 = b_3/b_4 \tag{16}$$

$$w_2 = b_2/a_2 = b_3'/b_4'. \tag{17}$$

Given the appropriate power meter readings and $a \cdots \rho$ for each six-port, the w_i may be obtained from (8) and (9). Excitation of the junction under three different conditions provides a system of linear equations based on (14) which may be solved for S_{11}, S_{22}, and Δ [10]. Following this, the product $S_{12}S_{21}$ may be obtained by use of (15). The technique does not permit the individual terms, S_{12} or S_{21} to be obtained, but this is not required. While a minimum of three measurements (under different excitation conditions, i.e., values of a_2/a_1) are required in each of the thru and line connections, only a single measurement is required with the reflect connection, since here $S_{12} = S_{21} = 0$. Moreover, in this case

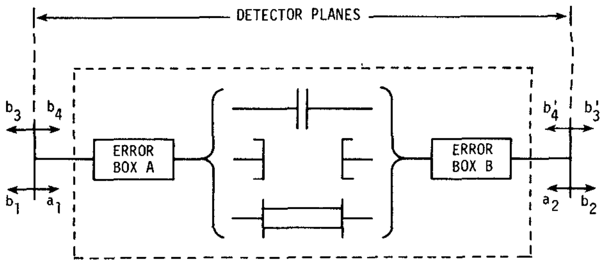

Fig. 4. The fictitious two-ports, from which the parameters of "error boxes" A, B are ultimately obtained, are formed from a "thru," "reflect," and "line" connection.

$$S_{ii} = w_i \qquad i = 1, 2. \tag{18}$$

IV. The TRL Solution

Given S_{11}, S_{22}, $S_{12}S_{21}$ for the three fictitious two-ports which result from the thru, reflect, and line connections, the final task is to determine, insofar as possible, the scattering parameters of the individual error boxes, A, B.

First, it is useful to solve (12) and (13) for b_1 and a_1 as functions of a_2 and b_2. This gives

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \frac{1}{S_{21}} \begin{pmatrix} -\Delta & S_{11} \\ -S_{22} & 1 \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = R \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}. \tag{19}$$

In (19) the matrix R is known as the *wave cascading matrix*. It has the important property that the R matrix for two or more two-ports in cascade is merely the product of the individual R matrices.

Let the cascading matrices of the error two-ports A, B be denoted by R_a, R_b, respectively, while R_t represents their cascade "thru" connection. Then

$$R_t = R_a R_b \tag{20}$$

while if R_d represents the "line" combination

$$R_d = R_a R_l R_b \tag{21}$$

where R_l represents the line which has been inserted.

Solving (20) for R_b gives

$$R_b = R_a^{-1} R_t \tag{22}$$

so that R_b may be obtained from R_a and R_t. Next, (22) is used to eliminate R_b from (21). This yields

$$TR_a = R_a R_l \tag{23}$$

where

$$T = R_d R_t^{-1} \tag{24}$$

and which may thus be found from the parameters of the thru and line "two-ports."

If γ and l represent respectively the propagation constant and length of the delay line, then assuming the line is nonreflecting,

$$R_l = \begin{pmatrix} e^{-\gamma l} & 0 \\ 0 & e^{\gamma l} \end{pmatrix}. \tag{25}$$

Finally, the elements of R_a and T will be represented by r_{ij} and t_{ij}, respectively. Expansion of (23) gives

$$t_{11}r_{11} + t_{12}r_{21} = r_{11}e^{-\gamma l} \tag{26}$$

$$t_{21}r_{11} + t_{22}r_{21} = r_{21}e^{-\gamma l} \tag{27}$$

$$t_{11}r_{12} + t_{12}r_{22} = r_{12}e^{\gamma l} \tag{28}$$

$$t_{21}r_{12} + t_{22}r_{22} = r_{22}e^{\gamma l}. \tag{29}$$

Next, taking the ratio of (26) to (27), and of (28) to (29) gives

$$t_{21}(r_{11}/r_{21})^2 + (t_{22} - t_{11})(r_{11}/r_{21}) - t_{12} = 0 \tag{30}$$

$$t_{21}(r_{12}/r_{22})^2 + (t_{22} - t_{11})(r_{12}/r_{22}) - t_{12} = 0. \tag{31}$$

