ECE 451
Advanced Microwave Measurements

Lossy Transmission Lines

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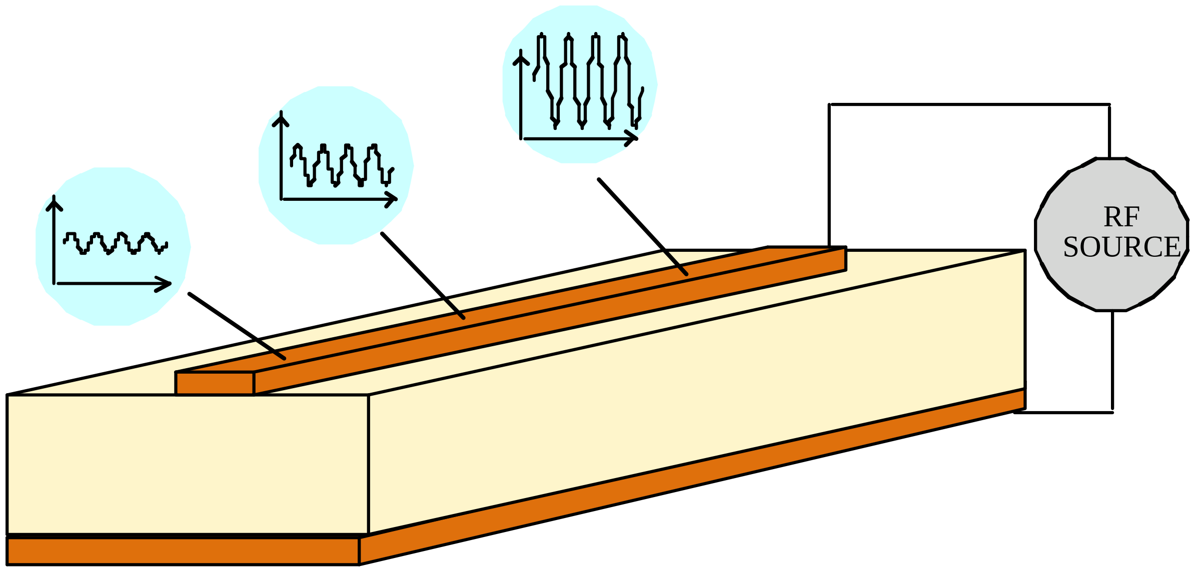
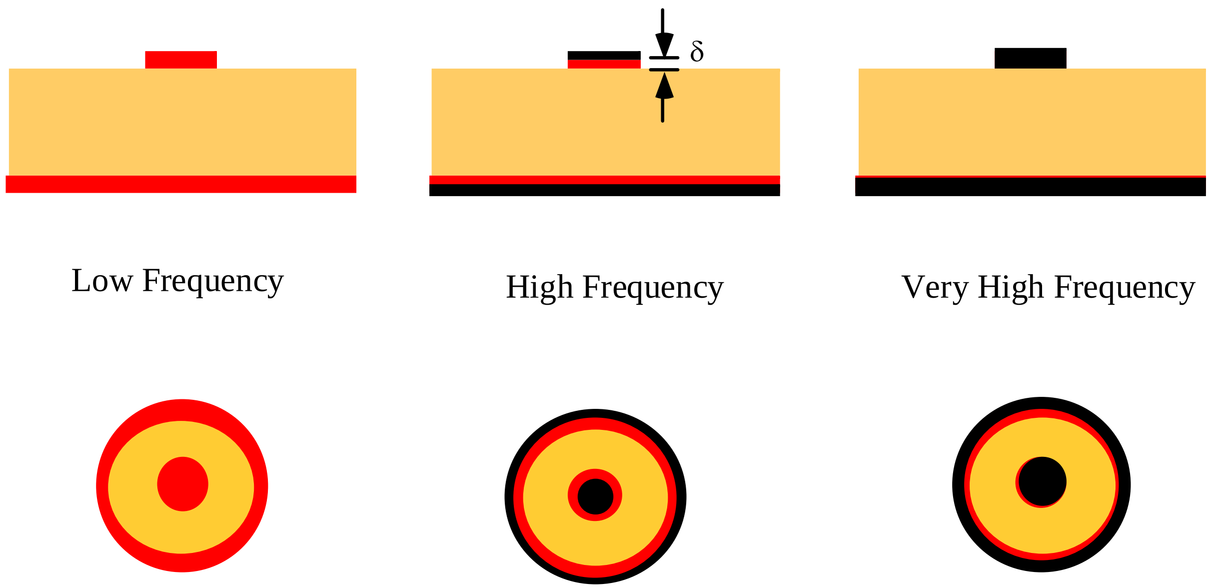
Loss in Transmission Lines

