ECE 451
Advanced Microwave Measurements

S Parameters

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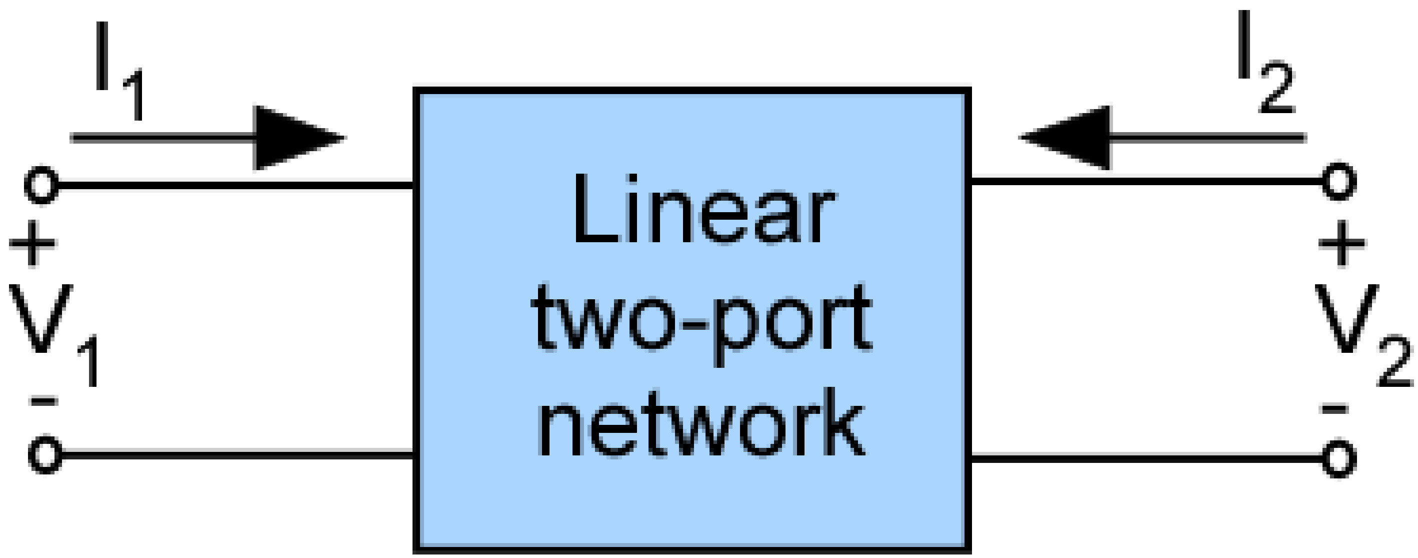
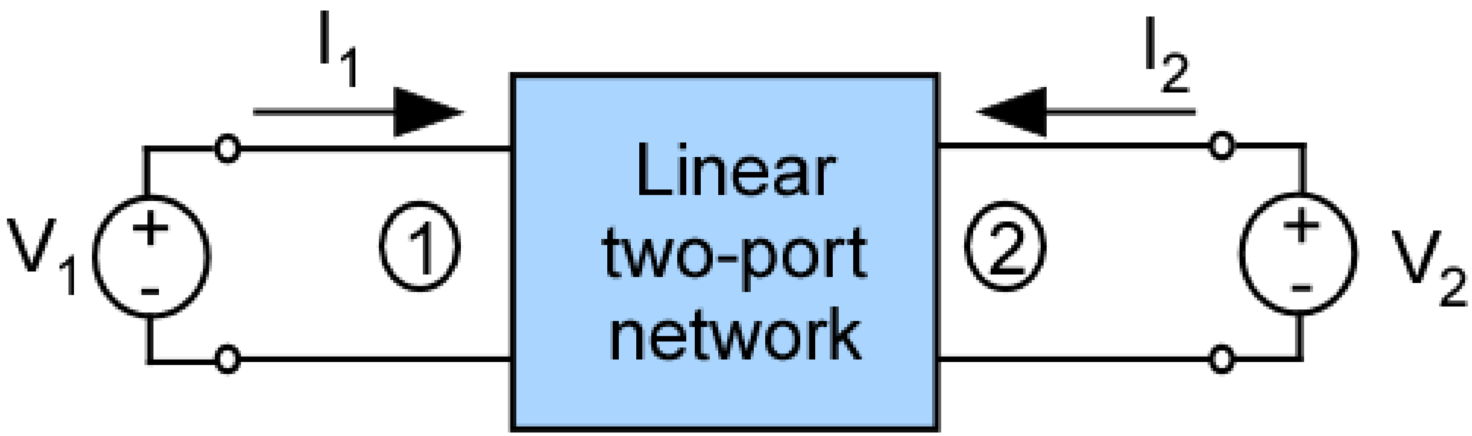
Transfer Function Representation