The ratios (r_{11}/r_{21}) and (r_{12}/r_{22}) are thus both given by a solution of the same quadratic equation, where the coefficients are parameters of the T-matrix. The problem of which root represents which ratio will be deferred to the following section. For the moment it will be assumed that (r_{11}/r_{21}) and (r_{12}/r_{22}) have been determined. Following this, by taking the ratio of (29) to (27) one has

$$e^{2\gamma l} = \frac{t_{21}(r_{12}/r_{22}) + t_{22}}{t_{12}(r_{21}/r_{11}) + t_{11}}. \tag{32}$$

Ordinarily, a system of four equations (26)–(29) can be solved to yield four unknowns. At this point, three have been obtained: (r_{11}/r_{21}), (r_{12}/r_{22}), and $e^{2\gamma l}$. By taking the determinant of (23), and noting that the determinant of a product is equal to the product of the determinants, it is easy to show that

$$t_{11}t_{22} - t_{12}t_{21} = 1 \tag{33}$$

thus there are only three independent parameters in the T matrix. It follows that there are only three independent experimental observations in (26)–(29), and since they have been solved for three unknowns, there is nothing more that can be learned from this system of equations.

It is perhaps desirable to stop and make certain observations on the practical application of the theory developed thus far. By means of (30), (31), and (32), it is possible to obtain (r_{11}/r_{21}), (r_{12}/r_{22}), and $e^{2\gamma l}$ from *ratios* among the elements of T. This matrix, in turn, is given by (24). At this point, it appears that there may be a problem since a complete determination of R_t, for example, calls for the complete set of scattering parameters which results from A and B in cascade, while only S_{11}, S_{22}, and $S_{12}S_{21}$ have been measured. Examination of (19) shows, however, that R_t can be determined from this partial information except for an unknown constant multiplier for each of its elements (in this case $1/S_{21}$). The same observation holds for R_d. Comparing this with (24), the experimental procedure does yield the *ratios* between the elements of T, and fortunately this is all that is required.[2]

[2] If desired, the remaining factor in T could be obtained, a part from a $\pm$ sign, by the use of (33). However, there is no immediate use for this information.

To continue, the reflection coefficient w_1 which obtains at the fictitious detector plane for error two-port A is related to the reflection coefficient of the load Γ_l by

$$w_1 = \frac{a\Gamma_l + b}{c\Gamma_l + 1} \tag{34}$$

where[3]

$$a = r_{11}/r_{22} \tag{35}$$

$$b = r_{12}/r_{22} \tag{36}$$

$$c = r_{21}/r_{22}. \tag{37}$$

Comparing these definitions for a, b, c with (30) and (31) indicates that b and a/c have already been determined by the solution to the quadratic equations. To complete the determination of the parameters of "error box A" and thus effect the calibration of six-port 1, it is sufficient to determine a. Rearranging (34) one has

$$a = \frac{w_1 - b}{\Gamma_l(1 - w_1 c/a)}. \tag{38}$$

If Γ_l is known and different from zero, a is determined by (38) since the remaining parameters are also known. This represents the "TSD" solution although the formulation is somewhat different from what has been previously published. To complete the "TSD" solution the corresponding parameters of "error box B" may be readily obtained by use of (20).

For the "TRL" procedure, Γ_l is, by hypothesis, unknown. Before proceeding, it may be useful to recap what has been accomplished thus far. Apart from the determination of the line parameters, which has no bearing on the immediate problem, one has obtained b and c/a. In addition a and Γ_l are connected by (38) but both are as yet unknown.

Returning to (20), this may be written

$$r_{22}\rho_{22}\begin{pmatrix} a & b \\ c & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha & \beta \\ \gamma & 1 \end{pmatrix} = g\begin{pmatrix} d & e \\ f & 1 \end{pmatrix} \tag{39}$$

where[4] α, β, γ, ρ_{22} and d, e, f, g correspond, respectively, to a, b, c, r_{22} in R_b and R_t. Premultiplying (39) by R_a^{-1} and expanding it is easy to show that

$$\gamma = \frac{f - dc/a}{1 - ec/a} \tag{40}$$

$$\beta/\alpha = \frac{e - b}{d - bf} \tag{41}$$

and

$$a\alpha = \frac{d - bf}{1 - ec/a}. \tag{42}$$

Since b and c/a are already known, and d, e, and f can be obtained from ratios among the elements of R_t, then γ, β/α, and $a\alpha$ may be obtained from (40)–(42).

