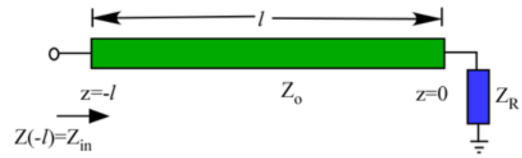
ECE 451 – Automated Microwave Measurement

Formula Sheet

Transmission line equations



$$\Gamma(-l) = \Gamma_R e^{-j2\beta l} = \frac{Z_R - Z_0}{Z_R + Z_0} e^{-j2\beta l}$$

$$Z(-l) = Z_0 \left[\frac{Z_R + jZ_0 tan\beta l}{Z_0 + jZ_R tan\beta l}\right]$$

$$Z(z) = Z_0 \frac{1 + \Gamma(z)}{1 - \Gamma(z)} \qquad \Gamma(z) = \frac{Z(z) - Z_0}{Z(z) + Z_0}$$

$$V(z) = V_+ e^{-j\beta z}\left[1 + \Gamma_R e^{+2j\beta z}\right]$$

$$I(z) = \frac{V_+}{Z_0} e^{-j\beta z}\left[1 - \Gamma_R e^{+2j\beta z}\right]$$

$$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{v}$$

Mason's non-touching loop rule

$$T = \frac{\sum_{i=1}^{N} P_i\left[1 + \sum_{j=1}^{M}(-1)^j L(j)^{(i)}\right]}{1 + \sum_{j=1}^{M}(-1)^j L(j)}$$

$$= \frac{P_1\left[1 - \sum L(1)^{(1)} + \sum L(2)^{(1)} - \ldots\right] + P_2\left[1 - \sum L(1)^{(2)} + \sum L(2)^{(2)} - \ldots\right] + \ldots}{1 - \sum L(1) + \sum L(2) - \sum L(3) + \ldots}$$

P_i – ith path from source to sink.

$L(j)$ – jth order loop.

$L(j)^{(i)}$ – jth order loop which doesn't touch ith path.

S – Z parameter transformation

$$\boldsymbol{Z} = \boldsymbol{Z_0}(\boldsymbol{U} + \boldsymbol{S})(\boldsymbol{U} - \boldsymbol{S})^{-\boldsymbol{1}}$$

$$\boldsymbol{S} = (\boldsymbol{Z} + \boldsymbol{Z_0 U})^{-\boldsymbol{1}}(\boldsymbol{Z} - \boldsymbol{Z_0 U})$$

$\boldsymbol{U}$**:** unit matrix

S – ABCD parameters conversion

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2S_{21}} \begin{bmatrix} (1 + S_{11})(1 - S_{22}) + S_{12}S_{21} & Z_0(1 + S_{11})(1 + S_{22}) - S_{12}S_{21} \\ \frac{1}{Z_0}\left[(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}\right] & (1 - S_{11})(1 + S_{22}) - S_{12}S_{21} \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} A + \frac{B}{Z_0} - CZ_0 - D & 2(AD - BC) \\ 2 & -A + \frac{B}{Z_0} - CZ_0 + D \end{bmatrix}, \quad \Delta = A + \frac{B}{Z_0} + CZ_0 + D$$

S – T parameters conversion

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} T_{12}T_{22}^{-1} & T_{11} - T_{12}T_{21}T_{22}^{-1} \\ T_{22}^{-1} & -T_{21}T_{22}^{-1} \end{bmatrix}$$

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} S_{12} - S_{11}S_{22}S_{21}^{-1} & S_{11}S_{21}^{-1} \\ S_{22}S_{21}^{-1} & S_{21}^{-1} \end{bmatrix}$$

Alternate S – T formulas:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} T_{21}T_{11}^{-1} & T_{22} - T_{21}T_{11}T_{11}^{-1} \\ T_{11}^{-1} & -T_{12}T_{11}^{-1} \end{bmatrix}$$

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} S_{21}^{-1} & -S_{22}S_{21}^{-1} \\ S_{11}S_{21}^{-1} & S_{12} - S_{11}S_{22}S_{21}^{-1} \end{bmatrix}$$

S Parameters of Transmission Line of Characteristic Impedance Z_c

Reference system has impedance Z_o

$$S_{11} = S_{22} = \frac{(1 - X^2)\Gamma}{1 - \Gamma^2 X^2} \qquad S_{12} = S_{21} = \frac{(1 - \Gamma^2)X}{1 - \Gamma^2 X^2}$$

$$\Gamma = \frac{Z_C - Z_0}{Z_C + Z_0} = Q \pm \sqrt{Q^2 - 1} \qquad Q = \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}}$$

$$X = e^{-j\beta l} = \frac{S_{11} + S_{21} - \Gamma}{1 - (S_{11} + S_{21})\Gamma}$$

Reference Plane Extension

$$S' = \Phi S \Phi$$

where S is for the DUT, S' is for the system with the port extensions.

$$\Phi = \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix}$$

ϕ_1 and ϕ_2 are the phase shifts in ports 1 and 2 respectively

Special Case for Lossless System

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

$$|S_{22}|^2 + |S_{12}|^2 = 1$$

If in addition, the network is reciprocal: $S_{12} = S_{21}$, the network is symmetrical: $S_{11} = S_{22}$.