ECE 451
Advanced Microwave Measurements

Error Correction

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Network Analyzer

Source provides RF/microwave signal and consists of high-frequency circuitry with internal impedance of 50 Ω.

Test set consists of couplers used to separate signals. There are also power dividers, switches all of which must operate at the RF/microwave frequency of interest.

Signals REF, A, & B are routed to analyzer which down-converts RF signals to intermediate frequency.

![Waveform Diagram]

Signals are then amplified using low-frequency amplifiers and detected using low-frequency detectors.

Display shows ratios B/REF or A/REF for $S_{11}$, $S_{21}$ in magnitude and/or phase format.
8510C Network Analyzer
Two-Port Measurement

Forward

\[ S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \]

\[ S_{21} = \frac{b_2}{a_1} \big|_{a_2=0} \]
Two-Port Measurement

Reverse

\[ S_{12} = \frac{b_1}{a_2} \bigg|_{a_1=0} \]
\[ S_{22} = \frac{b_2}{a_2} \bigg|_{a_1=0} \]
Two-Port Measurement

In general, the measured $S_{11}$, $S_{12}$, $S_{21}$ and $S_{22}$ are not the parameters of the actual DUT.

Need to remove the effects of $X_1$ and $X_2$.

Calibration
One-Port Measurement

The system of cables, couplers, etc… represents a 2-port and must be de-embedded in order to obtain the actual $S_{11}$ of the unknown $\rightarrow S_{11a}$
Assume that the network analyzer is perfectly matched. Then,

\[ S_{11m} = \frac{b_1}{a_1} = S_{11c} + \frac{S_{12c}S_{21c}S_{11a}}{1 - S_{22c}S_{11a}} \]

All quantities are complex and frequency-dependent!
One-Term Error Model

If we assume that the cables and couplers are a perfect 50-Ω system, then \( S_{11c} = S_{22c} = 0 \). We have a **one-term error model**.

\[
S_{11m} = S_{12c} S_{21c} S_{11a} = TS_{11a}
\]

where \( T = S_{12c} S_{21c} \)

\( T \) is not known. To determine \( T \), we first measure a short since for a short, \( S_{11a}^{(short)} = -1 \)

We get \( S_{11m}^{(short)} = TS_{11a}^{(short)} = -T \)

\[ \uparrow \text{known} \]
One-Term Error Model

Once $T$ is known, we can then measure the DUT

$$S_{11m}^{(DUT)} = TS_{11a}^{(DUT)}$$

From which

$$S_{11a}^{(DUT)} = \frac{S_{11m}^{(DUT)}}{T} = -\frac{S_{11m}^{(DUT)}}{S_{11m}^{(short)}}$$

### Practical Observations:

Since the correction involves a simple complex division, we can do the following

$$\left|S_{11a}^{(DUT)} \right| = \left|\frac{S_{11m}^{(DUT)}}{S_{11m}^{(short)}} \right| \quad \text{and} \quad \angle S_{11a}^{(DUT)} = \angle S_{11m}^{(DUT)} - \angle S_{11m}^{(short)} \pm 180^\circ$$
Three-Term Error Model
Three-Term Error Model

- $E_{DF}$: Directivity of couplers (leak through test ports)
- $E_{RF}$: reflection tracking error (signal path tracking error)
- $E_{SF}$: Source match error

$$S_{11m} = \Gamma_{in} = S_{11c} + \frac{S_{12c} S_{21c} S_{11a}}{1 - S_{22c} S_{11a}}$$

or

$$S_{11m} = E_{DF} + \frac{E_{RF} S_{11a}}{1 - E_{SF} S_{11a}}$$
Three-Term Error Model

**Step 1:**
Use matched load \((Z_o = 50 \, \Omega)\) as DUT \(\Rightarrow S_{11a} = 0\)

\[
S_{11m}^{(\text{load})} = E_{DF} = A
\]  

**Step 2:**
Use a perfect short as DUT \(\Rightarrow S_{11a} = -1\)

\[
S_{11m}^{(\text{short})} = E_{DF} - \frac{E_{RF}}{1 + E_{SF}} = B
\]  

**Step 3:**
Use a perfect open as DUT \(\Rightarrow S_{11a} = +1\)

\[
S_{11m}^{(\text{open})} = E_{DF} + \frac{E_{RF}}{1 - E_{SF}} = C
\]
Three-Term Error Model

Combining (1), (2), and (3) gives $E_{SF}$, $E_{RF}$ and $E_{DF}$

$$E_{DF} = S_{11m}^{(load)} = A$$

$$E_{SF} = \frac{B + C - 2A}{C - B} = \frac{S_{11m}^{(short)} + S_{11m}^{(open)} - 2S_{11m}^{(load)}}{S_{11m}^{(open)} - S_{11m}^{(short)}}$$

$$E_{RF} = \frac{-2(B - A)(C - A)}{C - B} = \frac{-2(S_{11m}^{(short)} - S_{11m}^{(load)})(S_{11m}^{(open)} - S_{11m}^{(load)})}{S_{11m}^{(open)} - S_{11m}^{(short)}}$$

**Step 4:** Measuring the unknown DUT

This is the actual $S_{11}$ with corrections.

$$S_{11a}^{(DUT)} = \frac{S_{11m}^{(DUT)} - E_{DF}}{E_{RF} + E_{SF}\left[S_{11m}^{(DUT)} - E_{DF}\right]}$$
Alternative Calibration Standards

At very high frequencies, it is difficult to make a good short, open or matched termination. We need to find alternative standards for calibration.