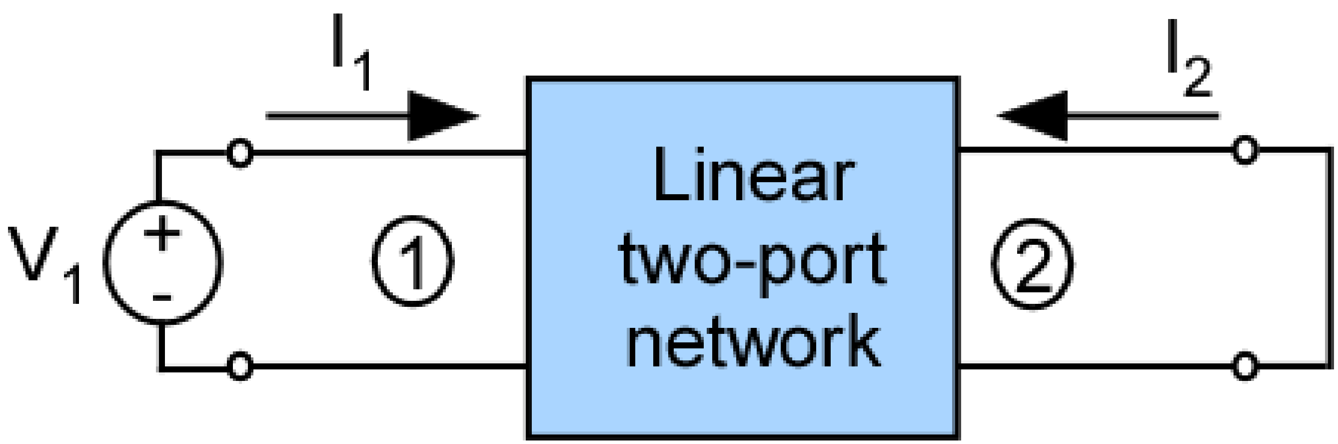
Use a two-terminal representation of system for input and output