[3] The motivation in the choice of notation, as in a preceding section, has been to provide continuity with prior work. However, this creates a double use of the symbols a, b, c. These symbols refer to (34)–(37) for the rest of the paper.

[4] This definition of γ is not to be confused with its prior use in the exponential associated with the delay line!

For "error box B" the counterpart of (38) may be written

$$\alpha = \frac{w_2 + \gamma}{\Gamma_l(1 + w_2 \beta/\alpha)} \tag{43}$$

where the reversed signs and interchange between the roles of β and γ are due to the reversed direction associated with "error box B."

Eliminating Γ_l between (38) and (43) one has

$$a/\alpha = \frac{(w_1 - b)(1 + w_2 \beta/\alpha)}{(w_2 + \gamma)(1 - w_1 c/a)} \tag{44}$$

so that by combining (42) and (44)

$$a = \pm \sqrt{\frac{(w_1 - b)(1 + w_2 \beta/\alpha)(d - bf)}{(w_2 + \gamma)(1 - w_1 c/a)(1 - ec/a)}} \tag{45}$$

and

$$\alpha = \frac{(d - bf)}{a(1 - ec/a)}. \tag{46}$$

Apart from the choice of sign in (45), the requirement that Γ_l be known has been eliminated. As a practical matter, a nominal short continues to be a convenient choice for the unknown reflector, although for some applications it appears that an "open" may provide better repeatability. In either case, a nominal value for the argument of the "unknown" reflection is available and with the help of (38) this permits the proper sign choice to be made in (45).

At this point, the system has been calibrated in sufficient detail to permit the measurement of reflection coefficient, and the S-parameters of reciprocal two-ports. Returning to Fig. 2, it easily follows from (38) that the reflection coefficient Γ_1 at the left measurement plane is given by

$$\Gamma_1 = \frac{w_1 - b}{-cw_1 + a} \tag{47}$$

while a similar expression for Γ_2 may be obtained from (43). This capability now permits the evaluation of two-ports in the manner described in the preceding section for the "fictitious" ones by simply replacing w_1 and w_2 by Γ_1 and Γ_2 in (14).

In order to measure the properties of nonreciprocal two-ports, and power, some additional theory is required. Referring to Fig. 5, for clarity, the reference planes at the input and output of error box A will be relabeled 1, 2 as shown. The wave amplitudes a_2, b_2 at measurement plane 2 are expressed in terms of a_1, b_1 at the detector plane by

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = R_a^{-1} \begin{pmatrix} b_1 \\ a_1 \end{pmatrix} \tag{48}$$

where

$$R_a^{-1} = \frac{1}{r_{22}(a - bc)} \begin{pmatrix} 1 & -b \\ -c & a \end{pmatrix}. \tag{49}$$

The net power output from error box A is given by

$$P = |b_2|^2 (1 - |\Gamma_2|^2) \tag{50}$$

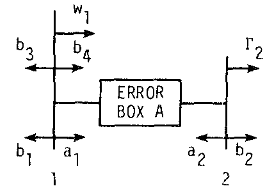

Fig. 5. It is convenient to redefine certain parameters associated with "error box" A as shown here.

where, with allowance for the change in notation, Γ_2 may be obtained from (47), while by use of (48) and (49)

$$|b_2|^2 = \frac{|a_1(a - cw_1)|^2}{|r_{22}(a - bc)|^2}. \tag{51}$$

Substituting (47) and (51) into (50), and noting [5] that $|a_1|^2 = |b_4|^2 \equiv P_4$, yields

$$P = \frac{P_4 |a - cw_1|^2}{|r_{22}|^2 |a - bc|^2} \left(1 - \left|\frac{w_1 - b}{a - cw_1}\right|^2\right). \tag{52}$$

The only system parameter which is undetermined at this point is $|r_{22}|$. It may be found by observing the system response P_4 and w_1 with P indicated by a "standard" power meter. After this has been done, the power delivered to other terminations may be found by use of (52). Moreover, by a simple extension, $|a_2|^2$ may also be obtained.

