An Introduction to X-Parameters*

ECE 451: Advanced Microwave Measurements

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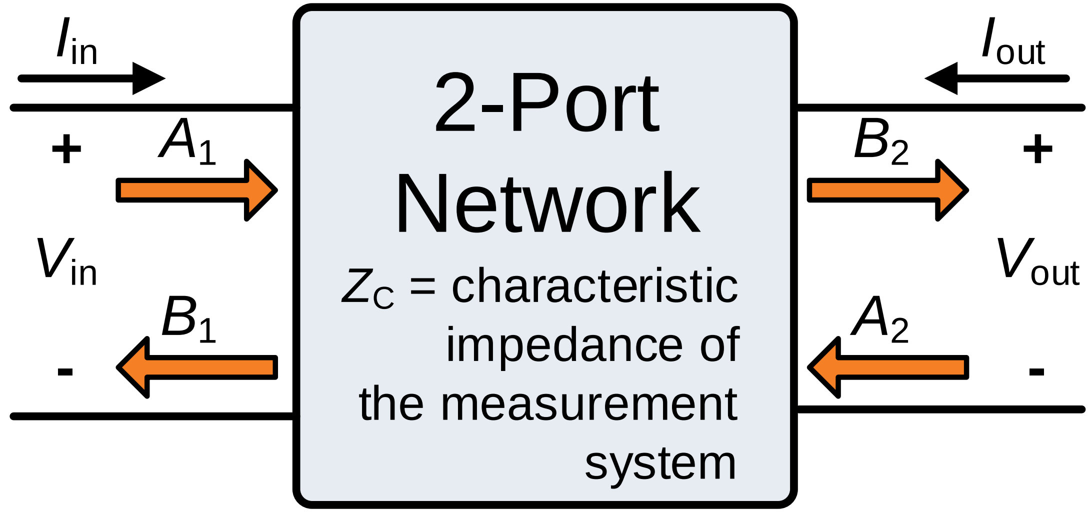
Scattering Parameters

$$A_1 = \frac{\left(V_{in} + Z_C I_{in}\right)}{2\sqrt{Z_C}} \qquad A_2 = \frac{\left(V_{out} + Z_C I_{out}\right)}{2\sqrt{Z_C}}$$

$$B_1 = \frac{\left(V_{in} - Z_C I_{in}\right)}{2\sqrt{Z_C}} \qquad B_2 = \frac{\left(V_{out} - Z_C I_{out}\right)}{2\sqrt{Z_C}}$$

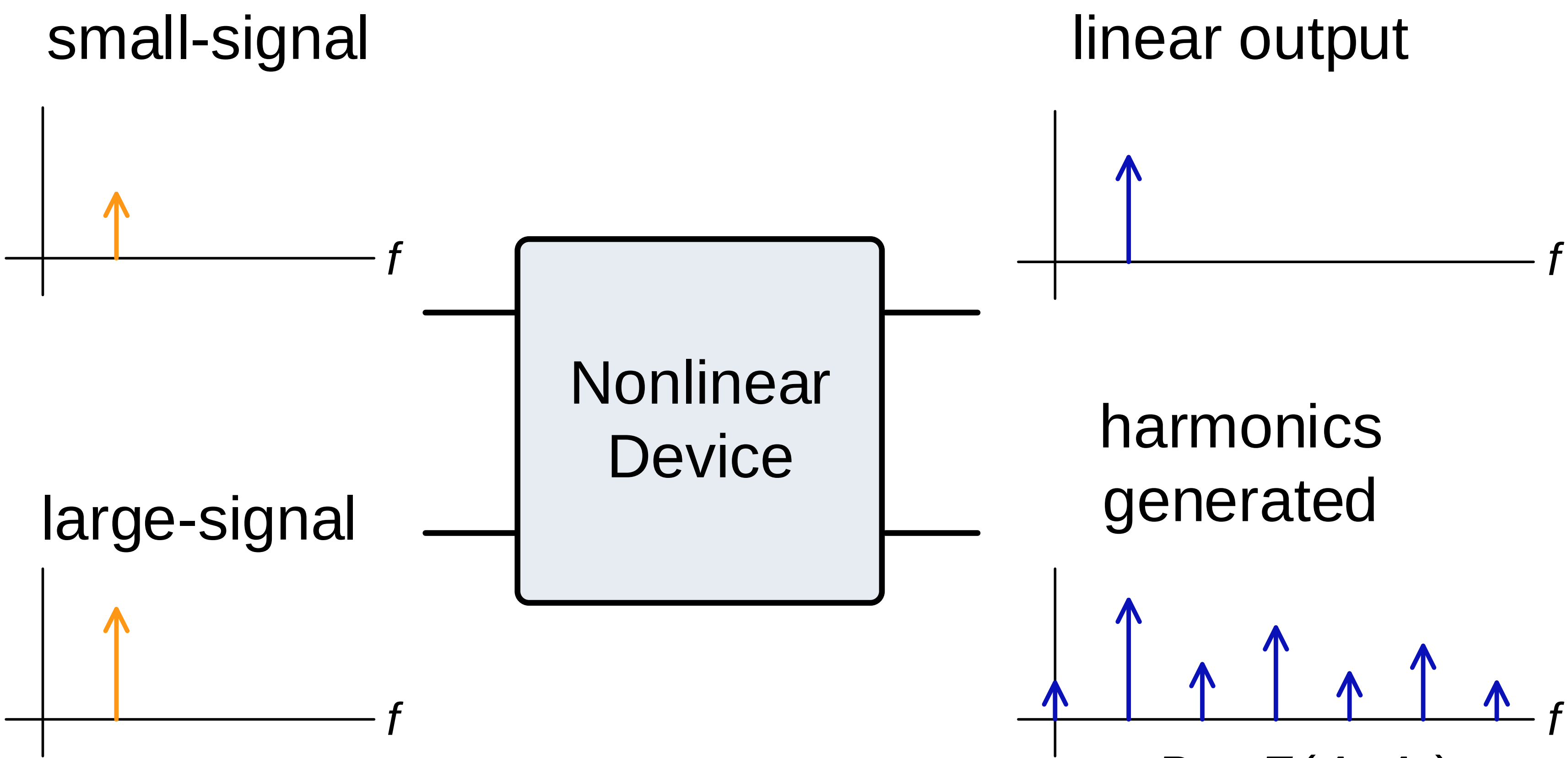
$$B_1 = S_{11}A_1 + S_{12}A_2$$

$$B_2 = S_{21}A_1 + S_{22}A_2$$



- Models all linear time-invariant behavior.
- Can model time-invariant nonlinear devices in the small-signal case.
- What about the large-signal case?

