1. A parallel-plate waveguide is filled with a dielectric $\varepsilon = 9\varepsilon_o$ and $\mu = \mu_o$. The separation between the plates is 2.5 cm. Determine the propagating modes for a wave of frequency 5 GHz. For each propagating mode, fill in the following table. Waves are incident on the plates with an angle θ with respect to the normal to the plates. . v_{pz} and v_g are the phase and group velocities respectively.

Mode	f_c (GHz)	θ (deg)	v_{pz} (m/s)	v_g (m/s)
TEM	0	90	0.1×10^9	0.1×10^9
TE_1	2	66.42	0.1091×10^9	0.0916×10^9
TM_1	2	66.42	0.1091×10^9	0.0916×10^9
TE_2	4	86.86	0.16×10^9	0.06×10^9
TM_2	4	36.86	0.16×10^9	0.06×10^9

$$\lambda = \frac{v_p}{f}, \quad f_c = \frac{m}{2a\sqrt{\mu\varepsilon}}, \quad \theta = \cos^{-1}\frac{\lambda}{\lambda_c}$$

$$\lambda_c = \frac{v_p}{f_c} = \frac{2a}{m}, \quad \lambda_g = \frac{v_{pz}}{f}, v_{pz} = \frac{v_p}{\sqrt{1-\frac{f_c^2}{f^2}}}, \quad v_g = v_p\sqrt{1-\frac{f_c^2}{f^2}}$$

2. An air-filled rectangular waveguide has largest side a = 1 cm. Find the length of the side b, necessary to have identical cutoff frequency for the TE_{20} and TM_{11} modes.

$$Set\ \frac{c}{a} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

$$\frac{c^2}{a^2} = \frac{c^2}{4}\left(\frac{1}{a^2} + \frac{1}{b^2}\right)$$

$$\frac{1}{a^2} - \frac{1}{4a^2} = \frac{1}{4b^2} \Rightarrow b = \frac{a}{\sqrt{3}} = \frac{1}{\sqrt{3}} = 0.57\ cm$$

3. An air-filled, X-band (8-12GHz), WC-94 circular waveguide has an inner diameter of 2.383 cm.

(a) Determine the cutoff frequencies of the TE_{11}, TM_{01}, and TE_{21} modes.

(b) Find the modes that will propagate through this guide at 10 GHz.

(c) Find the frequency range within which only the TE_{11} mode propagates.

Solution:

(a) For the TE_{11} mode, we have

$$f_{cTE_{11}} = \frac{1.8412c}{2\pi a} = \frac{1.8412 \times 3 \times 10^8}{\pi \times 2.383 \times 10^{-2}} \simeq 7.38GHz$$

For the TM_{01} and TE_{21} modes, we have

$$f_{cTM_{01}} = \frac{2.4049}{1.8412} f_{cTE_{11}} \simeq 9.64\ \text{GHz}$$

$$f_{cTE_{21}} = \frac{3.0542}{1.8412} f_{cTE_{11}} \simeq 12.2\ \text{GHz}$$

(b) At 10 GHz, the only mode that will propagate through this guide are the TE_{11} and TM_{01}

(c)The frequency range over which only the dominant TE_{11} mode propagates along the guide can be found as

Frequency range f_{cTM01}-f_{cTE11} = 9.64-7.38 = 2.26 GHz.