Offset Short

TL of length $l$ terminated with a short
Offset Short Standard

At \( z=0 \), \( \Gamma = \Gamma_L \)

At \( z=-l \), \( \Gamma(-l) = \Gamma_L e^{-2j\beta l} \)

Since \( \Gamma_L = -1 \), \( \Gamma(-l) = \Gamma_{in} = -e^{-2j\beta l} = e^{j\left(\pi - \frac{4\pi l}{\lambda}\right)} \)

\( \Gamma_{in} = e^{j\theta} \) where \( \theta = \pi \left(1 - \frac{4l}{\lambda}\right) \)

Therefore, when calibrating with an offset short, we use: \( S_{11a}^{(offset\ short)} = e^{j\theta} \)

where \( \theta \) is known:
Offset Short Restriction

The offset short will only work if the frequency range is such that \( 0 < l < \frac{\lambda}{2} \)

This corresponds to a frequency range of

\[
f < \frac{v}{2l}
\]

where \( v \) is the propagation velocity in the line.
Shielded Open Standard

The shielded open can be modeled as a controlled capacitor.

For a system with reference impedance of $Z_o$, the associated reflection coefficient is:

$$
\Gamma_{in} = \frac{1}{1/j\omega C + Z_o} = \frac{1 - j\omega CZ_o}{1 + j\omega CZ_o} = \frac{1 - ja}{1 + ja}
$$

with $a = \omega CZ_o$

$$
\Gamma_{in} = e^{-2j \tan^{-1} a}
$$

So, for shielded open, we use $S_{11a}^{(shielded \ open)} = e^{-2j \tan^{-1} a}$
Sliding Load

**Motivation:** Need to accurately measure the actual directivity error of the system.

**Observation:** If termination is imperfect, then the measured directivity is the vector sum of the actual directivity and the reflection coefficient of the load.
Sliding Load

With the sliding load, a small $\Gamma$ is willfully introduced and varied in terms of its phase.

By sliding the load at a given frequency point, a circle is defined about the tip of the directivity vector.

We find the best circle that fits the measured $S_{11}$. The center of that circle is the tip of the actual (desired) directivity vector.
Alternate Combinations

- Matched Load
- Offset Short
- Short

- Matched Load
- Short
- Shielded open

- Matched Load
- Offset Short
- Shielded open

- Sliding Load
- Offset Short
- Short

- Sliding Load
- Short
- Shielded open

- Sliding Load
- Offset Short
- Shielded open
8-Term Error Model

Calibration consists of determining the $i$ and $o$ terms by placing known standards as the $\alpha$ terms.
8-Term Error Model

Calibration Stage: Reflection/Port 1

Placing open, short and load as standards \((S_{11a})\) in port 1 yields

\[
S^{(op)}_{11m} = S_{11i} + \frac{S_{21i} S_{12i}}{e^{j\beta} - S_{21i}}
\]

\[
S^{(sh)}_{11m} = S_{11i} - \frac{S_{21i} S_{12i}}{1 + S_{21i}}
\]

\[
S^{(ld)}_{11m} = S_{11i}
\]
8-Term Error Model

Calibration Stage: Reflection/Port 1

The system is simultaneously solved to give

\[ S_{11i} = S_{11m}^{(ld)} \]

\[ S_{22i} = \frac{e^{j\beta} \left[ S_{11m}^{(op)} - S_{11m}^{(ld)} \right] - \left[ S_{11m}^{(ld)} - S_{11m}^{(sh)} \right]}{S_{11m}^{(op)} - S_{11m}^{(sh)}} \]

\[ S_{12i} S_{21i} = \frac{(1 + e^{j\beta}) \left[ S_{11m}^{(op)} - S_{11m}^{(ld)} \right] \left[ S_{11m}^{(sh)} - S_{11m}^{(ld)} \right]}{S_{11m}^{(sh)} - S_{11m}^{(op)}} \]
8-Term Error Model

Calibration Stage: Reflection/Port 2

Same principle can be applied to port 2 to give

\[ S_{11o} = S_{22m}^{(ld)} \]

\[ S_{22o} = \frac{e^{j\beta} \left[ S_{22m}^{(op)} - S_{22m}^{(ld)} \right] - \left[ S_{22m}^{(ld)} - S_{22m}^{(sh)} \right]}{S_{22m}^{(op)} - S_{22m}^{(sh)}} \]

\[ S_{12o}S_{21o} = \left(1 + e^{j\beta} \right) \left[ S_{22m}^{(op)} - S_{22m}^{(ld)} \right] \left[ S_{22m}^{(sh)} - S_{22m}^{(ld)} \right] \frac{S_{22m}^{(sh)} - S_{22m}^{(op)}}{S_{22m}^{(sh)} - S_{22m}^{(op)}} \]
8-Term Error Model