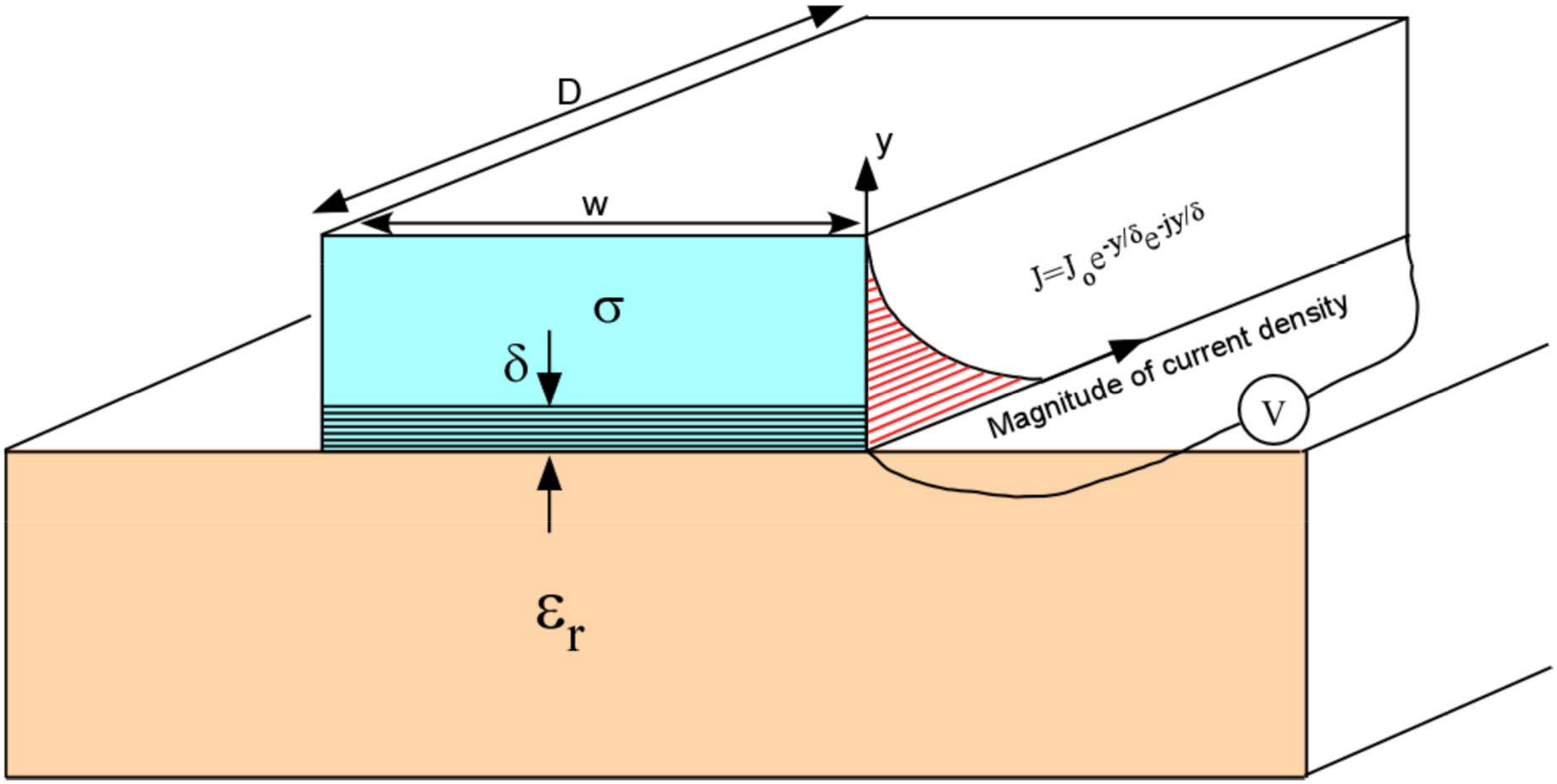


Signal amplitude decreases with distance from the source.