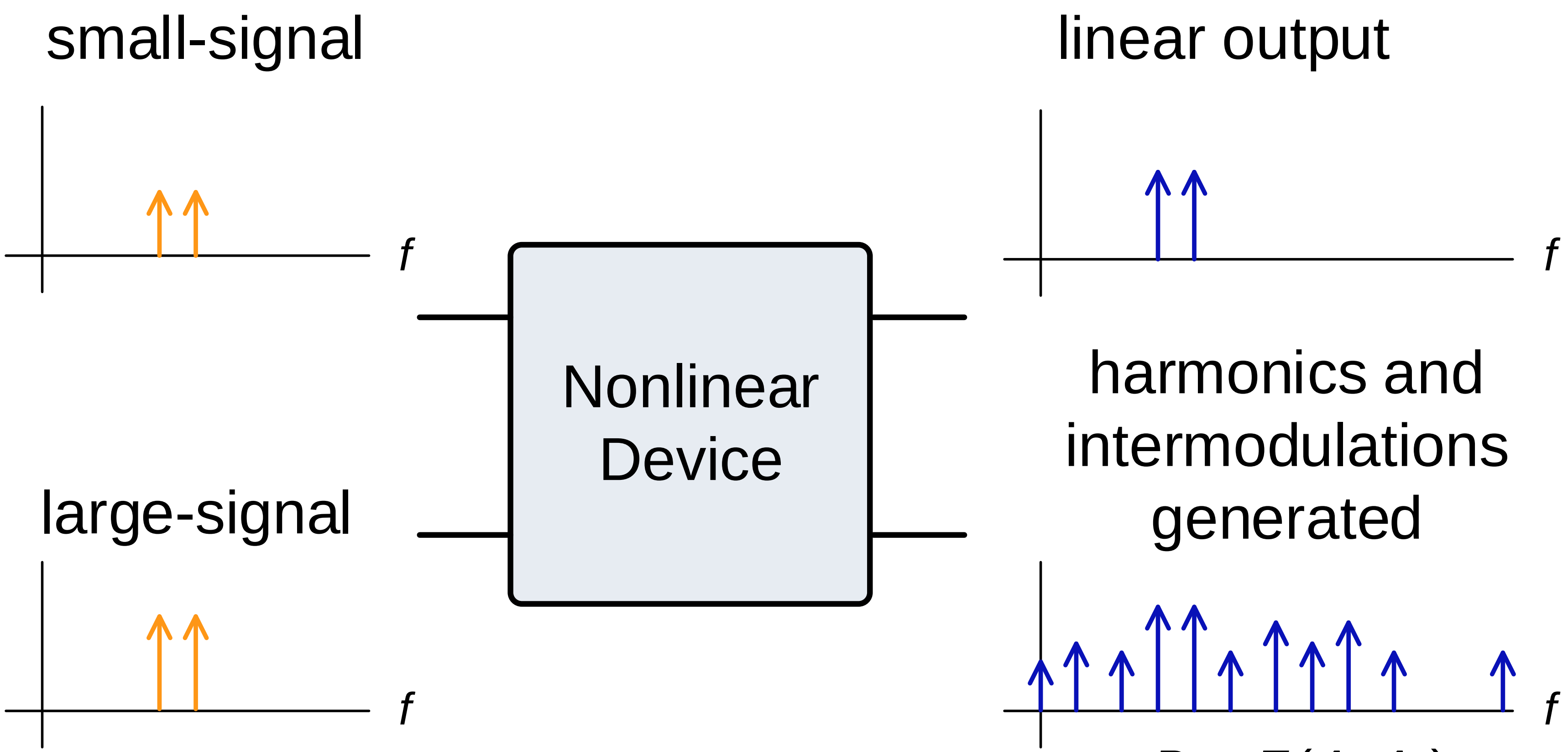
Nonlinear Functions with Single-Tone Stimuli



$$B_1 = F_1(A_1, A_2)$$

$$B_2 = F_2(A_1, A_2)$$

Nonlinear Functions with Multi-Tone Stimuli

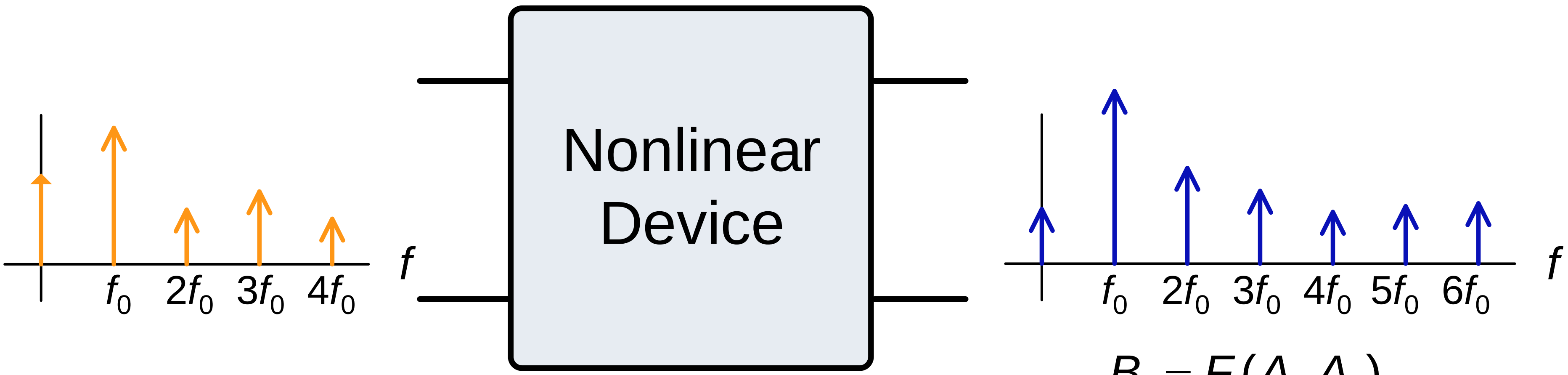


$$B_1 = F_1(A_1, A_2)$$

$$B_2 = F_2(A_1, A_2)$$

Nonlinear Functions with Commensurate Tone Stimuli

- A set of pure tones are commensurate if all the tones in the set are located on a frequency grid $f_k = kf_0$ defined by f_0, called the fundamental.
- Output tones will all land on the same frequency grid and have a same common period.

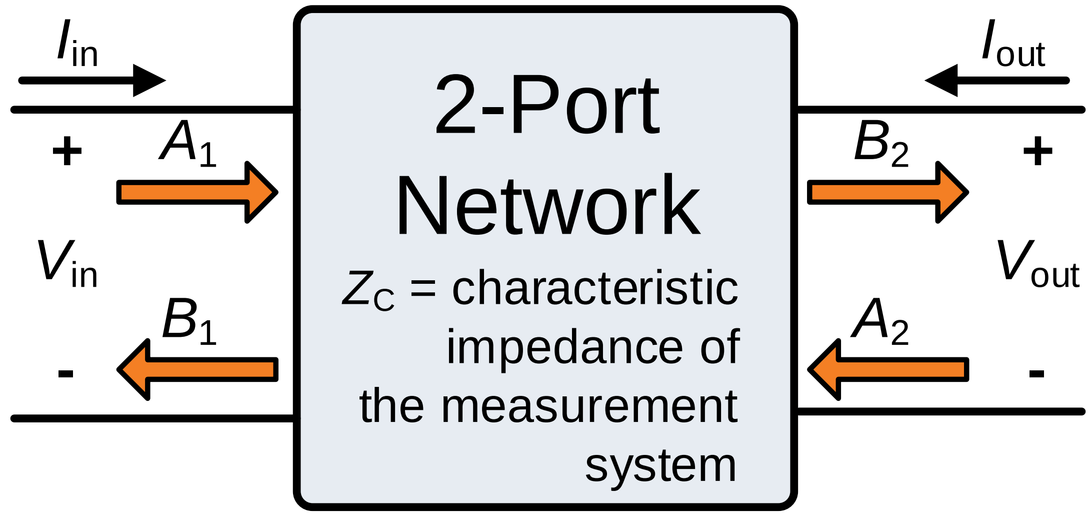


$$B_1 = F_1(A_1, A_2)$$

$$B_2 = F_2(A_1, A_2)$$

Nonlinear Scattering Waves

- Break incident and scattered waves into their commensurate tone components, called pseudowaves.

