ECE 451
Advanced Microwave Measurements

Transmission Lines - TDR

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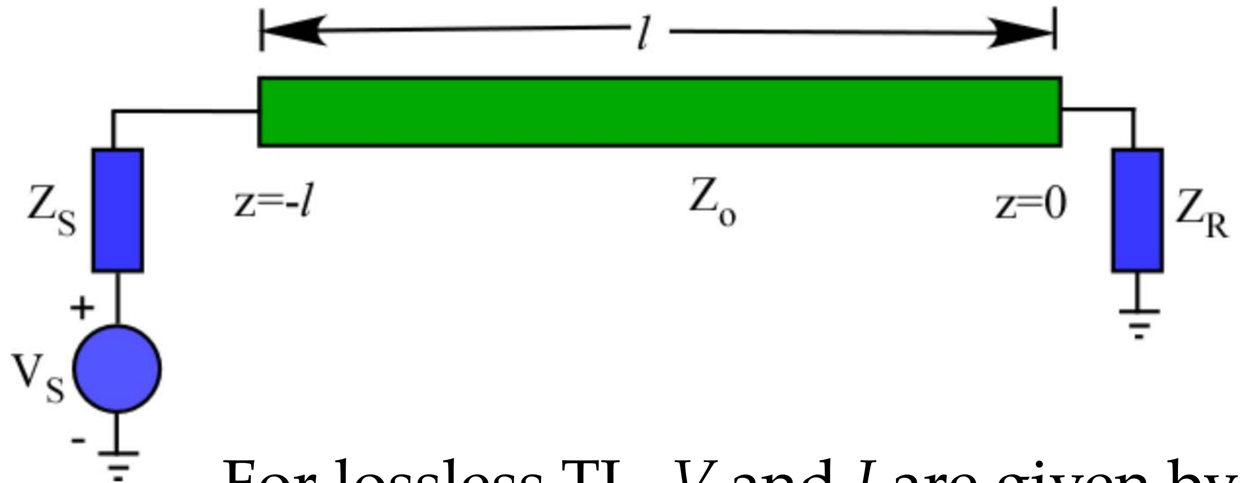
Determining V_+