As explained in [10] the evaluation of a nonreciprocal two-port calls for measurement of the ratio of the incident waves at the two-port, as well as the reflection coefficients. Since for the "thru" connection the powers at the measurement planes are the same (except for a sign reversal), taking the ratio of (52) to a similar expression for error two-port B yields the ratio $|r_{22}|/|\rho_{22}|$. This suffices for a determination of the ratio of the incident wave amplitudes, which in turn yields $|S_{12}|$ and $|S_{12}|$ of the nonreciprocal two-port. If the individual phases are also required, the method may be further developed along the lines described in [10].

V. The Problem of Root Choice

Comparison of (30) and (31) with (35)–(37) indicates that the two roots of either quadratic are a/c and b. Another possibility, which must be considered, is that one of the roots is equal to both a/c and b. The latter, however, requires $a = bc$. This, in turn, is only possible if for error box A, $S_{12}S_{21} = 0$ which obviously cannot be true for a practical measurement system. Thus the two roots a/c and b are distinct.

Next, it is useful to consider the ratio of the absolute values of the two roots, $|b|/|a/c| = |bc|/|a|$. Provided that $|bc|/|a| < 1$, then $|b| < |a/c|$ and this will serve as a basis for root choice. To demonstrate that this is ordinarily the case, it is convenient to expand (19),

$$b_1 = r_{12}\left(\frac{r_{11}}{r_{12}} a_2 + b_2\right) \tag{53}$$

$$a_1 = r_{22}\left(\frac{r_{21}}{r_{22}} a_2 + b_2\right). \tag{54}$$

In (53), r_{12} may be considered a scale factor while r_{11}/r_{12} is a measure of how tightly b_1 is coupled to a_2 as compared to b_2. A similar observation holds for (54).

Now, in most measurement systems, including the six-port, it is desirable to have one response nominally proportional to the wave amplitude b_2 incident upon the item under test. This calls for $|r_{21}/r_{22}| \ll 1$. In the case of the four-port reflectometer, it is further required that the remaining response be nominally proportional to a_2 which requires $|r_{11}/r_{12}| \gg 1$. For the six-port, however, $|r_{11}/r_{12}| \sim 1$ is more typical. In any event, except for gross departures from the usual design objectives, one has

$$\left|\frac{r_{21}}{r_{22}}\right| < \left|\frac{r_{11}}{r_{12}}\right| \tag{55}$$

which requires

$$|b| < |a/c|. \tag{56}$$

An alternative, and in principle more definitive, test may be obtained from (32). Solving (30) and (31) for r_{11}/r_{21}, and r_{12}/r_{22}, in terms of the t_{ij}, and substituting in (32) gives,

$$e^{2\gamma l} = \frac{t_{11} + t_{22} \pm R}{t_{11} + t_{22} \mp R} \tag{57}$$

where

$$R = \sqrt{(t_{11} - t_{22})^2 + 4t_{12}t_{21}} \tag{58}$$

and where the choice of sign depends upon how the root assignment has been made. The two corresponding values for $e^{2\gamma l}$, as given by (57), however, are reciprocals of each other. Thus as an alternative criterion, the root assignment should be made such that $|e^{2\gamma l}| < 1$.

As noted, this is in theory a more definitive test, however, since the difference between $|e^{2\gamma l}|$ and unity is usually small, it is possible for measurement error to mask this effect unless good quality power detectors are used.

Finally, the argument of $e^{2\gamma l}$, as obtained from (57), depends upon the root assignment. If the line length differs in a known direction from the design center of a quarter wavelength, this could also serve as a basis for root choice.

Ordinarily, however, the first criterion is more than adequate in a practical application.

VI. Summary

The foregoing provides the theoretical basis for a calibration procedure which promises to have a major impact upon the microwave art. In particular, the long-standing role of the short or open as an "impedance standard," if not obsolete, at least needs to be reexamined. This is in agreement with the observation by Allred [11] that "open and short circuits are not standards in the true sense." As brought out in a companion paper [12], the TRL technique, quite possibly, represents the best experimental method for evaluating a short or an open which has yet been devised. Moreover, the values thus obtained provide a useful monitor of the system performance.

Instead of a collection of offset shorts, or the single short in conjunction with sliding terminations, which have characterized the prior art, the only items now required to calibrate the measurement system are a section of the transmission line or waveguide in which the measurements are to be made (which implicitly defines Z_0), plus a termination for which a nominal value for the argument of the reflection is known. The method is thus immediately applicable to the solid dielectric line.