Calibration Stage: Transmission

Next, connect the two ports together for transmission calibration $S_{21a} = S_{12a} = 1$

$$S^{(thr)}_{21m} = \frac{S_{21i}S_{12o}}{1 - S_{22i}S_{22o}}$$

$$S^{(thr)}_{12m} = \frac{S_{12i}S_{21o}}{1 - S_{22i}S_{22o}}$$
8-Term Error Model

Measurement Stage

Insert unknown and provide unit reference signal at input

\[ a_1 = S_{21i} + S_{22i}a_4 \]
\[ a_4 = S_{11a}a_1 + S_{12a}a_3 \]
\[ a_3 = S_{22o}a_2 \]
\[ a_2 = S_{21a}a_1 + S_{22a}a_3 \]
\[ S_{11m} = S_{11i} + S_{12i}a_4 \]
\[ S_{21m} = S_{21o}a_2 \]
8-Term Error Model

Measurement Stage

\[ a_1 = S_{21i} + S_{22i} \left( \frac{S_{11m} - S_{11i}}{S_{21i}} \right) \]

Solve for the \( a \)'s

\[ a_2 = \frac{S_{21m}}{S_{12o}} \]

\[ a_3 = \frac{S_{22o}S_{21m}}{S_{12o}} \]

\[ a_4 = \frac{S_{11m}S_{11i}}{S_{12i}} \]
Insert unknown and provide unit reference signal at port 2

\[
b_1 = S_{22i} b_4
\]

\[
b_3 = S_{22o} b_2 + S_{12o}
\]

\[
S_{22m} = S_{11o} + S_{12o} b_2
\]

\[
S_{12m} = S_{12i} b_4
\]

\[
b_4 = S_{11a} b_1 + S_{12a} b_3
\]

\[
b_2 = S_{21a} b_1 + S_{22a} b_3
\]
8-Term Error Model

Measurement Stage

Solve for the $b$’s

\[
\begin{align*}
b_1 &= \frac{S_{22i}S_{12m}}{S_{12i}} \\
b_2 &= \frac{S_{22m} - S_{11o}}{S_{12o}} \\
b_3 &= S_{22o} \left[ \frac{S_{22m} - S_{11o}}{S_{12o}} \right] + S_{21o} \\
b_4 &= \frac{S_{21m}}{S_{12i}}
\end{align*}
\]
8-Term Error Model - Solution

\[ S_{11a} = \frac{S_{11m} - S_{11i}}{S_{21i}S_{12i}} \left[ 1 + \frac{S_{22o} (S_{22m} - S_{11o})}{S_{12o}S_{21o}} \right] - \frac{S_{22o}S_{21m}}{S_{21i}S_{12o}} \times \frac{S_{12m}}{S_{21o}S_{12i}} \]

\[ S_{12a} = \frac{S_{12m}}{S_{21o}S_{12i}} \]

\[ S_{21a} = \frac{S_{21m}}{S_{21i}S_{12o}} \]

\[ S_{22a} = \frac{S_{22m} - S_{11o}}{S_{21o}S_{12o}} \left[ 1 + \frac{S_{22i} (S_{11m} - S_{11i})}{S_{21i}S_{12i}} \right] - \frac{S_{22i}S_{21m}}{S_{21i}S_{12o}} \times \frac{S_{12m}}{S_{21o}S_{12i}} \]
8-Term Error Model - Solution

\[
D = \left[ 1 + \frac{S_{22i} \left( S_{11m} - S_{11i} \right)}{S_{21i} S_{12o}} \right] \left[ 1 + \frac{S_{22o} \left( S_{22m} - S_{11o} \right)}{S_{12o} S_{21o}} \right] - S_{22i} S_{22o} \frac{S_{21m}}{S_{21i} S_{12o}} \frac{S_{12m}}{S_{21o} S_{12i}}
\]

Reference

12-Term Error Model

Forward Mode

Reflection terms
- ESF
- EDF
- ERF
- Open
- Short
- Load
- 3 measurements of $S_{11m}$

Transmission terms
- ELF
- ETF
- Thru
- Measure $S_{11m}$ and $S_{21m}$

Isolation term
- EXF
- Loads
- Measure $S_{21m}$
12-Term Error Model

Reverse Mode

Reflection terms
- ESR: Open
- EDR: Short
- ERR: Load

Transmission terms
- ELR: Thru
- ETR: Measure $S_{22m}$ and $S_{12m}$

Isolation term
- EXR: Loads
  Measure $S_{12m}$

Measurements:
- 3 measurements of $S_{22m}$
12-Term Error Model

For equations and complete development, see

Doug Rytting, "Network Analyzer Error Models and Calibration Methods"