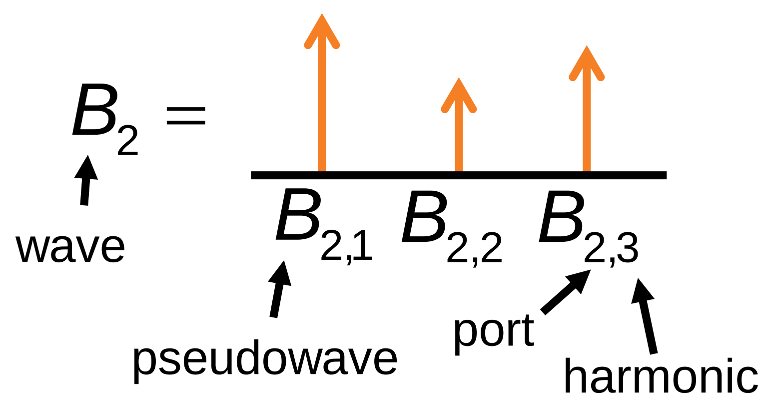


$$B_1 = F_1(A_1, A_2)$$

$$B_2 = F_2(A_1, A_2)$$

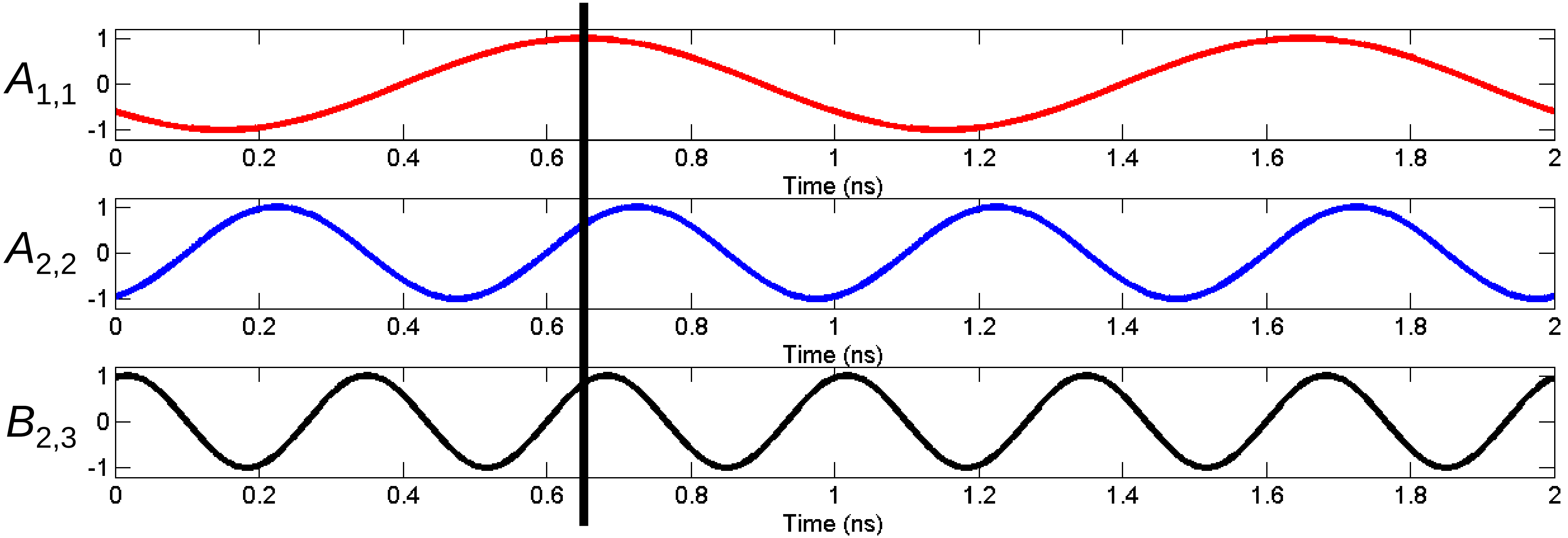
$$B_{1,k} = F_{1,k}(A_{1,1}, A_{1,2}, A_{1,3}, \ldots A_{2,1}, A_{2,2}, A_{2,3}, \ldots)$$

$$B_{2,k} = F_{2,k}(A_{1,1}, A_{1,2}, A_{1,3}, \ldots A_{2,1}, A_{2,2}, A_{2,3}, \ldots)$$

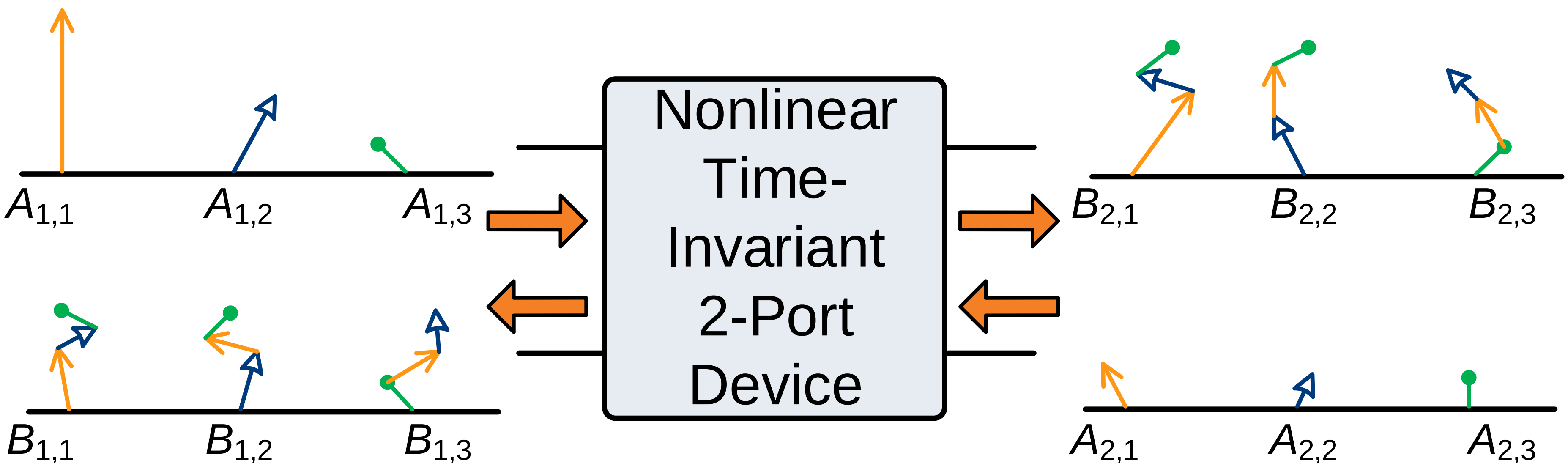


Cross-Frequency Phase for Commensurate Tones

- Defined as the phase of each pseudowave when the fundamental, $A_{1,1}$, has zero phase.
- $B_{2,3}$ can be related to $A_{2,2}$ in magnitude and phase.



Nonlinear Scattering Functions



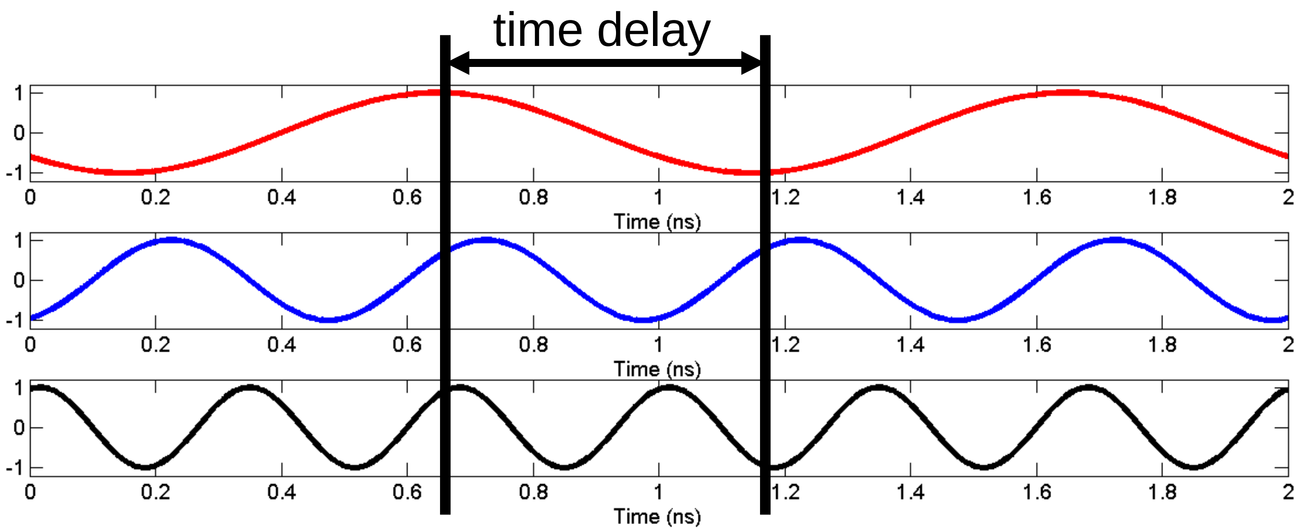
$$B_{p,k} = F_{p,k}(A_{1,1}, A_{1,2}, A_{1,3}, \ldots, A_{2,1}, A_{2,2}, A_{2,3}, \ldots)$$

- Scattered pseudowave determined by a complicated time-invariant scattering function that depends on the magnitude and phase of each incident pseudowave.

