

## REQUIREMENTS OF PHYSICAL CHANNELS

### 1. Introduction

Frequency-domain measurement of network parameters is an incredibly powerful technique. When the networks are linear and time-invariant, frequency-domain measurements can completely characterize these systems. From these measurements, the time-domain behavior of these systems can be predicted accurately through the use of transform techniques. In a previous experiment, we studied that the step and impulse responses of a linear network could be obtained from a broadband measurement of its scattering parameters.

For actual networks, the time-domain signals must exhibit certain properties that are associated with a physical system. For instance, a signal in the time domain must be real, stable and causal. Violation of these properties would lead to non-physical situations. These properties lead to some important constraints to the time-domain signals and their associated frequency-domain responses. This lab will examine these properties from the frequency-domain measurements of scattering parameters of linear systems. The subject of the passivity, stability, and causality of scattering parameter models is a very rich field. There is a basic but excellent paper on the topic at: <http://ieeexplore.ieee.org/document/4358038/>

#### A. Time-Domain Response

The time-domain response of a network must be real. From transform theory, we know that this correlates to the relationship

$$V(-f) = V^*(f) \Leftrightarrow v(t) \text{ real} \quad (1)$$

Since negative frequencies are not measured in a network analyzer, this has limited relevance to measurements; however, the relationship must be applied in determining the time-domain response from the measurement data. When we use S-parameters in a transient simulation, the simulator uses the above property of Hermitian symmetry to reproduce the negative frequencies and perform an inverse Fourier transform to generate an impulse response for the S-parameter measurements. The simulator then performs discrete convolution on this impulse response with whichever stimulus you provided to generate the output.

#### B. Passivity and Stability

The scattering matrix of a linear N-port,  $S$  can define a dissipation matrix  $P$  given by:

$$P = U - S^{*T} S \quad (2)$$

where the superscripts \* and T indicate a complex conjugate and transpose matrix respectively. Passivity insures that the system will always be stable provided that it is connected to another passive network. For passivity,

- The determinant of the dissipation matrix must be greater than or equal to zero
- The determinant of the principal minors must be greater than or equal to zero.

If any of the conditions are violated, then the network is active. The limiting case is when the dissipation matrix is zero. This corresponds to a lossless network. For a lossless network,

$$S^{*T} S = U \quad (3)$$

In other words, the S matrix of a lossless network is unitary. In terms of a two-port lossless network this leads to:

$$|S_{11}|^2 + |S_{21}|^2 = 1 \quad (4)$$

$$|S_{22}|^2 + |S_{12}|^2 = 1 \quad (5)$$

If in addition the network is reciprocal, then,  $S_{12}=S_{21}$ , which leads to:

$$|S_{11}| = |S_{22}| \quad (6)$$

### C. Causality

Time-domain signals associated with physical networks must be causal. This means that the signal is null for  $t < 0$ . In other words,

$$h(t) = 0, \quad t < 0;$$

The time-domain function can be considered as the superposition of an even function and an odd function:

$$h_e(t) = \frac{1}{2} [h(t) + h(-t)] \quad (7)$$

$$h_o(t) = \frac{1}{2} [h(t) - h(-t)] \quad (8)$$

Also, it can be shown that the transform of the transfer function is given by:

$$H(f) = H_e(f) - j\hat{H}_e(f) \quad (9)$$

and

$$H(f) = \hat{H}_o(f) + jH_o(f) \quad (9)$$

where  $H(f)$ ,  $H_e(f)$ , and  $H_o(f)$  are the Fourier transforms of  $h(t)$ ,  $h_e(t)$ , and  $h_o(t)$  respectively and  $\hat{H}_e(f)$  and  $\hat{H}_o(f)$  are the Hilbert transforms of  $H_e(f)$  and  $H_o(f)$ . The Hilbert transform relationship is given by:

$$\hat{x}(t) = x(t) * \frac{1}{\pi t} = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau} d\tau \quad (10)$$

From this we see that in order for causality to exist, the imaginary part of the transform must be related to the real part through the Hilbert transform and vice versa. When dealing with discrete data points, the discrete version of the Hilbert transform relationship is more appropriate. It is given by:

$$\hat{f}_k = \begin{cases} \frac{2}{\pi} \sum_{n \text{ odd}} \frac{f_n}{k - n}, & k \text{ even} \\ \frac{2}{\pi} \sum_{n \text{ even}} \frac{f_n}{k - n}, & k \text{ odd} \end{cases} \quad (11)$$

So now your discrete S-parameter measurement (say  $S_{21}$ ) is a function of frequency and can be written as a vector.