For lossless TL, *V* and *I* are given by

$$V(z) = V_+ e^{-j\beta z}\left[1+\Gamma_R e^{+2j\beta z}\right]$$

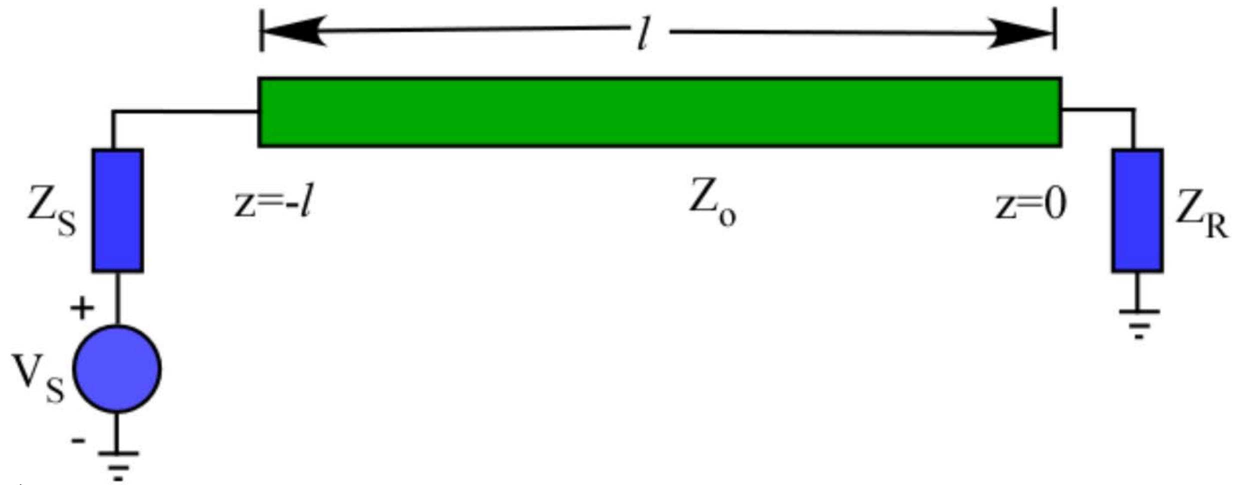
reflection coefficient at the load

$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o}$$

$$I(z) = \frac{V_+ e^{-j\beta z}}{Z_o}\left[1-\Gamma_R e^{+2j\beta z}\right]$$

$$At\ z = \text{-}l, \quad V_S = Z_S I(-l) + V(-l)$$

Determining V_+



this leads to

$$V_S = V_+ e^{+j\beta l}\left(1+\Gamma_R e^{-2j\beta l}\right) + \frac{Z_S}{Z_o} V_+ e^{+j\beta l}\left(1-\Gamma_R e^{-2j\beta l}\right)$$

or

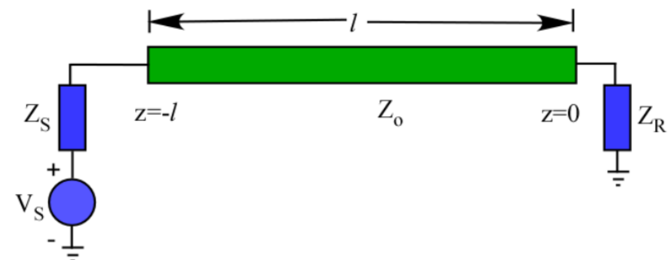
$$V_S = V_+\left(e^{+j\beta l} + \Gamma_R e^{-j\beta l} + \frac{Z_S}{Z_o} e^{+j\beta l} - \Gamma_R \frac{Z_S}{Z_o} e^{-j\beta l}\right)$$

$$V_S = V_+\left(e^{+j\beta l}\left(1+\frac{Z_S}{Z_o}\right) + \Gamma_R e^{-j\beta l}\left(1-\frac{Z_S}{Z_o}\right)\right)$$

Determining V_+

Divide through by $\left(1+\frac{Z_S}{Z_o}\right)=\frac{1}{T_S}$

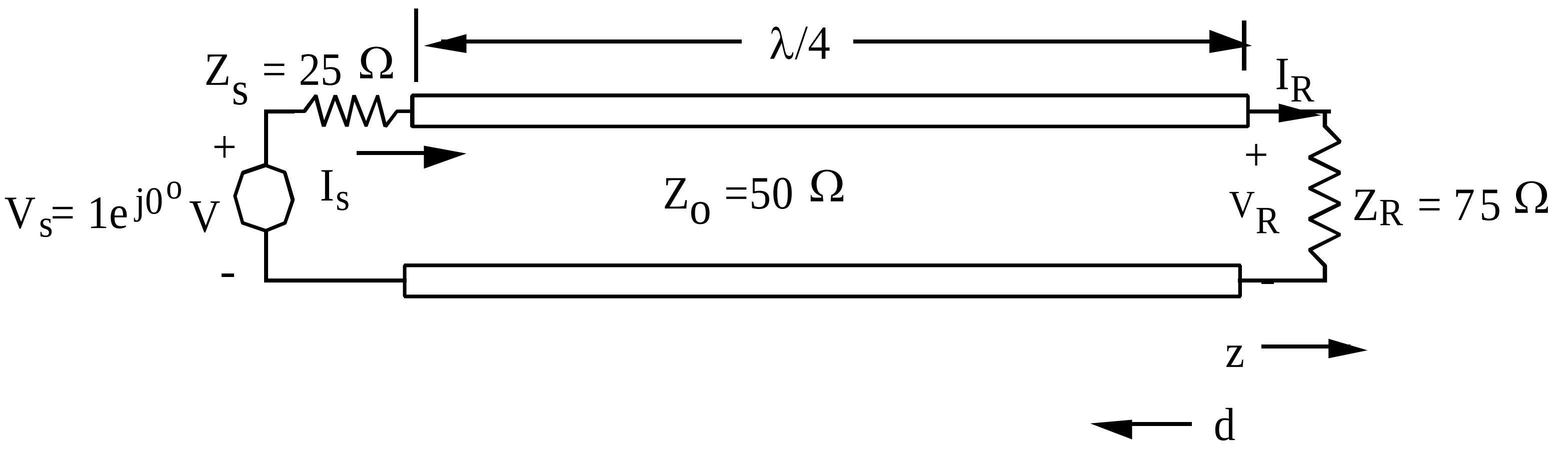
$$V_+\left(e^{+j\beta l}-\Gamma_S\Gamma_R e^{-j\beta l}\right)=T_S V_S$$