While the method is, in principle, equally applicable to a dual four-port reflectometer, the latter has found but limited application to date because of the requirement for duplicating the complex ratio detector, which is usually a costly item, or alternatively, the need for an elaborate switching scheme and its associated errors. The advantages of the TRL procedure are thus more readily realized in the six-port environment; this represents another advantage of the six-port technique.

For a report on the performance of a measuring system in which this calibration technique plays a key role, the reader is referred to a companion paper [12]. While the effect of residual imperfections, such as line reflections, remains to be evaluated, the cited results [12] leave little room for doubt that the method is capable of high accuracy!

This calibration technique may also be applied to the problem of adapter evaluation. Returning again to Fig. 4, let error boxes A, B represent a pair of adapters. If the entire assembly of Fig. 4 is inserted between the measurement planes of Fig. 3, then a_1, b_1, a_2, b_2 (Fig. 4) may be measured by the six-ports where it is assumed the calibration has already been completed. The procedure thus calls for a repeat of the TRL technique with the adapters in place and yields the adapter parameters in place of those of a fictitious error box.

Finally, it may be noted that while TRL may be considered an extension of TSD, another variant of TSD, namely "super TSD," also exists [13]. The latter, however, was developed to take account of a crosstalk problem which, for all practical purposes, is nonexistant in the six-port environment.

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Performance of a Dual Six-Port Automatic Network Analyzer

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***Abstract*—Initial results of the performance of an experimental dual six-port automatic network analyzer operating in the 2–18-GHz range with thermistor-type power detectors are given. The imprecision in measuring reflection coefficients of one-port devices, or the scattering parameters of two-port devices is 4×10^{-5}, excluding connector repeatability. At 3 GHz, the imprecision in measuring attenuation varies from 0.0003 dB at low values of attenuation to 0.15 dB at 60 dB. The systematic error in measuring attenuation appears to be less than the imprecision. The systematic error in measuring reflection coefficient appears to be less than 0.0004. Additional systematic errors caused by changes in the calibration constants over a 20-week period were observed to be less than 0.003 dB in attenuation and less than 0.002 in reflection coefficient.**

I. Introduction

An EXPERIMENTAL automatic network analyzer (ANA) incorporating two six-port reflectometers has been constructed at NBS for measuring the network parameters of one-port and two-port devices from 2–18 GHz. The precision, accuracy, and stability of the ANA are now being investigated. Results obtained so far are summarized in this paper.

II. System Description

A block diagram of the dual six-port ANA is shown in Fig. 1. Measurements of the reflection coefficient Γ of one-port devices are made by connecting the termination to either six-port reference plane. The network parameters of a two-port device are measured by inserting the two-port between the two six-port reflectometers. The theory of operation and a description of the basic system have already been published [1].

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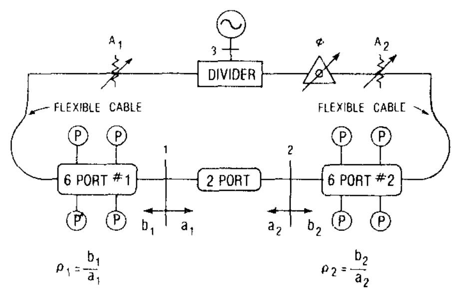

Fig. 1. Block diagram of a dual six-port automatic network analyzer, where P indicates power detector. When a termination is connected to either measurement plane, ρ_1 or ρ_2 becomes the usual reflection coefficient Γ_1 or Γ_2 of the termination.

The accuracy of a six-port measurement is primarily a function of the quality of the connectors, quality of the standard transmission line used in the calibration, and of the resolution, stability, and linearity of the four sidearm power detectors. Greatest accuracy has been obtained with NBS Type IV power meters [2] using thermistor type power detectors. The thermistor detectors are housed in an aluminum block whose temperature is held constant to 0.01°C. The present system has a phase-locked source whose output power is externally leveled. Connectors at the measurement planes are GPC-7. The system is controlled by a programmable calculator.

III. Calibration Techniques

The technique used to calibrate the dual six-port ANA is the "thru-reflect-line" (TRL) technique [4] augmented by including a nominal 10-dB pad in the set of measurements. The steps in the calibration are shown in Fig. 2 and outlined below.