ECE 451
Advanced Microwave Measurements

ABCD Parameters

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ABCD - Parameters

\[ V_A = AV_B + BI_B \]
\[ I_A = CV_B + DI_B \]
ABCD -Parameters

\[ V_A = V_1 \]
\[ V_B = V_2 \]
\[ I_A = I_1 \]
\[ I_B = -I_2 \]

Relationship with Z parameters is obtained by first expressing ABCD parameters in terms of Z parameters.
ABCD - Parameters

From

\[ V_A = Z_{11} I_A - Z_{12} I_B \]
\[ V_B = Z_{21} I_A - Z_{22} I_B \]

We get

\[ A = \frac{Z_{11}}{Z_{21}} \quad B = \frac{\Delta}{Z_{21}} \]
\[ C = \frac{1}{Z_{21}} \quad D = \frac{Z_{22}}{Z_{21}} \]

\[ \Delta = Z_{11} Z_{22} - Z_{12} Z_{21} \]
ABCD -Parameters

\[
Z_{11} = \frac{A}{C} \quad Z_{11} = \frac{(AD - BC)}{C}
\]

\[
Z_{21} = \frac{1}{C} \quad Z_{22} = \frac{1}{C}
\]

For a reciprocal network, \(Z_{21} = Z_{12}\), therefore

\[
AD - BC = 1 \quad \text{Reciprocity condition for ABCD parameters}
\]
When cascading two-ports, it is best to use ABCD parameters. Put voltage and currents in cascadable form with the input variables in terms of the output variables

$$ABCD = (ABCD)_1 \cdot (ABCD)_2$$
Scattering Transfer Parameters

In T-Parameters, traveling waves at the input are related to those at the output

\[ b_1 = S_{11}a_1 + S_{12}a_2 \quad \quad \quad \quad \quad \quad b_1 = T_{11}a_2 + T_{12}b_2 \]

\[ b_2 = S_{21}a_1 + S_{22}a_2 \quad \quad \quad \quad \quad \quad a_1 = T_{21}a_2 + T_{22}b_2 \]

\[
\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} T_{12}T_{22}^{-1} & T_{11} - T_{12}T_{21}T_{22}^{-1} \\ T_{22}^{-1} & -T_{21}T_{22}^{-1} \end{pmatrix}
\]

\[
\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} S_{12} - S_{11}S_{22}S_{21}^{-1} & S_{11}S_{21}^{-1} \\ -S_{22}S_{21}^{-1} & S_{21}^{-1} \end{pmatrix}
\]

T parameters can be cascaded \( T = T_A \cdot T_B \)