Time-Invariance Property of Nonlinear Scattering Function

$$F_{p,k}(A_{1,1}e^{j\theta}, A_{1,2}(e^{j\theta})^2, A_{1,3}(e^{j\theta})^3, \ldots)$$
$$= F_{p,k}(A_{1,1}, A_{1,2}, A_{1,3}, \ldots)(e^{j\theta})^k$$

- Shifting all of the inputs by the same time means that different harmonic components are shifted by different phases.



f_0 180º phase shift

$2f_0$ 360º phase shift

$3f_0$ 540º phase shift

Defining Phase Reference

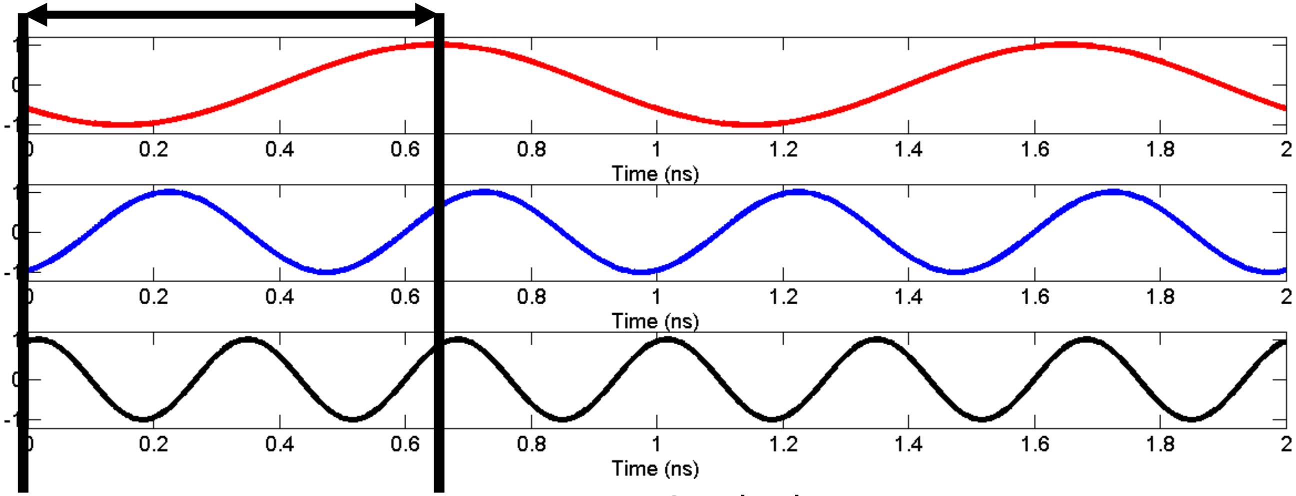
- Can use time-invariance to separate magnitude and phase dependence of one incident pseudowave.

$$B_{p,k} = F_{p,k}(A_{1,1}, A_{1,2}, A_{1,3}, \ldots)$$
$$= F_{p,k}(|A_{1,1}|, A_{1,2}P^{-2}, A_{1,3}P^{-3}, \ldots)P^{k}$$

using

$$P = \frac{A_{1,1}}{|A_{1,1}|} = e^{j\arg(A_{1,1})}$$

Shifting reference to zero phase of $A_{1,1}$.



Commensurate Tones X-Parameter Formalism

- Define $X_{p,k}^{(FB)}(|A_{1,1}|, A_{1,2}P^{-2}, A_{1,3}P^{-3}, \ldots)$

$$= F_{p,k}(A_{1,1}, A_{1,2}, A_{1,3}, \ldots)P^{-k}$$

⇩

$$B_{p,k} = X_{p,k}^{(FB)}(|A_{1,1}|, A_{1,2}P^{-2}, A_{1,3}P^{-3}, \ldots)P^{k}$$

- Still difficult to characterize this nonlinear term.
- If only one incident pseudowave, $A_{1,1}$, is large then the other smaller inputs can be linearized about the large-signal response of $F_{p,k}$ to only $A_{1,1}$.

Linearization of $F_{p,k}$ about $A_{1,1}$

$$B_{p,k} = F_{p,k}\left(\left|A_{1,1}\right|, A_{1,2}P^{-2},\ldots,A_{1,K}P^{-K},\ldots\right)P^{k}$$

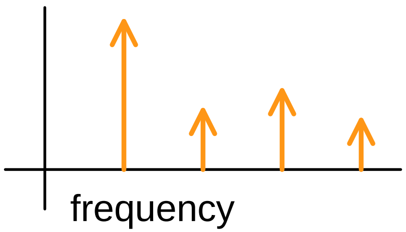
$$\approx F_{p,k}\left(\left|A_{1,1}\right|,0,\ldots,0,\ldots\right)P^{k}$$

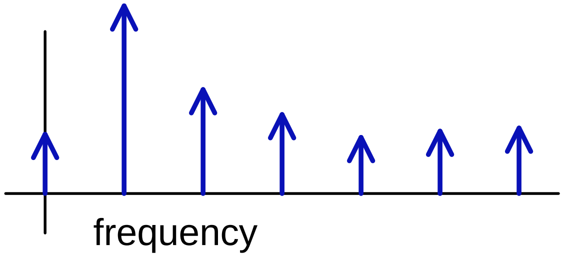
$$+\sum_{\substack{q=1,l=1\\(q,l)\neq 1}}^{q=N,l=K}\left[\underbrace{\left.\frac{\partial F_{p,k}}{\partial\left(A_{q,l}P^{-l}\right)}\right|_{\left|A_{1,1}\right|}}_{X^{(S)}_{p,k;q,l}} A_{q,l}P^{k-l}+\underbrace{\left.\frac{\partial F_{p,k}}{\partial\left(\left(A_{q,l}P^{-l}\right)^{*}\right)}\right|_{\left|A_{1,1}\right|}}_{X^{(T)}_{p,k;q,l}} A_{q,l}{}^{*}P^{k+l}\right]$$

1-Tone X-Parameter Formalism

Incident Waves Approximates **Scattered Waves**

$$B_{p,k} = X^{(FB)}_{p,k}\left(\left|A_{1,1}\right|, A_{1,2}P^{-2}, A_{1,3}P^{-3}, \ldots\right)$$

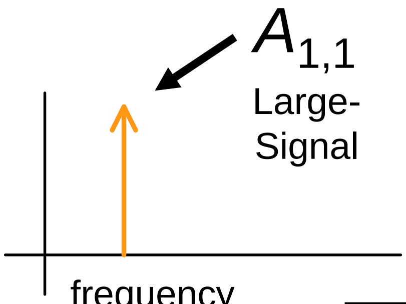


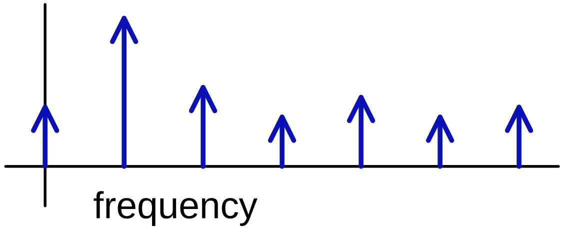


Nonlinear Mapping

$$\approx$$

$$X^{(FB)}_{p,k}\left(\left|A_{1,1}\right|, 0, 0, \ldots\right)$$

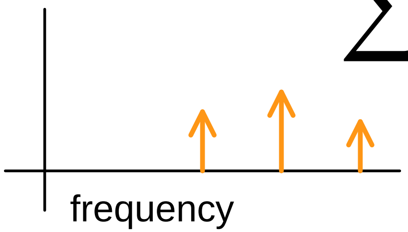


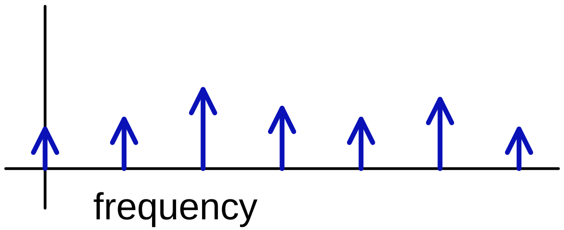


Simple Nonlinear Mapping

$$+$$

$$\sum\left[X^{(S)}_{p,k;q,l}\cdot A_{q,l} + X^{(T)}_{p,k;q,l}\cdot A^{*}_{q,l}\right]$$





