NAME ______**Solutions**______________________________ NETID ____________

MIDTERM EXAM

ECE 451 March 10, 2021

Instructions: Write your name and NetID where indicated. This examination consists of 4 problems. This is an open-book and open-notes exam. Use 50 Ω as the reference impedance for all measurement systems.

| Problem 1 (25 pts) | Problem 2 (25 pts) | Problem 3 (25 pts) | Problem 4 (25 pts) | Total (100 pts) |
|---|---|---|---|---|
| | | | | |

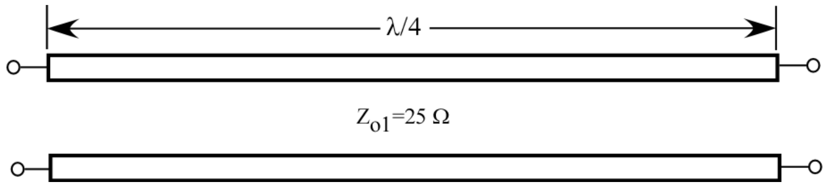
Mason's non-touching loop rule:

$$T = \frac{P_1\left[1-\sum L(1)^{(1)} + \sum L(2)^{(1)} - \ldots\right] + P_2\left[1-\sum L(1)^{(2)} + \sum L(2)^{(2)} - \ldots\right] + \ldots}{1-\sum L(1) + \sum L(2) - \sum L(3) + \ldots}$$

1. The matrices below are measured scattering parameters. In each case, indicate the characteristics that apply by checking in the appropriate boxes.

| | $\begin{bmatrix} 0.8 & 0.6 \\ 0.6 & j0.8 \end{bmatrix}$ | $\begin{bmatrix} 0 & 0.1 \\ 10 & 0 \end{bmatrix}$ | $\begin{bmatrix} 0 & e^{-(\alpha+j\beta)d} \\ e^{-(\alpha+j\beta)d} & 0 \end{bmatrix}, \alpha, \beta > 0$ |
|---|---|---|---|
| **active** | **No** | **Yes** | **No** |
| **reciprocal** | **Yes** | **No** | **Yes** |
| **lossy** | **No** | **No** | **Yes** |

2. For the transmission line shown below, write the scattering parameter matrix as measured on a 50-Ω network analyzer.




**Solutions**

$$S_{11} = \frac{\left(1-X^2\right)\Gamma}{1-X^2\Gamma^2} \quad \text{and} \quad S_{21} = \frac{\left(1-\Gamma^2\right)X}{1-X^2\Gamma^2}$$

with $\Gamma = \dfrac{Z_{o1}-Z_o}{Z_{o1}+Z_o}$ and $X = e^{-j\frac{2\pi}{\lambda}l}$

$$X = e^{-j\frac{2\pi}{\lambda}\frac{\lambda}{4}} = e^{-j\pi/2} = -j$$

$$\Gamma = \frac{25-50}{25+50} = \frac{-25}{100} = -\frac{1}{4}$$

$$S_{11} = \frac{\left(1-(-j)^2\right)(-1/3)}{1-(-j)^2(1/9)} = \frac{-2(1/3)}{1+1/9} = -0.6$$

$$S_{21} = \frac{(1-1/9)(-j)}{1-(-j)^2(1/9)} = \frac{-8j/9}{1+1/9} = -j0.8$$

$$S = \begin{bmatrix} -0.6 & -j0.8 \\ -j0.8 & -0. \end{bmatrix}$$

3. A transmission line of characteristic impedance $Z_o$, length $d$, propagation velocity $v$, and propagation constant $\beta$ is terminated with an open.

(a) Find the input impedance $Z_{in}$. Express your answers in terms of $Z_o$, $\beta$, and $d$

(b) Draw a rough sketch of $Z_{in}/Z_o$ for $\beta d$ ranging from 0 to $\pi$ and label the frequency bands where the transmission line looks capacitive and where it looks inductive.

(c) At what frequencies does this open transmission line look like a short circuit?

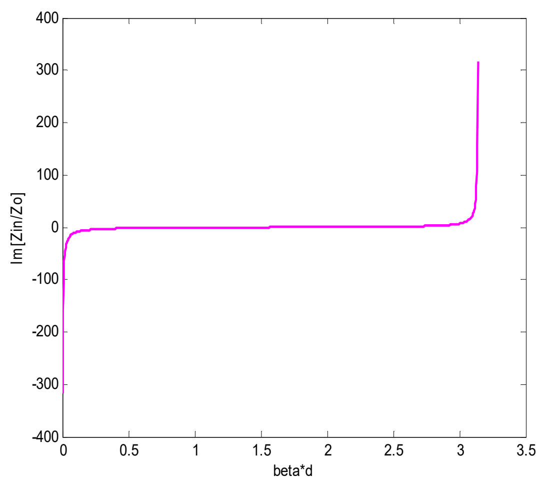
**Solutions**

(a) For a transmission line of length $d$, we have:

$$Z_{in} = Z_o \left\{ \frac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} \right\}$$

If $Z_L \to \infty$, then $Z_{in} = -jZ_o \operatorname{cotan} \beta d$

(b)




(c ) The TL looks like a short for $\beta d = \frac{(2n+1)\pi}{2}, n = 0,1,2,...$ or

$$\frac{2\pi fd}{v} = \frac{(2n+1)\pi}{2}, n = 0,1,2,...$$

where $v$ is the propagation velocity in the TL. This leads to: $f = \frac{(2n+1)v}{4d}, n = 0,1,2,...$

If $n = 0, f = v/4d$

4. A lossless transmission line has the following per unit length parameters: $L$ = 80 nH-m$^{-1}$ , $C$ = 200 pF·m$^{-1}$ Consider a traveling wave on the transmission line with a frequency of 1 GHz.

(a) What is the attenuation constant?

(b) What is the phase constant?

(c) What is the phase velocity?

(d) What is the characteristic impedance of the line?

(e) When the dielectric in the transmission line is replaced with air ($\varepsilon_r$ = 1), the capacitance per unit length of the line is found to be $C$(air) =50 pF.m$^{-1}$ . What was the effective relative permittivity of the dielectric?

(a) $\alpha = 0$

(b) $\beta = \omega\sqrt{LC} = 25.13 \; radians/m$

(c) $v_p = \omega/\beta = \dfrac{1}{\sqrt{LC}} = 2.5\times10^8 \; m/s$

(d) $Z_o = \sqrt{L/C} = 20\,\Omega$

(e) $\varepsilon_r = \dfrac{200}{50} = 4$