Y-parameter Representation



$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

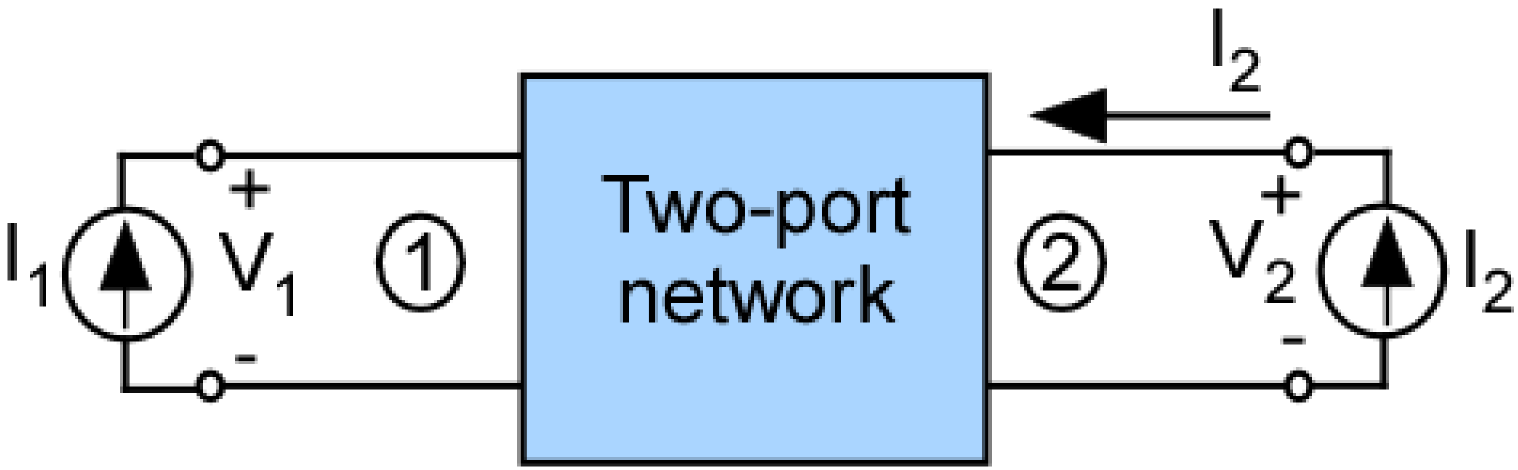
Y Parameter Calculations



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

To make $V_2 = 0$, place a short at port 2

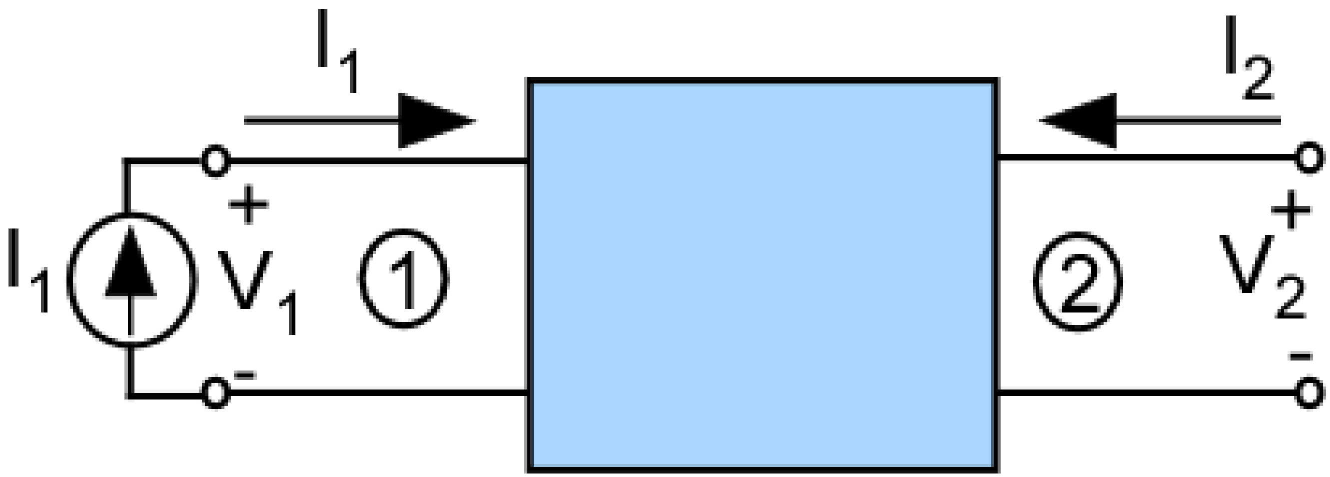
Z Parameters



$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

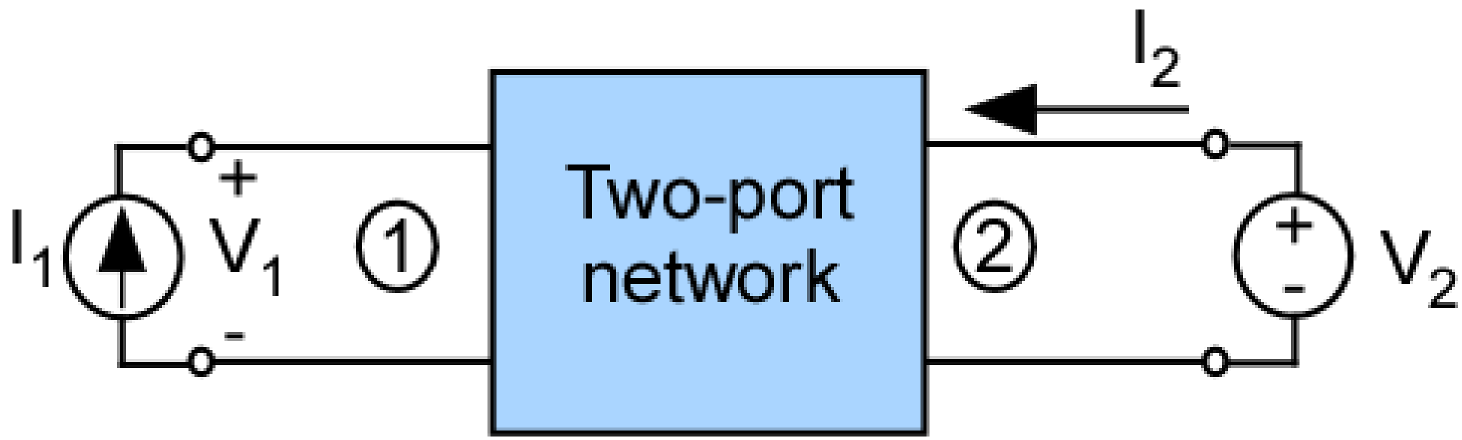
Z-parameter Calculations



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

To make $I_2 = 0$, place an open at port 2

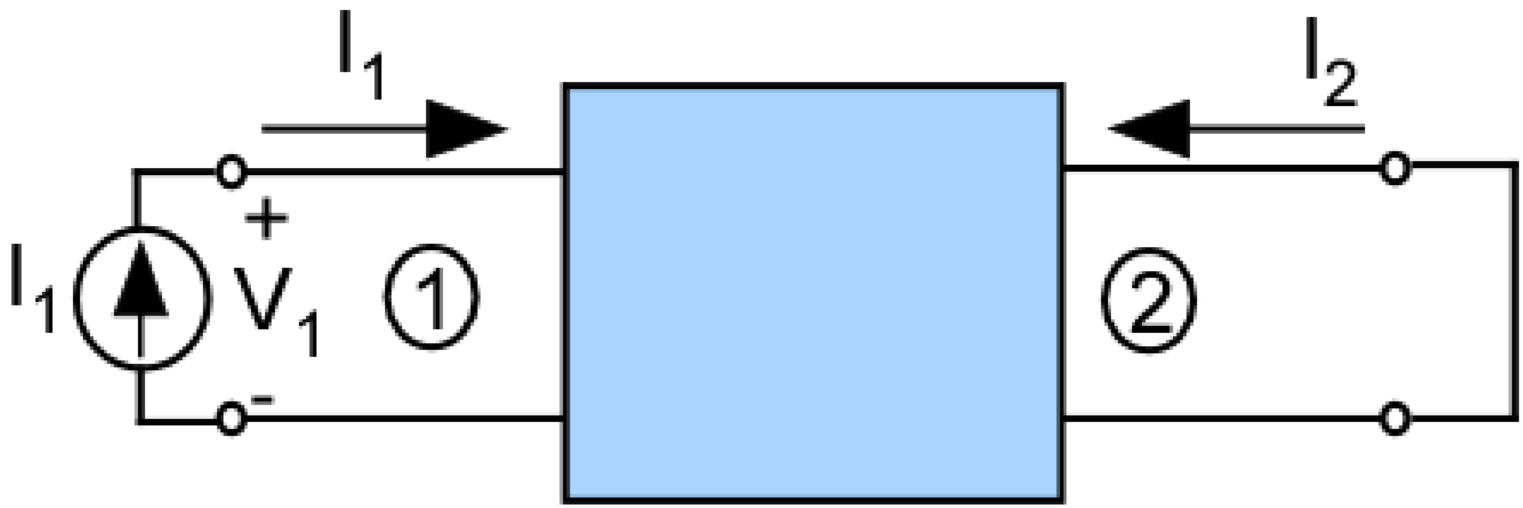
H Parameters



$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

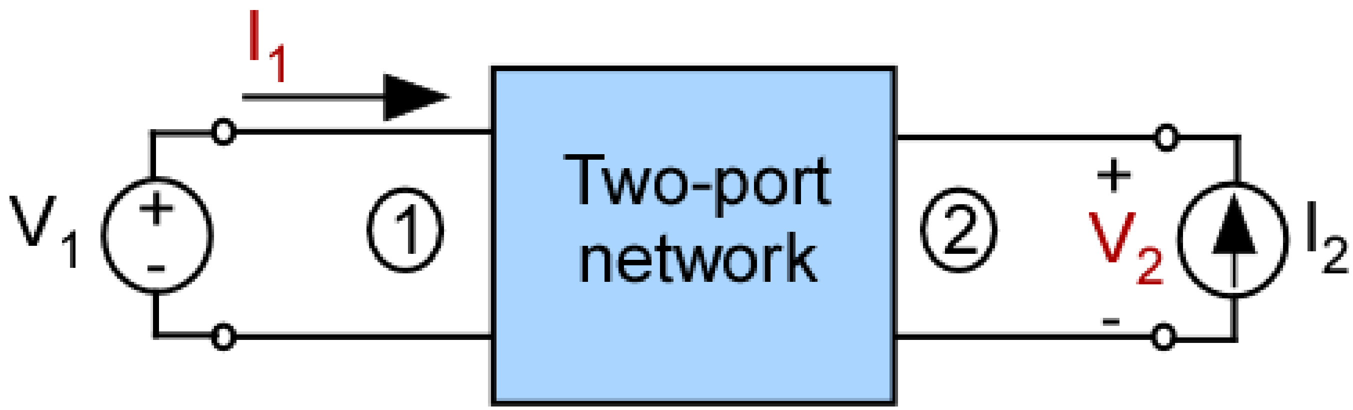