Nonanalytic Harmonic Superposition

1-Tone X-Parameter Formalism

$$B_{p,k} \approx \underline{X^{(FB)}_{p,k}} \cdot P^{k} + \sum_{\substack{q=1,\, l=1 \\ (q,l)\neq(1,1)}}^{q=N,\, l=K} \underline{X^{(S)}_{p,k;q,l}} \cdot A_{q,l} \cdot P^{k-l} + \sum_{\substack{q=1,\, l=1 \\ (q,l)\neq(1,1)}}^{q=N,\, l=K} \underline{X^{(T)}_{p,k;q,l}} \cdot A^{*}_{q,l} \cdot P^{k+l}$$

Simple nonlinear map

Linear harmonic map function of incident wave

Linear harmonic map function of conjugate of incident wave

- X-parameters of type FB, S, and T fully characterize the nonlinear function.
- Depend on
  - frequency
  - large signal magnitude, $|A_{1,1}|$
  - DC bias

$$X^{(S)}_{p,k;q,l}$$

output port, output harmonic, input port, input harmonic

$$P = \frac{A_{1,1}}{|A_{1,1}|}$$

X-Parameters Collapse to S-Parameters in Small-Signal Limit

$$B_{p,k} \approx X^{(FB)}_{p,k} \cdot P^{k} + \sum_{\substack{q=1,\, l=1 \\ (q,l)\neq(1,1)}}^{q=N,\, l=K} X^{(S)}_{p,k;q,l} \cdot A_{q,l} \cdot P^{k-l} + \sum_{\substack{q=1,\, l=1 \\ (q,l)\neq(1,1)}}^{q=N,\, l=K} X^{(T)}_{p,k;q,l} \cdot A^{*}_{q,l} \cdot P^{k+l}$$

As $A_{1,1}$ shrinks, the conjugate terms and harmonic terms vanish:

$$B_{p,1} \approx X^{(FB)}_{p,1} \cdot P + \sum_{q=2}^{q=N} X^{(S)}_{p,1;q,1} \cdot A_{q,1}$$

Remove unnecessary harmonic index and assume 2-port:

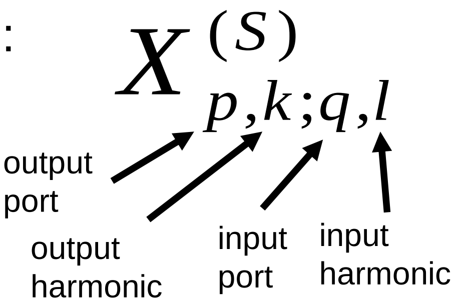
$$B_1 \approx X^{(FB)}_{1} \cdot P + X^{(S)}_{1,2} \cdot A_2$$

$$B_2 \approx X^{(FB)}_{2} \cdot P + X^{(S)}_{2,2} \cdot A_2$$

$X^{(FB)}_{p} \cdot P = S_{p1}\left|A_1\right| P = S_{p1} A_1$ for small A_1 and $P \equiv \arg(A_1)$:

$$B_1 = S_{11} A_1 + S_{12} A_2$$

$$B_2 = S_{21} A_1 + S_{22} A_2$$



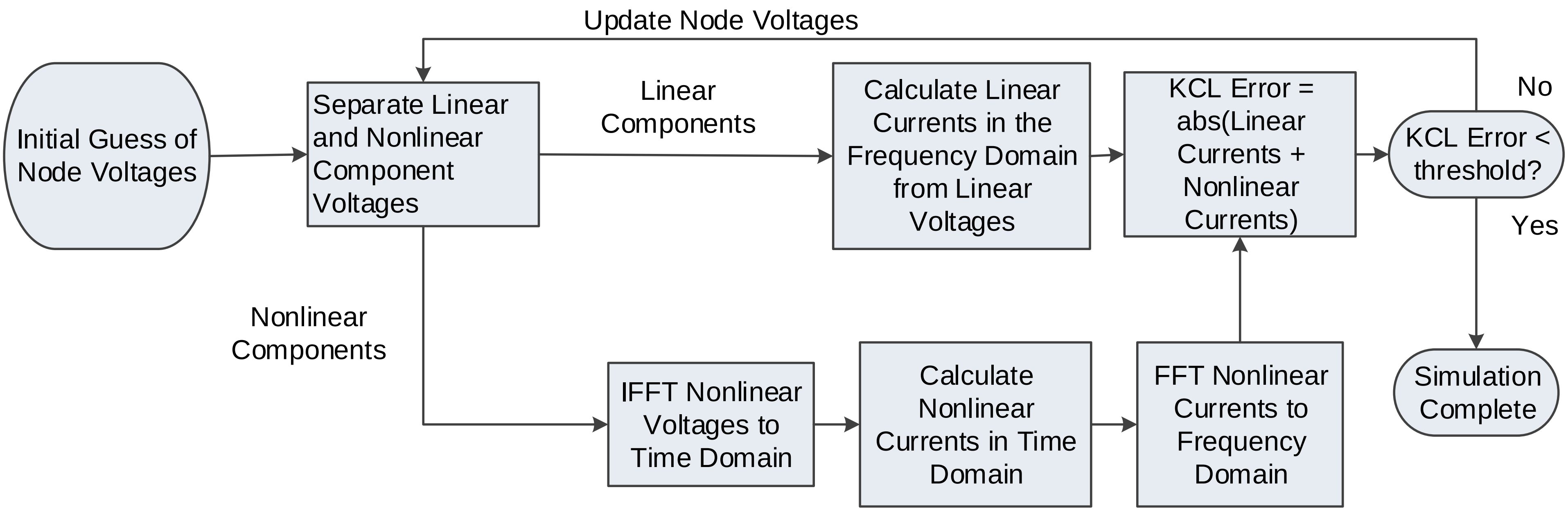
Generating X-Parameters

- Traditional Generation
  - Simulated using harmonic balance.
  - Measured with a nonlinear vector network analyzer (NVNA).

Harmonic Balance

Assume nodal voltages can be represented with Fourier series and solve for the Fourier coefficients.