Your S-parameter files are frequency-domain measurements, so in these equations,  $f_n$  is the discrete frequency data point that you have from your S parameters.  $\hat{f}_n$  is the Hilbert transform point.

## 2. Procedure

Equipment Used:

Agilent Advanced Design System 2016.01

MathWorks MATLAB R2016a

Previously-acquired measurement data for 2-port microstrip line with discontinuities (allfreq.s2p).

Previously-acquired measurement data for 2-port microstrip line with discontinuities with low-frequency data removed (highfreq.s2p).



**Figure 1: The 2-port microstrip line with discontinuities measured in previous experiments.**

### A. Time-Domain Response

The purpose of this section is twofold. The first is to demonstrate how S-parameter data files can be used in a transient (time-domain) simulation. The second is to point out how limited measurement points (particularly at low frequencies) might adversely affect the quality of the results of these transient simulations. Use the transient simulator in ADS as shown in **Figure 2** below to construct a set-up to simulate Time Domain Reflectometry (TDR). The idea is that you are providing a step function to the S-parameter model of your line, which you will terminate with a short (by grounding it). You are making two circuits so that you can easily plot both .s2p files side-by-side.

You can find the VtStep (this is a voltage step) in Sources-Time Domain, the SNP1 (this is a Touchstone Data Item) in Data Items, and the Tran1 (this is the Transient Simulation Engine) in Simulation-Transient. Be sure to set the Rise Time of step function to 0.1 nsec and the MaxTimeStep to 1 psec. Run this simulation for both Touchstone files to generate plots for each like the one shown in **Figure 3**.

Required plots (1):

- Transient Simulation for allfreq.s2p and transient simulation for highfreq.s2p plotted on the same axes with the stimulus step function.

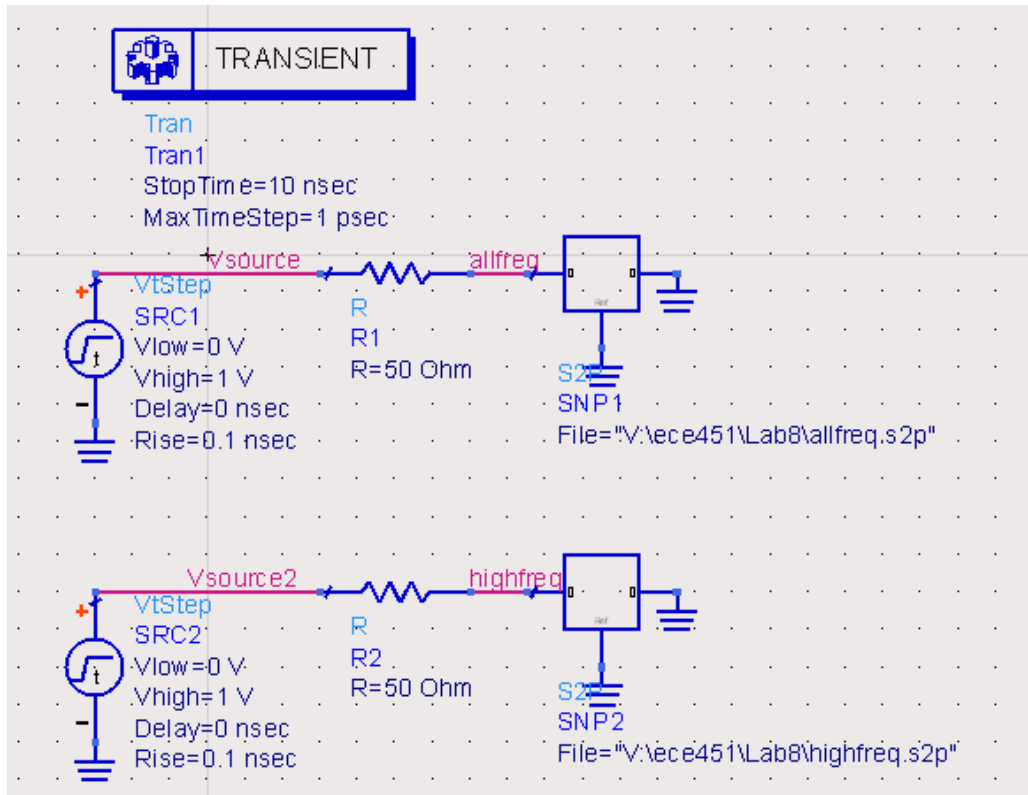


Figure 2: Using ADS to simulate TDR using the transient simulation engine.