H Parameter Calculations



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

To make $V_2 = 0$, place a short at port 2

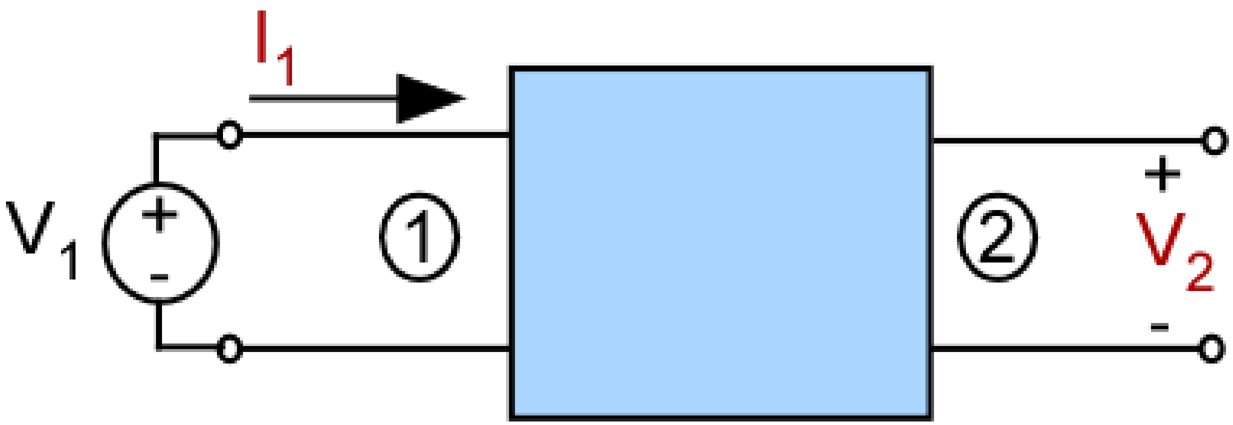
G Parameters



$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

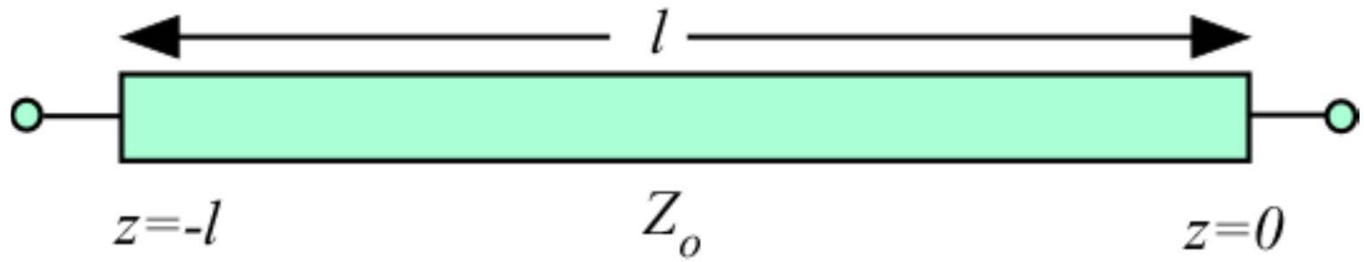
G-Parameter Calculations



$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \qquad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$

To make $I_2 = 0$, place an open at port 2

Y-Parameters of TL



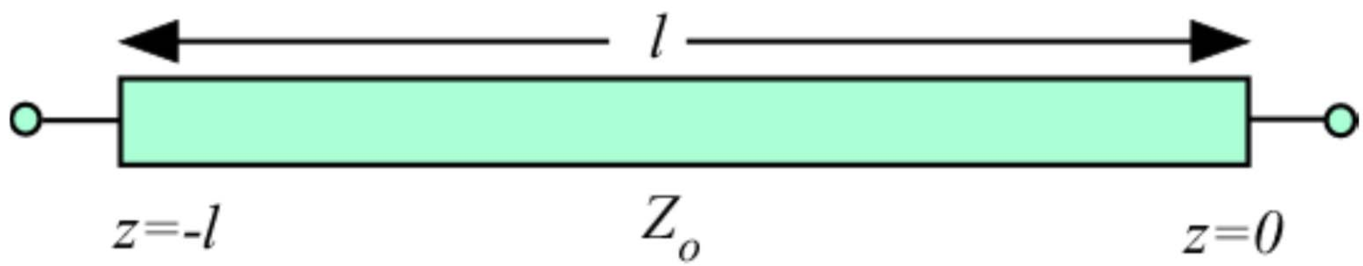
Find the Y-parameters of a lossless transmission line with propagation constant β and characteristic impedance Z_o (admittance Y_o)