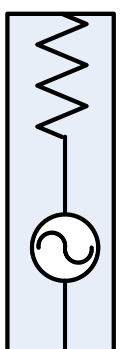
$$v(t) = \mathrm{Re}\left[\sum_{k_1=0}^{K_1}\sum_{k_2=0}^{K_2}\cdots\sum_{k_n=0}^{K_n} V_{k_1,k_2,\ldots,k_n} e^{j2\pi(k_1 f_1+\ldots+k_n f_n)t}\right]$$



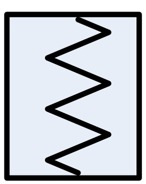
Generating X-Parameters with Harmonic Balance

- Need to set proper values for:
  - Frequency range
  - Fundamental power
  - DC bias
- X-parameter measurements are unidirectional because of large-signal fundamental $|A_{1,1}|$ on one port.
- Different types of X-parameter ports:

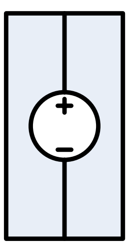
Source



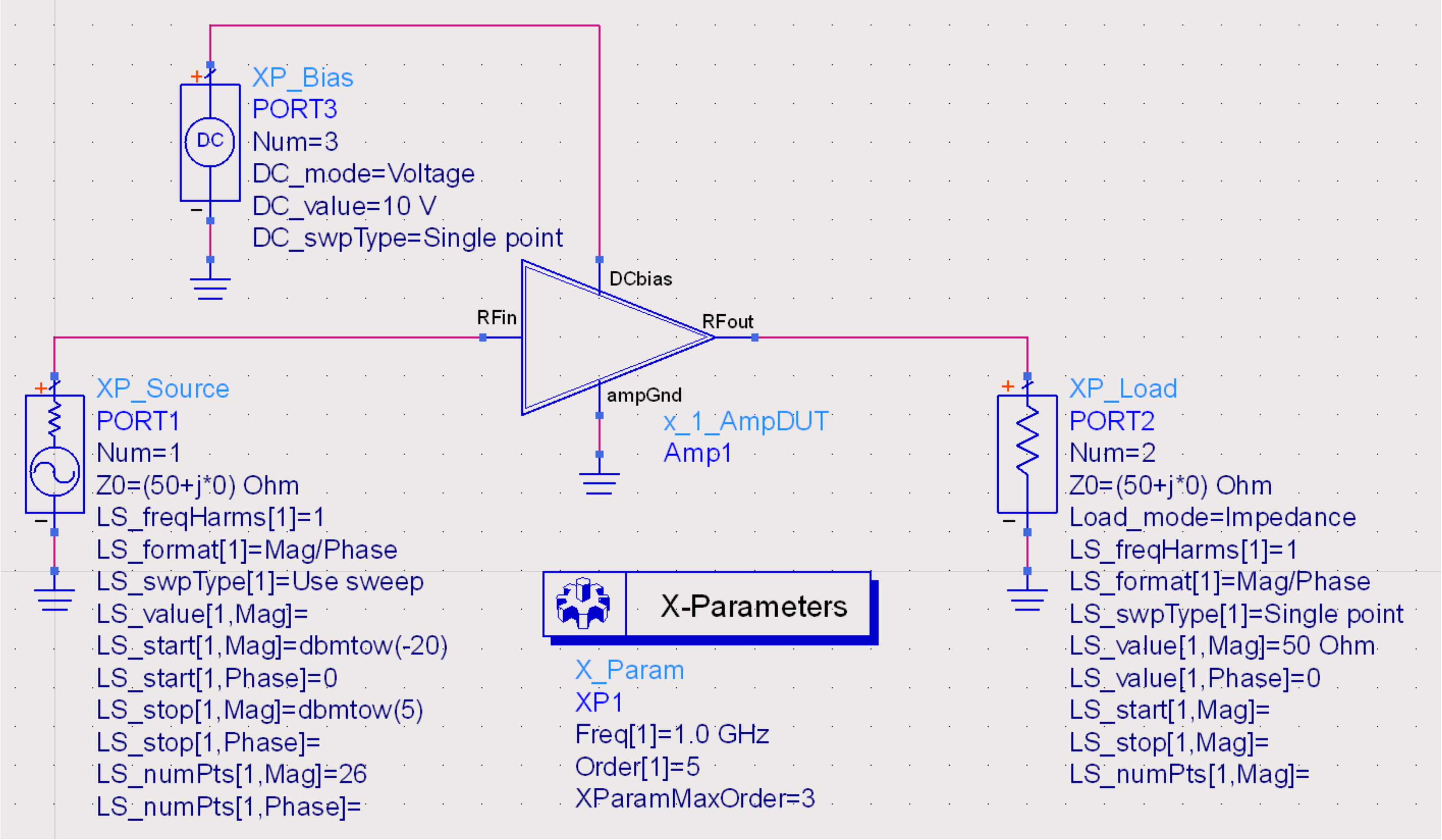
Load



Bias



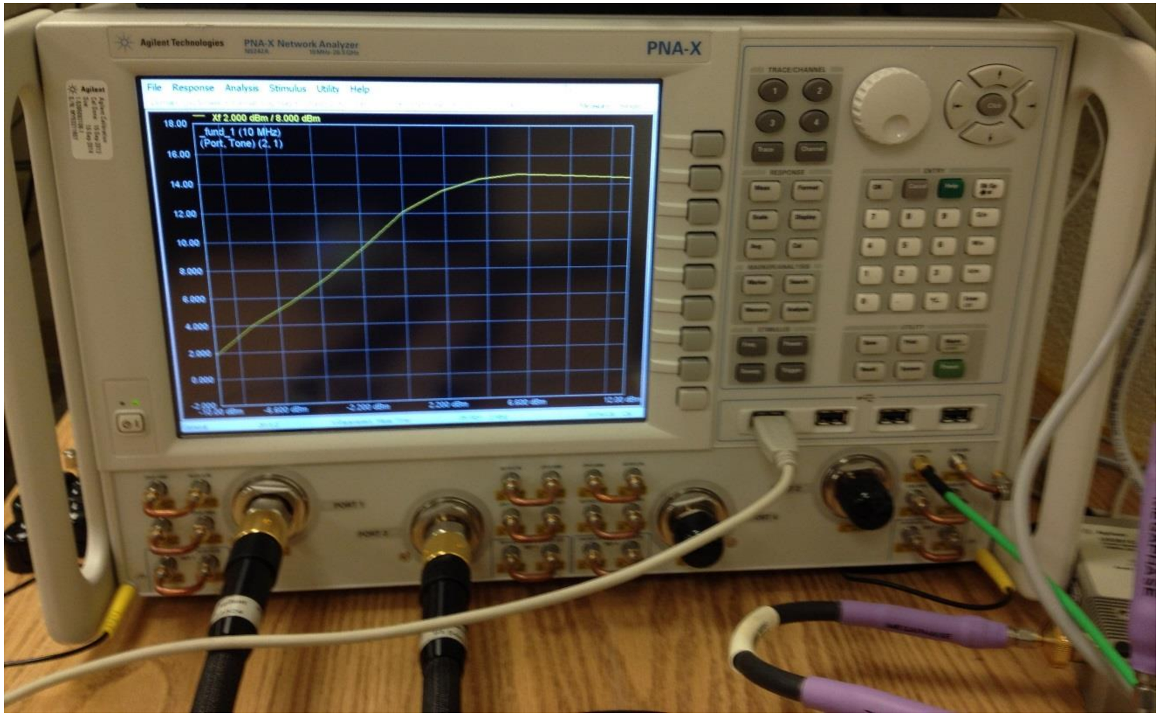
X-Parameter Generation Example



Nonlinear Vector Network Analyzer (NVNA)

PNA-X

- Four ports.
- Two filtered microwave sources.
- Microwave combiner.