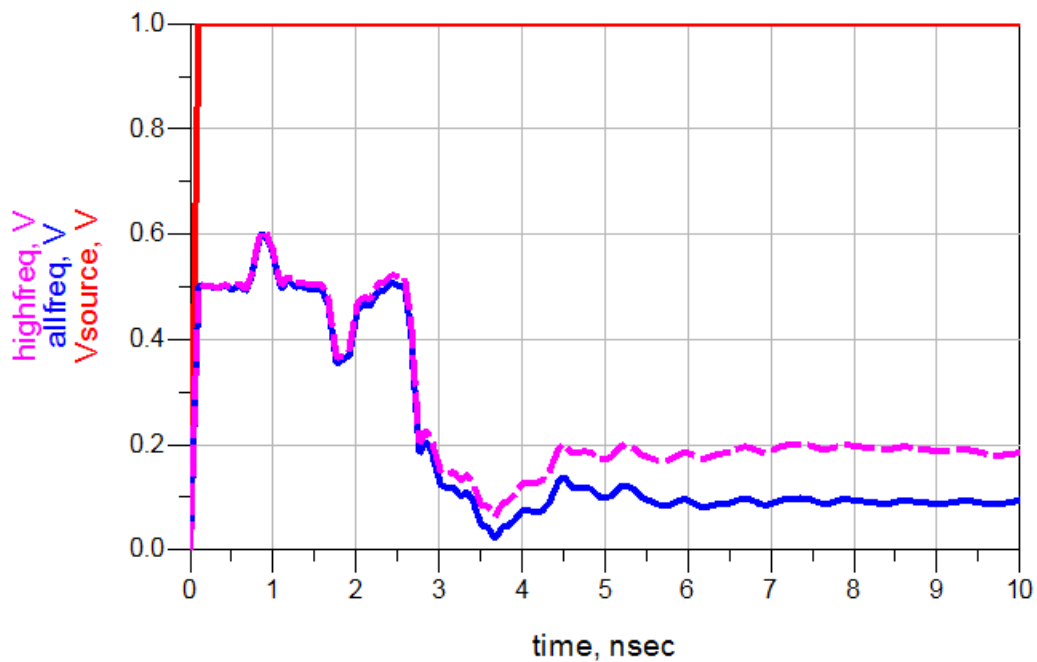


Figure 3: Sample plot of data for part A.

## B. Passivity and Stability

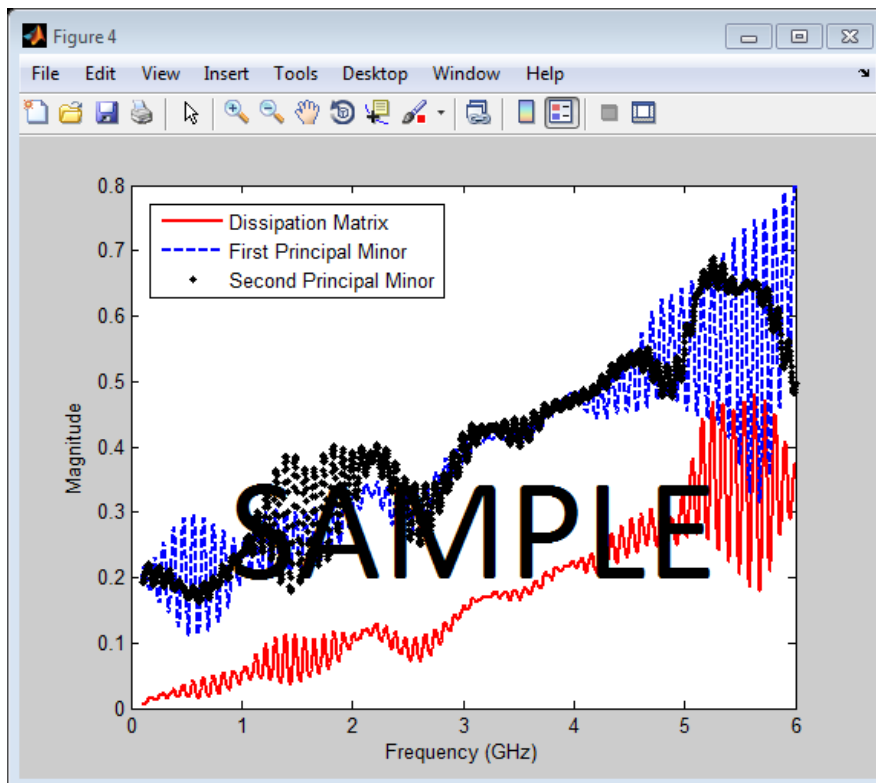
NOTE: In the next two sections, you will be using MATLAB. The key to becoming confident with MATLAB is to use its ‘help’ keyword to learn about different functions. For example, typing ‘help length’ on the command line and pressing enter will display information about the ‘length’ function and how to use it.