$$V(z) = V_+ e^{-j\beta z} + V_- e^{+j\beta z}$$

$$I(z) = Y_o\left(V_+ e^{-j\beta z} - V_- e^{+j\beta z}\right)$$

Let port 1 be at $z=-l$ and port 2 at $z=0$

Y-Parameters of TL



at port 1

$$V_1 = V_+ e^{+j\beta l} + V_- e^{-j\beta l}$$

$$I_1 = Y_o \left(V_+ e^{+j\beta l} - V_- e^{-j\beta l} \right)$$

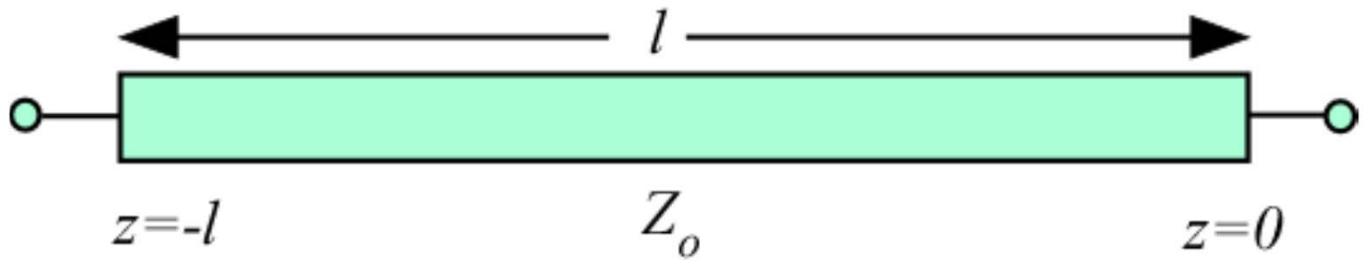
at port 2 (*z = 0*)

$$V_2 = V_+ + V_-$$

$$I_2 = -Y_o \left(V_+ - V_- \right)$$

$$V_+ = \frac{V_2 - Z_o I_2}{2} \quad \text{and} \quad V_- = \frac{V_2 + Z_o I_2}{2}$$

Y-Parameters of TL



So that

$$V_1 = \left(\frac{V_2 - Z_o I_2}{2}\right)e^{+j\beta l} + \left(\frac{V_2 + Z_o I_2}{2}\right)e^{-j\beta l}$$

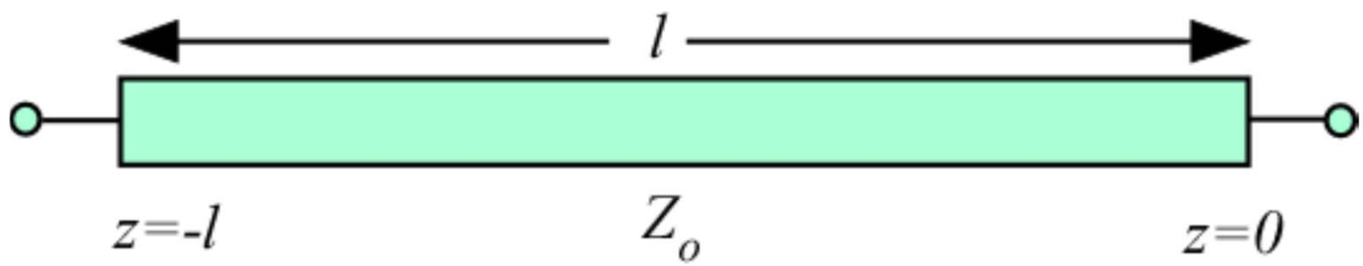
$$I_1 = Y_o\left(\frac{V_2 - Z_o I_2}{2}\right)e^{+j\beta l} - Y_o\left(\frac{V_2 + Z_o I_2}{2}\right)e^{-j\beta l}$$

and

$$V_1 = V_2 \cos\beta l - Z_o I_2 j \sin\beta l$$

$$I_1 = +Y_o V_2 j \sin\beta l - I_2 \cos\beta l$$

Y-Parameters of TL



Using definitions for Y_{11}

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{-I_2\cos\beta l}{-jZ_oI_2\sin\beta l} = \frac{-jY_o\cos\beta l}{\sin\beta l}$$

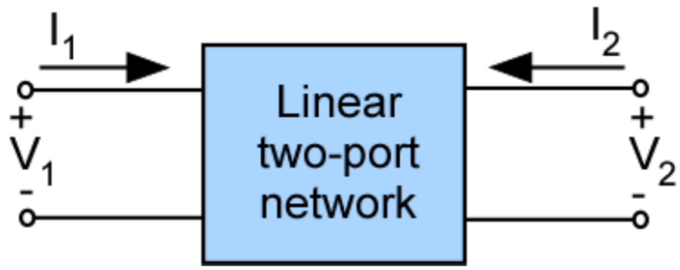
and

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-I_2}{-jZ_oI_2\sin\beta l} = \frac{+jY_o}{\sin\beta l}$$