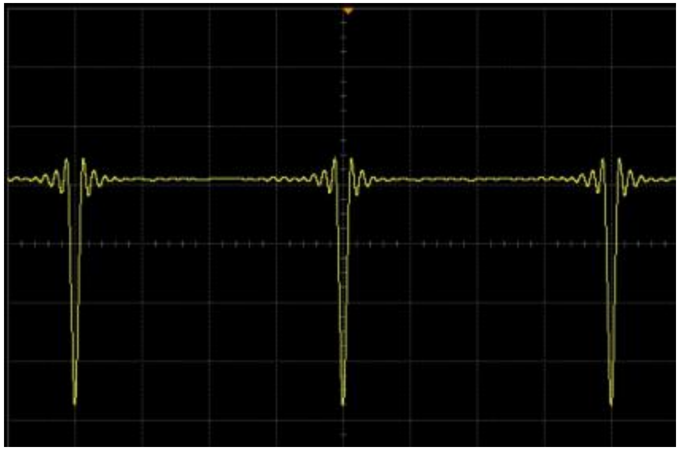


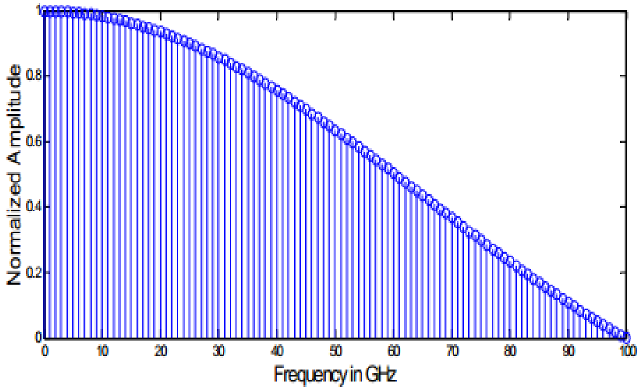
Amplitude Calibration

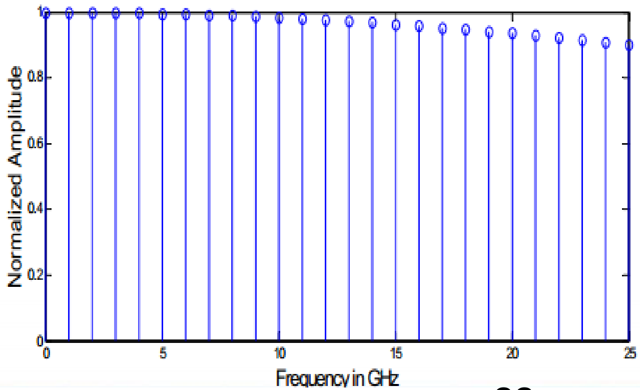
- Necessary for any nonlinear measurement because linear property of homogeneity does not apply.
- Measures power and is controlled via GPIB.

Phase Calibration

- Enables cross-frequency phase measurement.
- Takes frequency input from external microwave source.

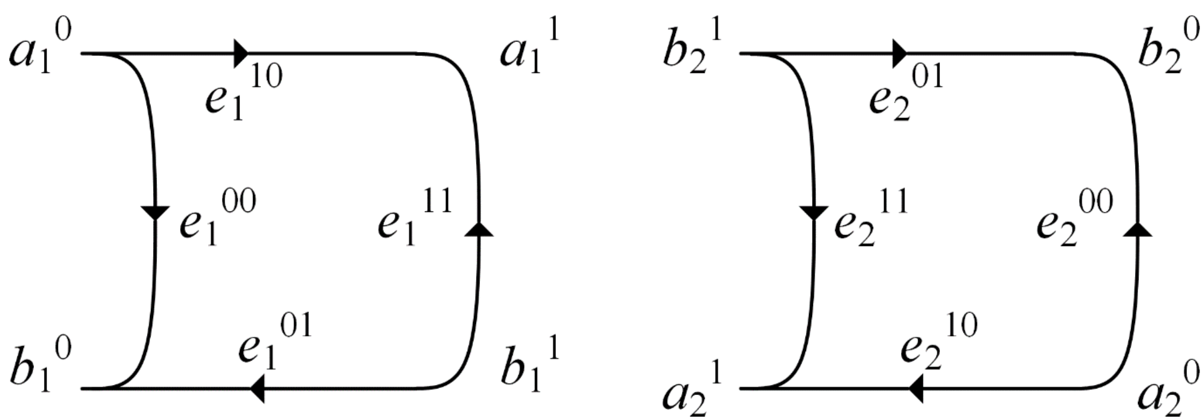






Vector Calibration

- Can use ECal.
- Based on eight-term error model.
- Works for forward, reverse, and combined stimuli.



Large-Signal X-Parameter Extraction

- Apply large-signal stimulus $A_{1,1}$ without any small-signal stimulus.
- Measure the response at all ports and harmonics of interest.
- $X_{p,k}^{(FB)}$ term is the measured response to the large-signal stimulus at port *p* and harmonic *k*

Offset-Phase Small-Signal X-Parameter Extraction

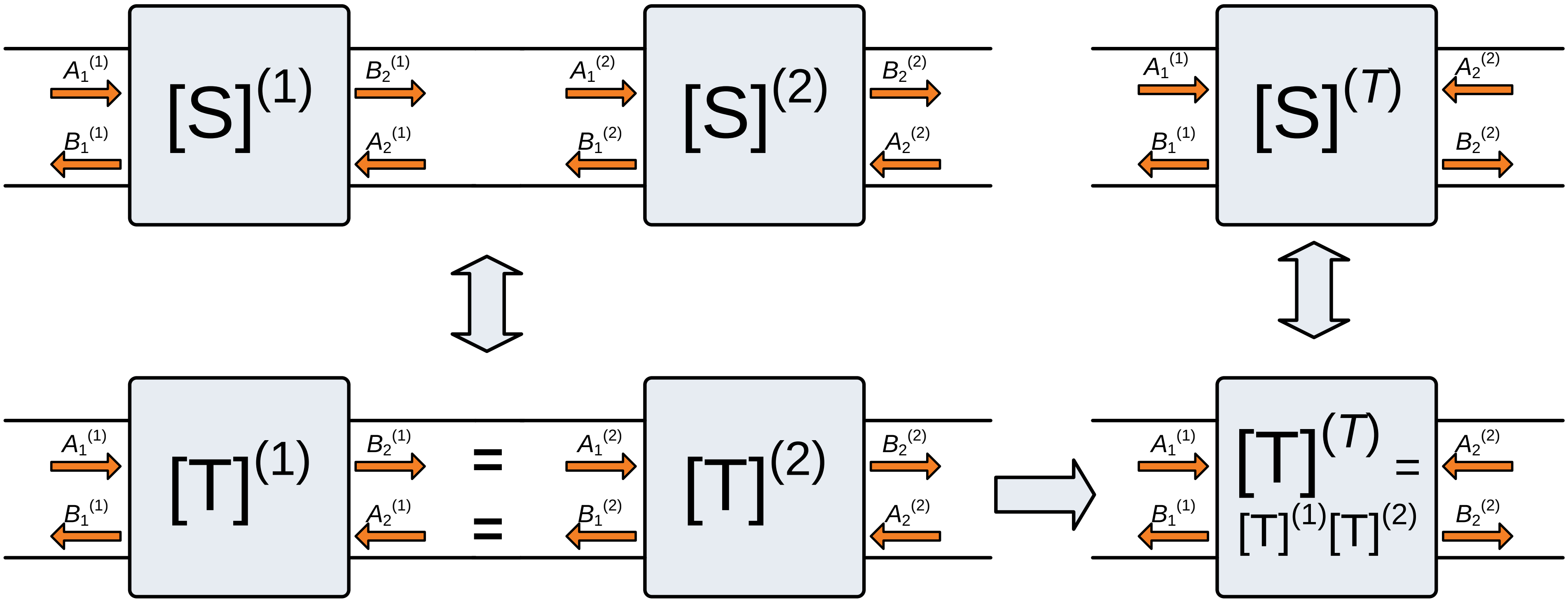
- Apply large-signal stimulus $A_{1,1}$ and one small-signal stimulus $A_{q,l}$ at zero phase.
- Measure the response at all ports and harmonics of interest.
- Apply large-signal stimulus $A_{1,1}$ and one small-signal stimulus $A_{q,l}$ at 90^{o} phase.
- Measure the response at all ports and harmonics of interest.
- Use both measurements to extract

$$X^{(S)}_{p,k;q,l} \text{ and } X^{(T)}_{p,k;q,l}.$$

Using X-Parameters

- Traditional Uses
  - Modeling mixers and amplifiers in steady-state simulations for RF systems.
  - Can be used to determine nonlinear figures of merit.
    - 1-dB Compression Point
    - AM/AM and AM/PM
    - Third Order Intercept