NOTE: MATLAB script files have the extension ‘.m’. This extension is also used by Mathematica, another mathematical program which we are not using. Mathematica is the default program for ‘.m’ files so when you try to open these files from Windows, Mathematica will be opened. Therefore, either select ‘Open with...’ in Windows and select MATLAB or open the ‘.m’ file from the MATLAB program.

NOTE: MATLAB indexes its arrays starting with 1. So for an array  $a$  of length  $n$ , the indices of the array go from 1 to  $n$ .

Write a Matlab script named Sub\_P to plot the determinants of the dissipation matrix, its first principal minor, and its second principal minor (how do you calculate these minors?) at every frequency for allfreq.s2p. You will need to load S-parameter into Matlab workspace first, the Matlab RF Toolbox will provide such tools. Refer to [here](#) for details.

An incomplete skeleton of the function Sub\_P is provided on the website. Feel free to use this to start writing your Sub\_P function. Generate a figure like that of **Figure 4**.



**Figure 4: Passivity check plot.**

Required plot (1):

- Plots (all on one figure) of the determinant of the dissipation matrix, the determinant of the first principal minor, and the determinant of the second principal minor, all versus frequency.

### C. Causality

Extend your code from Part B to write a script named `Sub_dht` to calculate the discrete Hilbert transform as in (11) to compare the Hilbert transform of the imaginary part of the  $S_{12}$  to the real part of  $S_{12}$  and the Hilbert transform of the real part of the  $S_{12}$  to the imaginary part of the  $S_{12}$  for both the `allfreq.s2p` and the `highfreq.s2p` files.

An incomplete skeleton of the `Sub_dht` function is also provided on the website. Feel free to use this to start writing your `Sub_dht` function.

Required plots (4):

For `allfreq.s2p`:

- Hilbert transform of the imaginary part of the  $S_{12}$  and the real part of  $S_{12}$ , both vs. frequency.
- Hilbert transform of the real part of the  $S_{12}$  and the imaginary part of  $S_{12}$ , both vs. frequency.

For `highfreq.s2p`:

- Hilbert transform of the imaginary part of the  $S_{12}$  and the real part of  $S_{12}$ , both vs. frequency.
- Hilbert transform of the real part of the  $S_{12}$  and the imaginary part of  $S_{12}$ , both vs. frequency.

Required code attachments (included in your homework write-up, do **not** submit separate `.m` files) (2):

- Include a copy of your passivity code, including your function that implements the determinant calculation for the  $P$  matrix and its minors.
- Include a copy of your causality code, including your function that implements the discrete Hilbert transform.

## TA Theory and Conclusion Questions

### Theory:

1. A network analyzer can measure only positive frequencies, but in order to convert frequency data into time-domain data, we have to use the IFFT, which requires both positive and negative frequency data points. How, given that we are dealing with data that we acquired in the real world, might we get the negative frequency data that we need? Explain any signal processing concepts behind your answer. (Hint: data measured in the *real* world has certain properties.)
2. For a two-port device, what are the principal minors?

### Conclusion

1. How does your TDR simulation from Part A compare to the one you generated in Experiment 5?
2. How do the TDR traces from Part A for allfreq.s2p and highfreq.s2p differ?
3. Is the S-parameter model used in Part B passive? How do your data support your answer?
4. Which of the S-parameter models used in Part C is causal, if either? How do your data support your answer?

Here is a link to an excellent white-paper with information about using the Hilbert transform to evaluate causality:

<http://cp.literature.agilent.com/litweb/pdf/5990-5266EN.pdf>