$Y_{22} = Y_{11}$ by symmetry

$Y_{12} = Y_{21}$ by reciprocity

TWO-PORT NETWORK REPRESENTATION



<u>Z Parameters</u>

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

<u>Y Parameters</u>

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

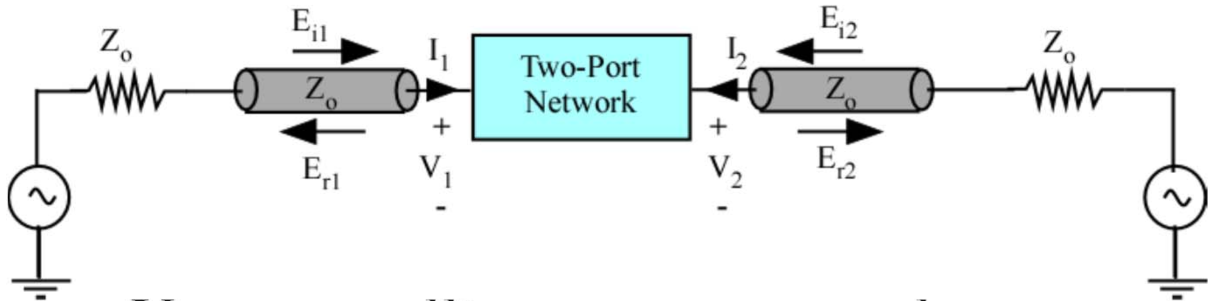
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

- At microwave frequencies, it is more difficult to measure total voltages and currents.

- Short and open circuits are difficult to achieve at high frequencies.

- Most active devices are not short- or open-circuit stable.

Wave Approach



Use a travelling wave approach

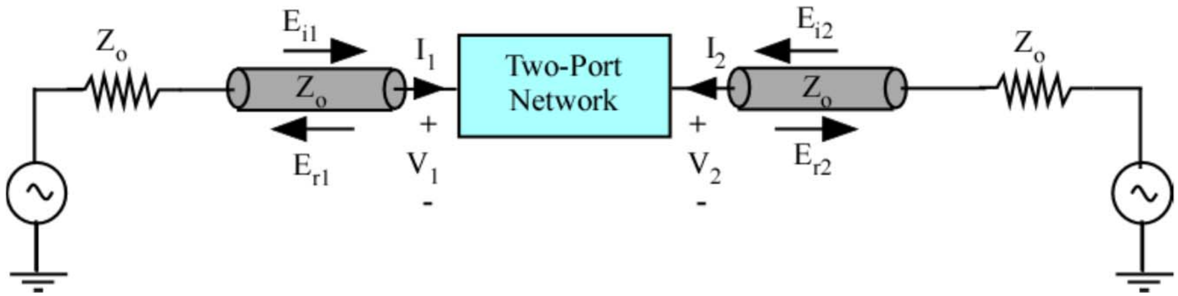
$$V_1 = E_{i1} + E_{r1} \qquad V_2 = E_{i2} + E_{r2}$$

$$I_1 = \frac{E_{i1} - E_{r1}}{Z_o} \qquad I_2 = \frac{E_{i2} - E_{r2}}{Z_o}$$

- **Total voltage and current are made up of sums of forward and backward traveling waves.**

- **Traveling waves can be determined from standing-wave ratio.**

Wave Approach



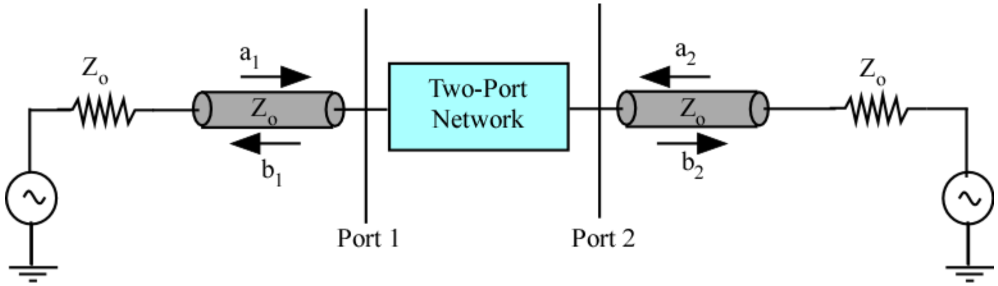
$$a_1 = \frac{E_{i1}}{\sqrt{Z_o}} \qquad a_2 = \frac{E_{i2}}{\sqrt{Z_o}}$$

$$b_1 = \frac{E_{r1}}{\sqrt{Z_o}} \qquad b_2 = \frac{E_{r2}}{\sqrt{Z_o}}$$

Z_o is the reference impedance of the system

$$\mathbf{b_1 = S_{11}\, a_1 + S_{12}\, a_2}$$
$$\mathbf{b_2 = S_{21}\, a_1 + S_{22}\, a_2}$$

Wave Approach



$$S_{11} = \frac{b_1}{a_1}\Big|_{a2=0} \qquad S_{21} = \frac{b_2}{a_1}\Big|_{a2=0}$$

$$S_{12} = \frac{b_1}{a_2}\Big|_{a1=0} \qquad S_{22} = \frac{b_2}{a_2}\Big|_{a1=0}$$

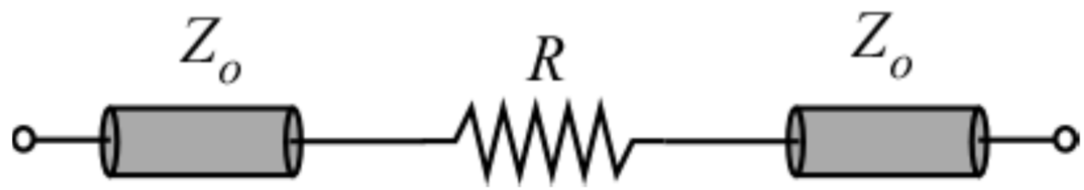
To make $a_i = 0$

1) Provide no excitation at port i

2) Match port i to the characteristic impedance of the reference lines.

CAUTION : a_i and b_i are the traveling waves in the <u>reference lines</u>.