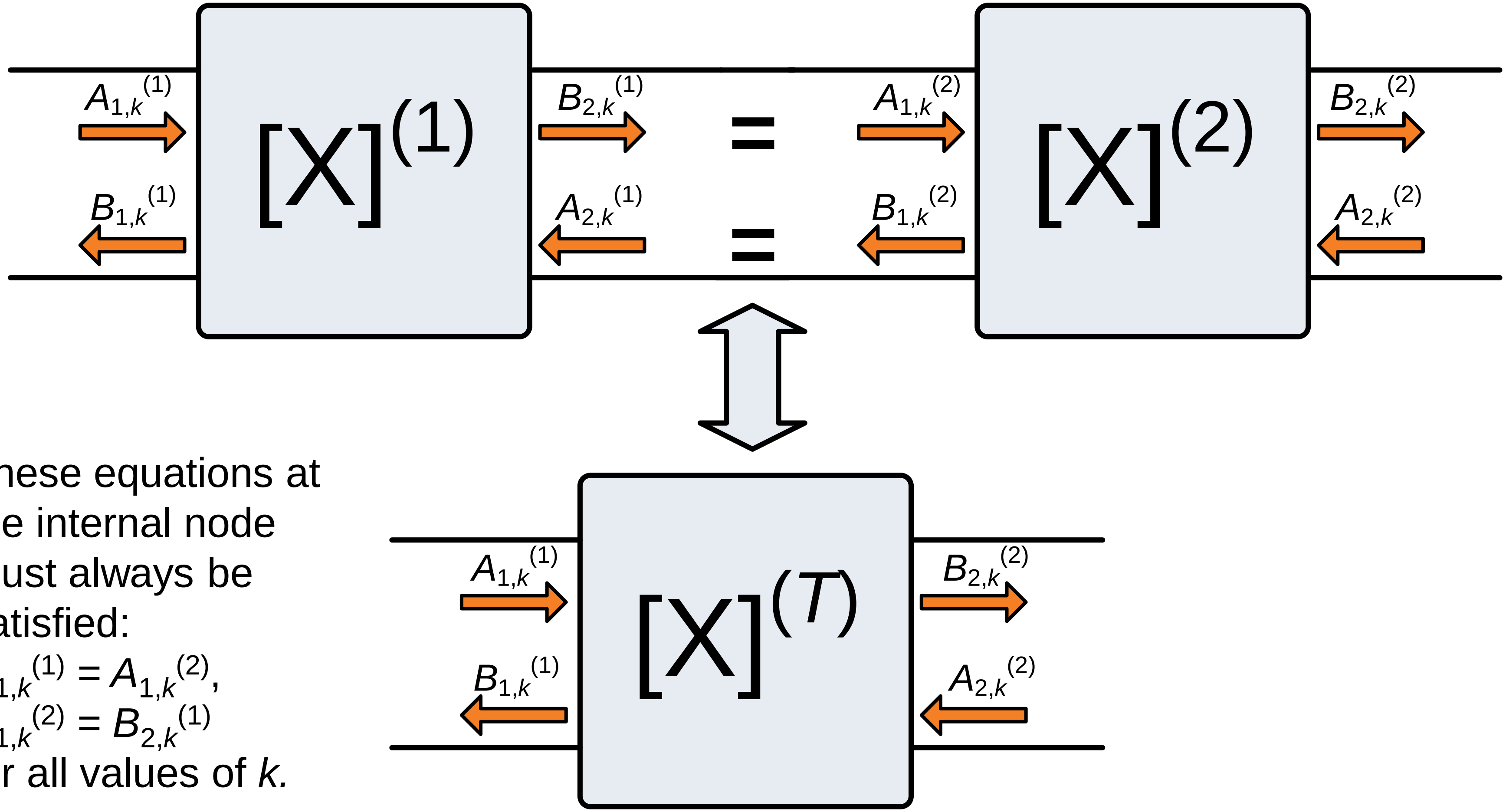
Cascading S-Parameter Blocks



[T] = transfer scattering parameters.

Can disregard circuit behavior at internal node.

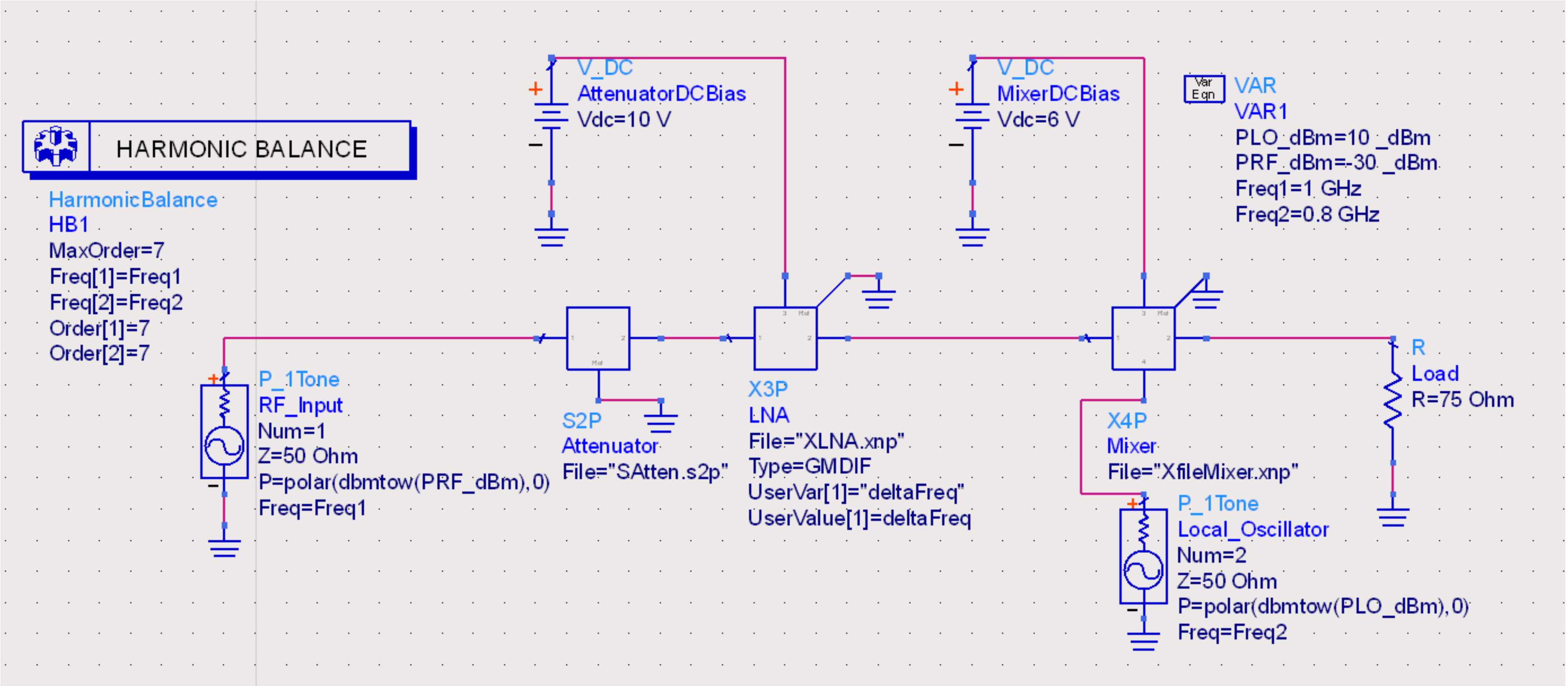
Cascading X-Parameter Blocks



These equations at the internal node must always be satisfied:
$B_{1,k}^{(1)} = A_{1,k}^{(2)}$,
$A_{1,k}^{(2)} = B_{2,k}^{(1)}$
for all values of *k*.

Using X-Parameters in Simulation

Can construct entire receiver chains made of S- and X-parameter blocks.



X-Parameter Extensions

- Multiple Large Signals
- DC Components of Scattered Waves
  - DC current port bias: $X^{(Z)}_{p,k}$
  - DC voltage port bias: $X^{(Y)}_{p,k}$
- Memory Effects