Skin Effect in Lines



Skin Effect in Microstrip



H. A. Wheeler, "Formulas for the skin effect," Proc. IRE, vol. 30, pp. 412-424,1942

Skin Effect in Microstrip

Current density varies as

$$J = J_o e^{-y/\delta} e^{-jy/\delta}$$

Note that the phase of the current density varies as a function of y

$$I = \int_0^\infty J_o w e^{-y/\delta} e^{-jy/\delta} dy = \frac{J_o w\delta}{1+j}$$

$$\sigma E_o = J_o \Rightarrow E_o = \frac{J_o}{\sigma}$$

The voltage measured over a section of conductor of length D is:

$$V = E_o D = \frac{J_o D}{\sigma}$$

Skin Effect in Microstrip

The skin effect impedance is

$$Z_{skin} = \frac{V}{I} = \frac{J_o D}{\sigma} \frac{(1+j)}{J_o w\delta} = \frac{D}{w}(1+j)\sqrt{\pi f \mu\rho}$$

where $\rho = \frac{1}{\sigma}$ is the bulk resistivity of the conductor

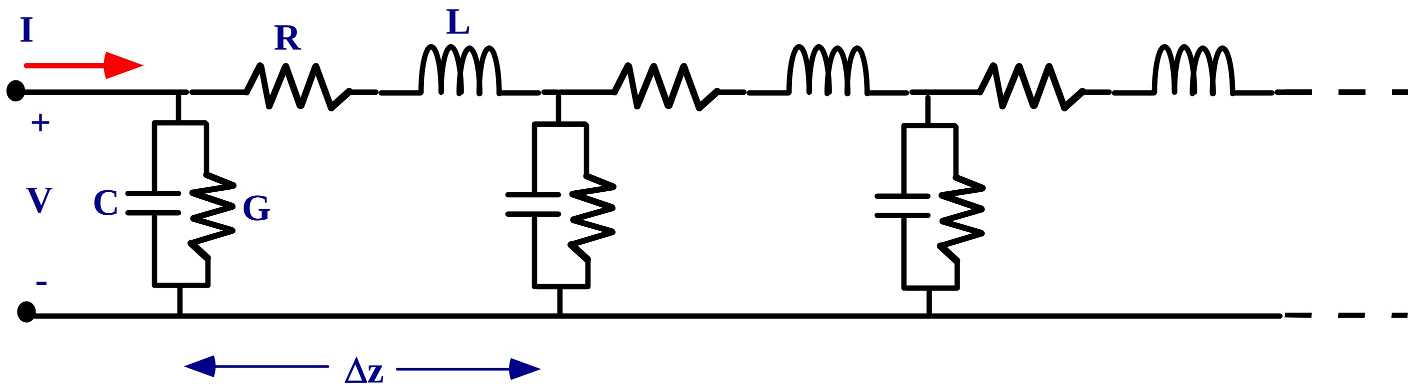
$$Z_{skin} = R_{skin} + jX_{skin}$$

with

$$R_{skin} = X_{skin} = \frac{D}{w}\sqrt{\pi f \mu\sigma}$$

➔ Skin effect has reactive (inductive) component

Lossy Transmission Line

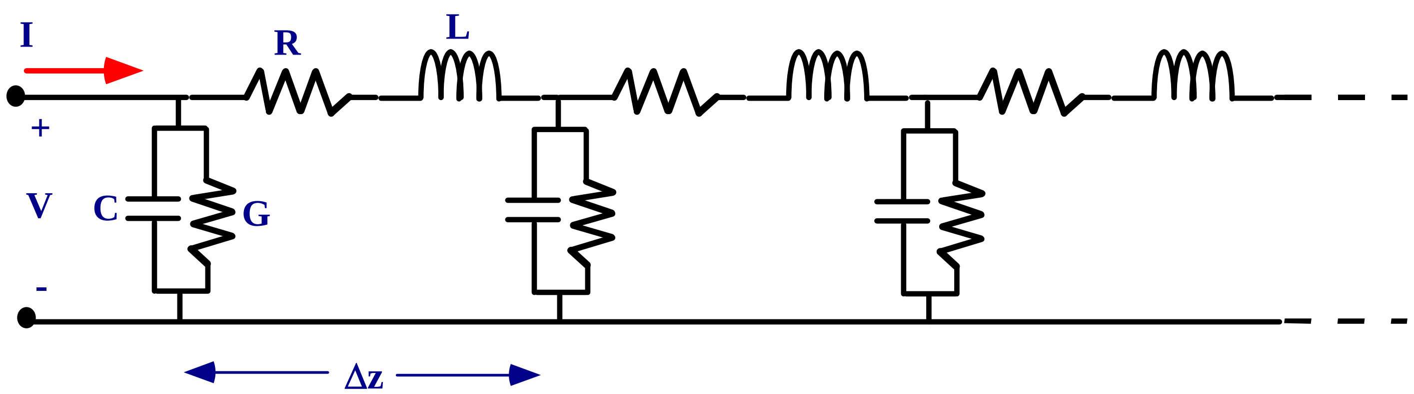


Telegraphers Equation: Time Domain

$$-\frac{\partial V}{\partial z} = RI + L\frac{\partial I}{\partial t}$$

$$-\frac{\partial I}{\partial z} = GV + C\frac{\partial V}{\partial t}$$

Lossy Transmission Line

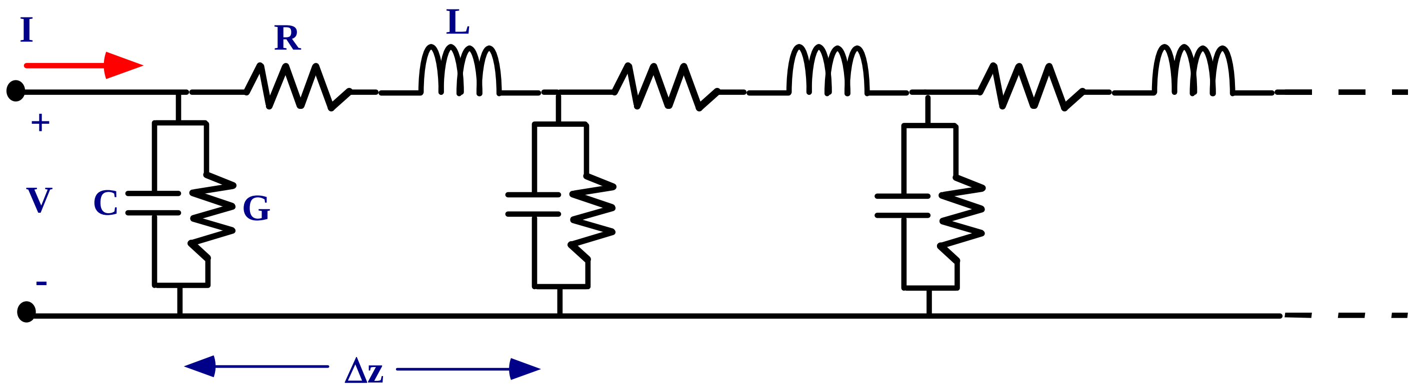


Telegraphers Equation: Frequency Domain