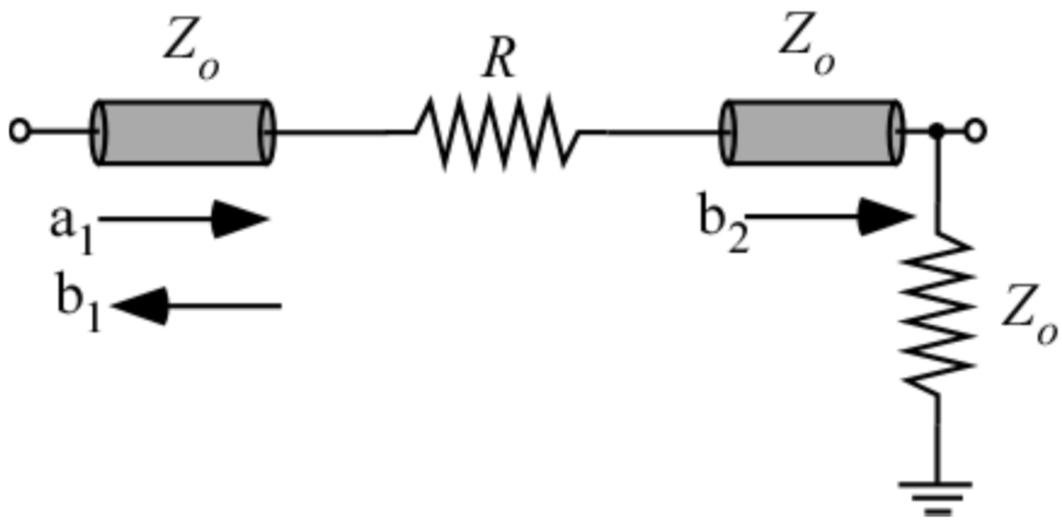
S-Parameters of Resistor



Determine S-Parameter of 2-port resistance

- Insert R between two reference TL
- Provide excitation at port 1 for S_{11} and S_{21}
- Provide excitation at port 2 for S_{12} and S_{22}
- Can use symmetry and reciprocity

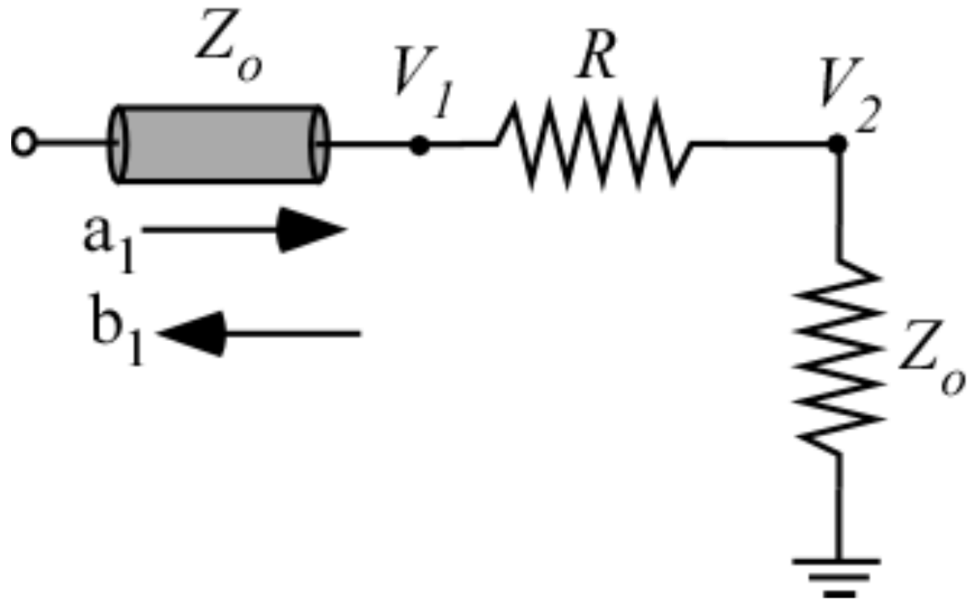
S-Parameters of Resistor



$$S_{11} = \frac{b_1}{a_1} = \Gamma = \frac{(R+Z_o)-Z_o}{(R+Z_o)+Z_o} = \frac{R}{R+2Z_o}$$

$$S_{11} = \frac{R}{R+2Z_o}$$ **and by symmetry,** $$S_{22} = \frac{R}{R+2Z_o}$$

Calculating S_{21} of Resistor

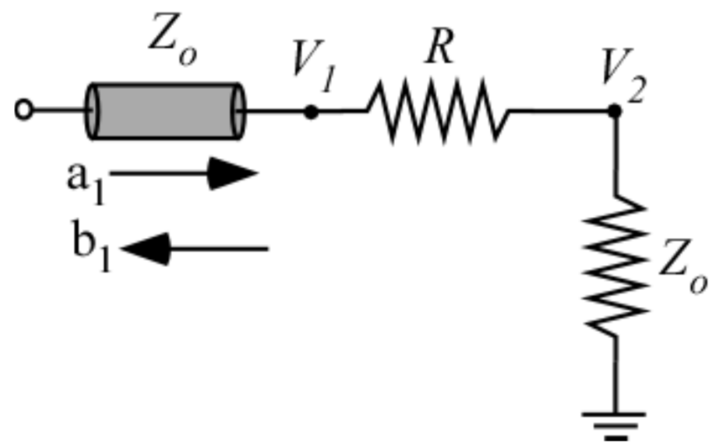


Since a_2=0, the total voltage in port 2 is: $V_2 = b_2\sqrt{Z_o}$

$$V_2 = \frac{V_1 Z_o}{R_1 + Z_o} = \frac{\sqrt{Z_o}\left(a_1 + b_1\right)Z_o}{R_1 + Z_o} = \frac{\sqrt{Z_o}\left(a_1 + S_{11}a_1\right)Z_o}{R_1 + Z_o}$$