with $T_S=\left(1+\frac{Z_S}{Z_o}\right)^{-1}=\frac{Z_o}{Z_S+Z_o}$ and $\Gamma_S=\frac{Z_S-Z_o}{Z_S+Z_o}$

From which $$V_+=\frac{T_S V_S e^{-j\beta l}}{1-\Gamma_S\Gamma_R e^{-2j\beta l}}$$

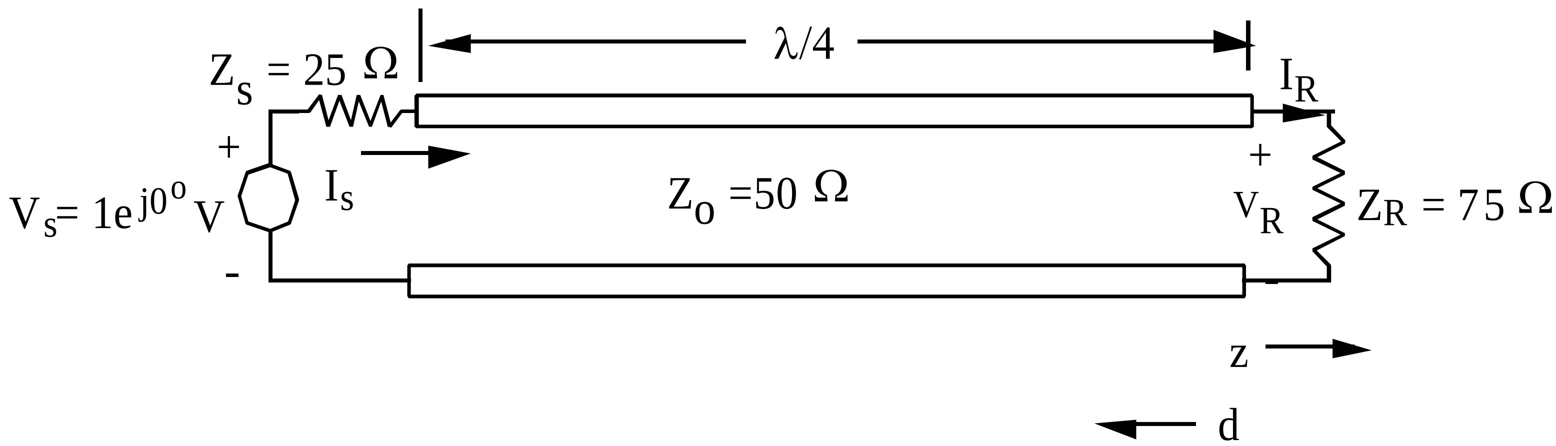
TL Example



A signal generator having an internal resistance $Z_s = 25\ \Omega$ and an open circuit phasor voltage $V_s = 1e^{j0}$ volt is connected to a 50-Ω lossless transmission line as shown in the above picture. The load impedance is $Z_R = 75\ \Omega$ and the line length is $\lambda/4$.

Find the magnitude and phase of the load current I_R.