$$-\frac{\partial V}{\partial z} = (R + j\omega L)I = ZI$$

$$-\frac{\partial I}{\partial z} = (G + j\omega C)V = YV$$

Lossy Transmission Line

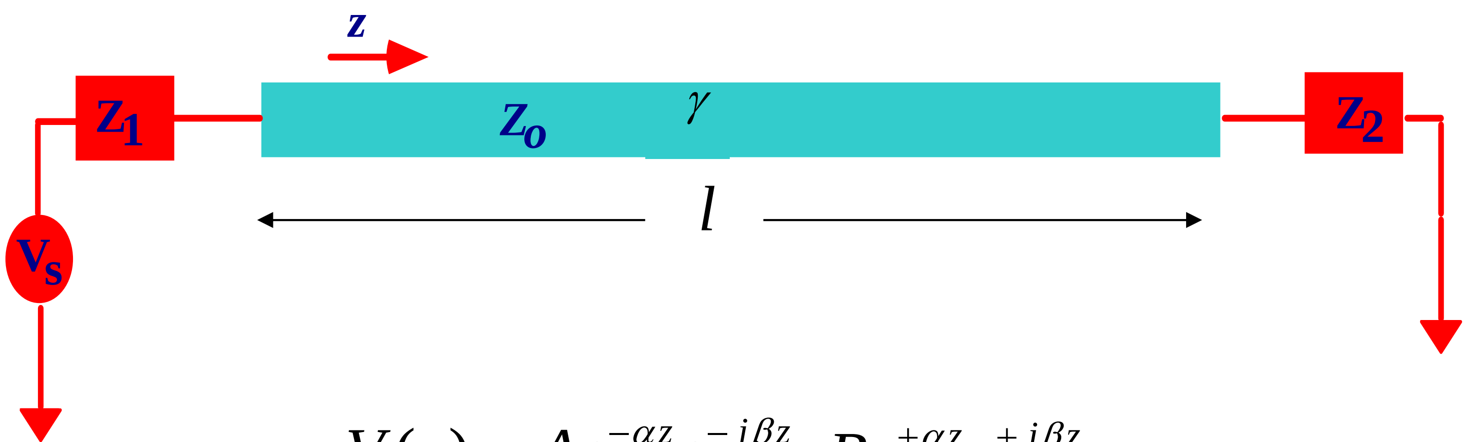


Telegraphers Equation: Frequency Domain

$$-\frac{\partial^2 V}{\partial z^2} = (R + j\omega L)(G + j\omega C)V \;=\; ZYV = \gamma^2 V$$

$$-\frac{\partial^2 I}{\partial z^2} = (G + j\omega C)(R + j\omega L)I \;=\; YZI = \gamma^2 I$$

Lossy Transmission Line

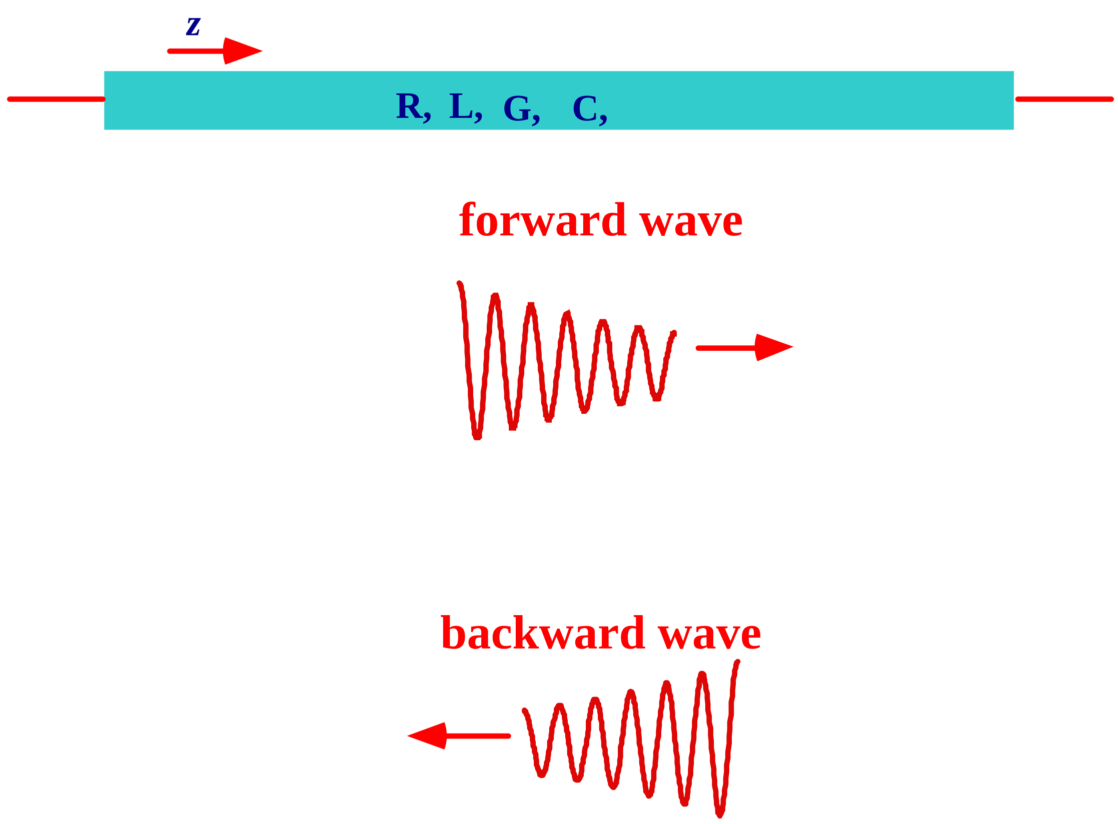


$$V(z) = Ae^{-\alpha z}e^{-j\beta z} + Be^{+\alpha z}e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\left[Ae^{-\alpha z}e^{-j\beta z} - Be^{+\alpha z}e^{+j\beta z}\right]$$