S-Parameters of Resistor

$$V_2 = \frac{Z_o \sqrt{Z_o}\left(1 + S_{11}\right) a_1}{R_1 + Z_o} = \frac{2 Z_o a_1 \sqrt{Z_o}}{R_1 + 2Z_o}$$



$$S_{21} = \frac{b_2}{a_1} = \frac{V_2}{\sqrt{Z_o}} \frac{1}{a_1} = \frac{2Z_o}{R + 2Z_o}$$

$$S_{21} = \frac{2Z_o}{R + 2Z_o}$$ and by reciprocity, $$S_{12} = \frac{2Z_o}{R + 2Z_o}$$

S parameters of resistor R

$$S = \begin{bmatrix} \frac{R}{R + 2Z_o} & \frac{2Z_o}{R + 2Z_o} \\ \frac{2Z_o}{R + 2Z_o} & \frac{R}{R + 2Z_o} \end{bmatrix}$$

N-Port S Parameters

$$\begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdot & \cdot \\ S_{21} & S_{22} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ a_n \end{bmatrix}$$

$$\mathbf{b} = \mathbf{Sa}$$

If $b_i = 0$, then no reflected wave on port i ➔ port is matched

$$a_i = \frac{V_i^+}{\sqrt{Z_{oi}}}$$

$$b_i = \frac{V_i^-}{\sqrt{Z_{oi}}}$$

V_i^+ : **incident voltage wave in port i**

V_i^- : **reflected voltage wave in port i**

Z_{oi} : **impedance in port i**

N-Port S Parameters

$$\mathbf{v} = \sqrt{Z_o}\left(\mathbf{a}+\mathbf{b}\right) \quad (1) \qquad \mathbf{i} = \frac{1}{\sqrt{Z_o}}\left(\mathbf{a}-\mathbf{b}\right) \quad (2) \qquad \mathbf{v} = \mathbf{Z}\mathbf{i} \quad (3)$$

Substitute (1) and (2) into (3)

$$\sqrt{Z_o}\left(\mathbf{a}+\mathbf{b}\right) = \mathbf{Z}\frac{1}{\sqrt{Z_o}}\left(\mathbf{a}-\mathbf{b}\right)$$

Defining S such that b = Sa and substituting for b

$$Z_o\left(\mathbf{U}+\mathbf{S}\right)\mathbf{a} = Z_o\left(\mathbf{U}-\mathbf{S}\right)\mathbf{a}$$

U : unit matrix

S→Z

$$\mathbf{Z} = Z_o\left(\mathbf{U}+\mathbf{S}\right)\left(\mathbf{U}-\mathbf{S}\right)^{-1}$$

Z→S

$$\mathbf{S} = \left(\mathbf{Z}+Z_o\mathbf{U}\right)^{-1}\left(\mathbf{Z}-Z_o\mathbf{U}\right)$$

N-Port S Parameters

If the port reference impedances are different, we define k as

$$\mathbf{k} = \begin{bmatrix} \sqrt{Z_{o1}} & & & \\ & \sqrt{Z_{o2}} & & \\ & & \cdot & \\ & & & \sqrt{Z_{on}} \end{bmatrix}$$

$\mathbf{v} = \mathbf{k}(\mathbf{a}+\mathbf{b})$ and $\mathbf{i} = \mathbf{k}^{-1}(\mathbf{a}-\mathbf{b})$ and $\mathbf{k}(\mathbf{a}+\mathbf{b}) = \mathbf{Z}\mathbf{k}^{-1}(\mathbf{a}-\mathbf{b})$

Z→S

$$\mathbf{S} = \left(\mathbf{Z}\mathbf{k}^{-1}+\mathbf{k}\right)\left(\mathbf{Z}\mathbf{k}^{-1}-\mathbf{k}\right)$$

S→Z

$$\mathbf{Z} = \mathbf{k}\left(\mathbf{U}+\mathbf{S}\right)\left(\mathbf{U}-\mathbf{S}\right)^{-1}\mathbf{k}$$

Normalization

Assume original S parameters as S_1 with system k_1. Then the representation S_2 on system k_2 is given by

Transformation Equation

$$\mathbf{S_2 = \left[k_1(U+S_1)(U-S_1)^{-1}k_1k_2 + k_2\right]^{-1}\left[k_1(U+S_1)(U-S_1)^{-1}k_1k_2 - k_2\right]}$$

If Z is symmetric, S is also symmetric

Dissipated Power

$$P_d = \frac{1}{2}\mathbf{a}^{\mathrm{T}}\left(\mathbf{U} - \mathbf{S}^{\mathrm{T}}\mathbf{S}^{*}\right)\mathbf{a}^{*}$$

The dissipation matrix D is given by:

$$\mathbf{D} = \mathbf{U} - \mathbf{S}^{\mathrm{T}}\mathbf{S}^{*}$$

Passivity insures that the system will always be stable provided that it is connected to another passive network

For passivity

- **(1) the determinant of D must be** ≥ 0
- **(2) the determinant of the principal minors must be** ≥ 0

Dissipated Power

When the dissipation matrix is 0, we have a lossless network➔

$$\mathbf{S}^T\mathbf{S}^* = \mathbf{U}$$

The S matrix is unitary.

For a lossless two-port:

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

$$|S_{22}|^2 + |S_{12}|^2 = 1$$

If in addition the network is reciprocal, then

$$S_{12} = S_{21} \text{ and } |S_{11}| = |S_{22}| = \sqrt{1-|S_{12}|^2}$$