TL Example – Cont'



$$V_{+} = \frac{T_s V_s e^{-j\beta l}}{1 - \Gamma_R \Gamma_s e^{-2j\beta l}} \qquad T_s = \frac{Z_o}{Z_s + Z_o} = \frac{50}{50+25} = 2/3$$

$$\Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o} = \frac{25-50}{25+50} = -1/3 \qquad \Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o} = \frac{75-50}{75+50} = 1/5$$

TL Example – Cont’

$$\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2} \Rightarrow e^{-j\beta l} = -j$$

$$V_+ = \frac{(2/3)(1)(-j)}{1-(-1/3)(1/5)(-1)} = \frac{-j2/3}{1-1/15} = -j5/7$$

$$V_+ = -j0.714285 \text{ V}$$

$$\mathrm{I_R} = \frac{V_+}{Z_o}\left[1-\Gamma_R\right] = -j\frac{0.714285}{50}\left[1-0.2\right] = -j\frac{0.714285\times 0.8}{50}$$

$$\mathrm{I_R} = -j0.014285 \text{ A}$$

Geometric Series Expansion

$$\text{Since } \left|\Gamma_S \Gamma_R e^{-2j\beta l}\right| \le 1$$

V_+ can be expanded in a geometric series form

$$V_+ = \frac{T_S V_S e^{-j\beta l}}{1 - \Gamma_S \Gamma_R e^{-2j\beta l}} \qquad V_+ = T_S \sum_{k=0}^{\infty} V_S \Gamma_S^k \Gamma_R^k e^{-2j\beta k l} e^{-j\beta l}$$

$$V(z) = T_S \sum_{k=0}^{\infty} V_S \Gamma_S^k \Gamma_R^k e^{-j\beta l(2k+1)} e^{-j\beta z} + T_S \sum_{k=0}^{\infty} V_S \Gamma_S^k \Gamma_R^{k+1} e^{-j\beta l(2k+1)} e^{+j\beta z}$$

$$\beta = \frac{\omega}{v}$$

$$V(z) = T_S \sum_{k=0}^{\infty} V_S \Gamma_S^k \Gamma_R^k e^{-j\frac{\omega}{v}\left[z+(2k+1)l\right]} + T_S \sum_{k=0}^{\infty} V_S \Gamma_S^k \Gamma_R^{k+1} e^{-j\frac{\omega}{v}\left[z+(2k+1)l\right]}$$

TL Time-Domain Solution

$$v(z,t) = T_S \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_R^k v_s \left(t - \frac{(2k+1)l + z}{v_o} \right)$$

$$+ T_S \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_R^{k+1} v_s \left(t - \frac{(2k+1)l - z}{v_o} \right)$$

at z=0

$$v(0,t) = T_S \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_R^k v_s \left(t - \frac{(2k+1)l}{v_o} \right)$$

$$+ T_S \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_R^{k+1} v_s \left(t - \frac{(2k+1)l}{v_o} \right)$$

TL Time-Domain Solution

At z=-l

$$v(-l,t)=T_S\sum_{k=0}^{\infty}\Gamma_S^k\Gamma_R^k v_s\left(t-\frac{(2k+1)l+l}{v_o}\right)$$

$$+T_S\sum_{k=0}^{\infty}\Gamma_S^k\Gamma_R^{k+1} v_s\left(t-\frac{(2k+1)l+l}{v_o}\right)$$

$$v(-l,t)=T_S\sum_{k=0}^{\infty}\Gamma_S^k\Gamma_R^k v_s\left(t-\frac{2kl}{v_o}\right)$$

$$+T_S\sum_{k=0}^{\infty}\Gamma_S^k\Gamma_R^{k+1} v_s\left(t-\frac{2(k+1)l}{v_o}\right)$$

TL - Time-Domain Reflectometer

For TDR, $Z_S = Z_o \rightarrow \Gamma_S = 0$, and retain only $k=1$

$$v(-l,t) = T_S v_s\left(t\right) + T_S \Gamma_R v_s\left(t - \frac{2l}{v_o}\right)$$