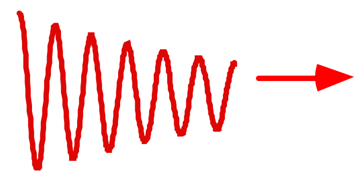
$$Z_o = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \qquad \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Lossy Transmission Line



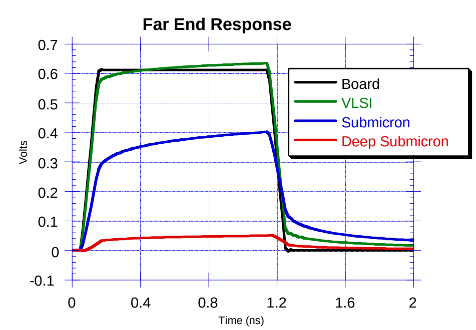
Effects of Losses

- Signal attenuation

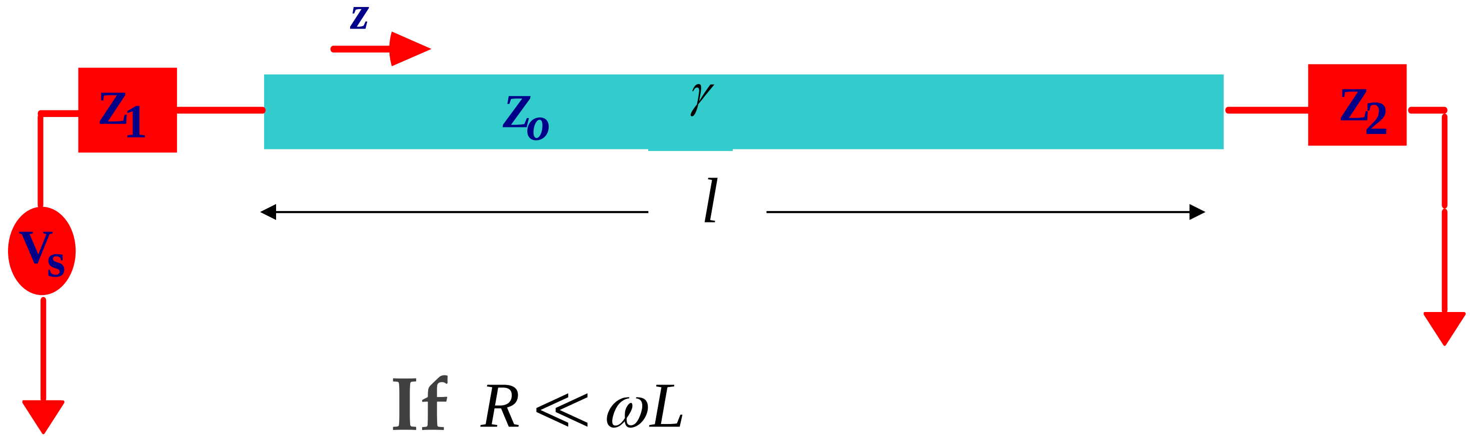


- Dispersion $\gamma = \alpha(\omega) + j\beta(\omega) = \sqrt{(R + j\omega L)(G + j\omega C)}$

- Rise time degradation



Special Case – Low Loss



If $R \ll \omega L$

and $G \ll \omega C$

$$Z_o = \sqrt{\frac{j\omega L\left(1+\frac{R}{j\omega L}\right)}{j\omega C\left(1+\frac{G}{j\omega C}\right)}} \simeq \sqrt{\frac{L}{C}}$$

$$\gamma = \alpha + j\beta$$

$$\alpha \simeq \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)$$

$$\beta \simeq \omega\sqrt{LC} \qquad v_p = \frac{\omega}{\beta} \simeq \frac{1}{\sqrt{LC}}$$