ECE 451
Advanced Microwave Measurements

Error Correction

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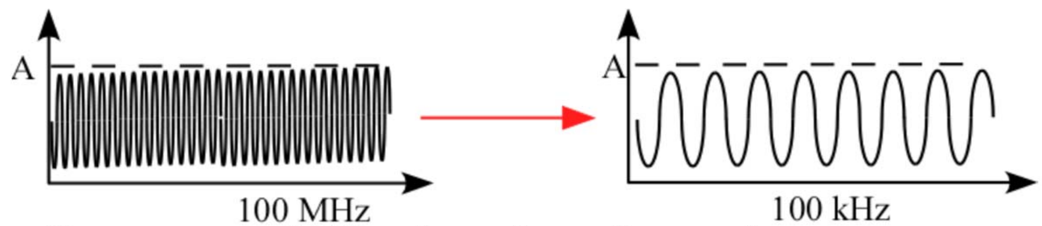
Network Analyzer

Source provides RF/microwave signal and consists of high-frequency circuitry with internal impedance of 50 Ω.

Test set consists of couplers used to separate signals. There are also power dividers, switches all of which must operate at the RF/microwave frequency of interest.

Signals REF, A, & B are routed to analyzer which down-converts RF signals to intermediate frequency.



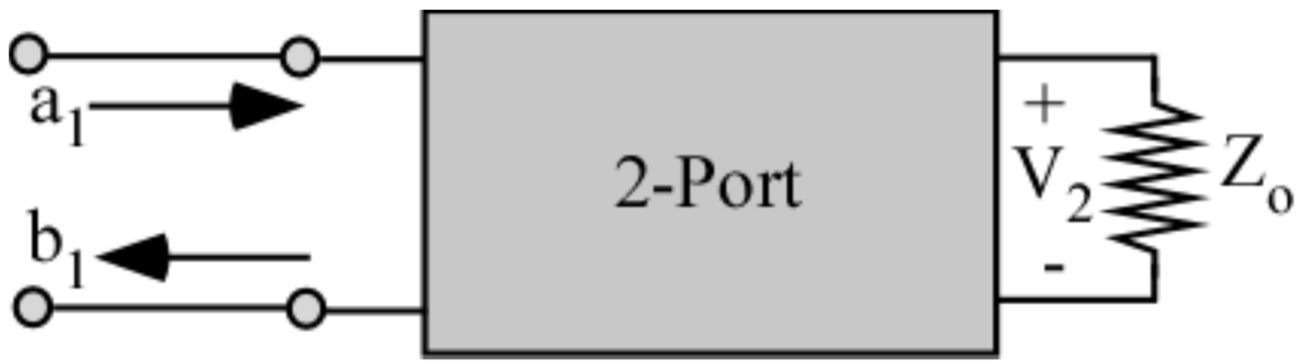
Signals are then amplified using low-frequency amplifiers and detected using low-frequency detectors.

Display shows ratios B/REF or A/REF for S_{11}, S_{21} in magnitude and/or phase format

8510C Network Analyzer

Two-Port Measurement

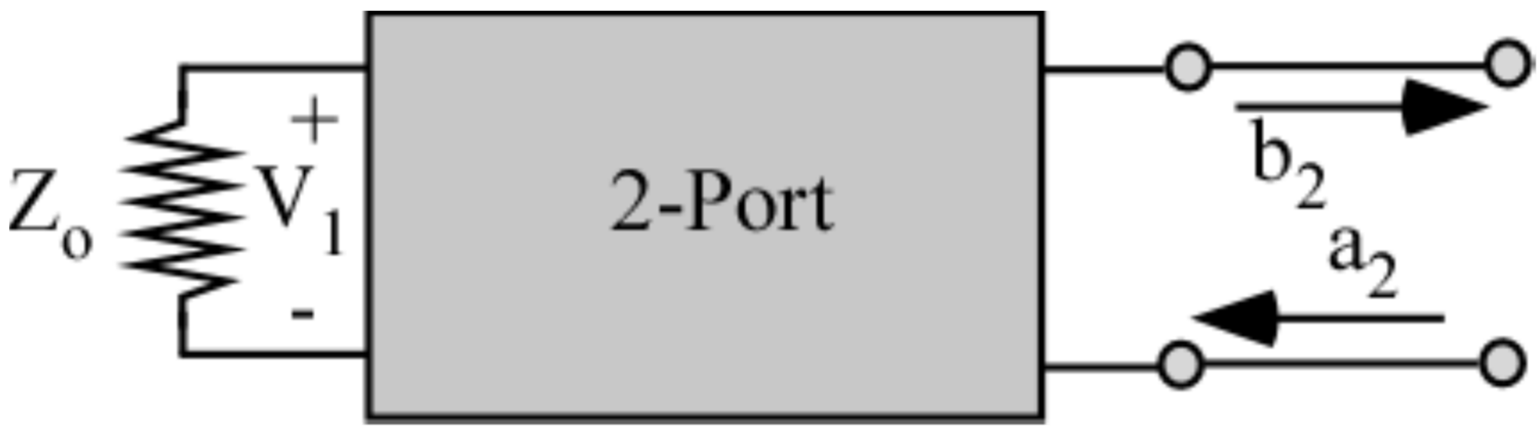
Forward



$$S_{11} = \frac{b_1}{a_1}\bigg|_{a2=0} \qquad S_{21} = \frac{b_2}{a_1}\bigg|_{a2=0}$$

Two-Port Measurement

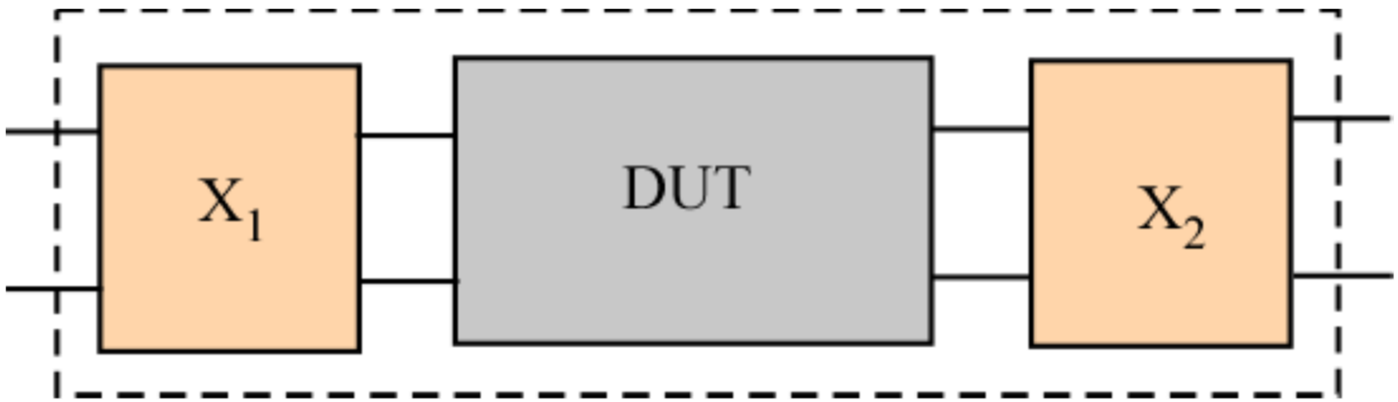
Reverse



$$S_{12} = \frac{b_1}{a_2}\Big|_{a1=0} \qquad S_{22} = \frac{b_2}{a_2}\Big|_{a1=0}$$

Two-Port Measurement

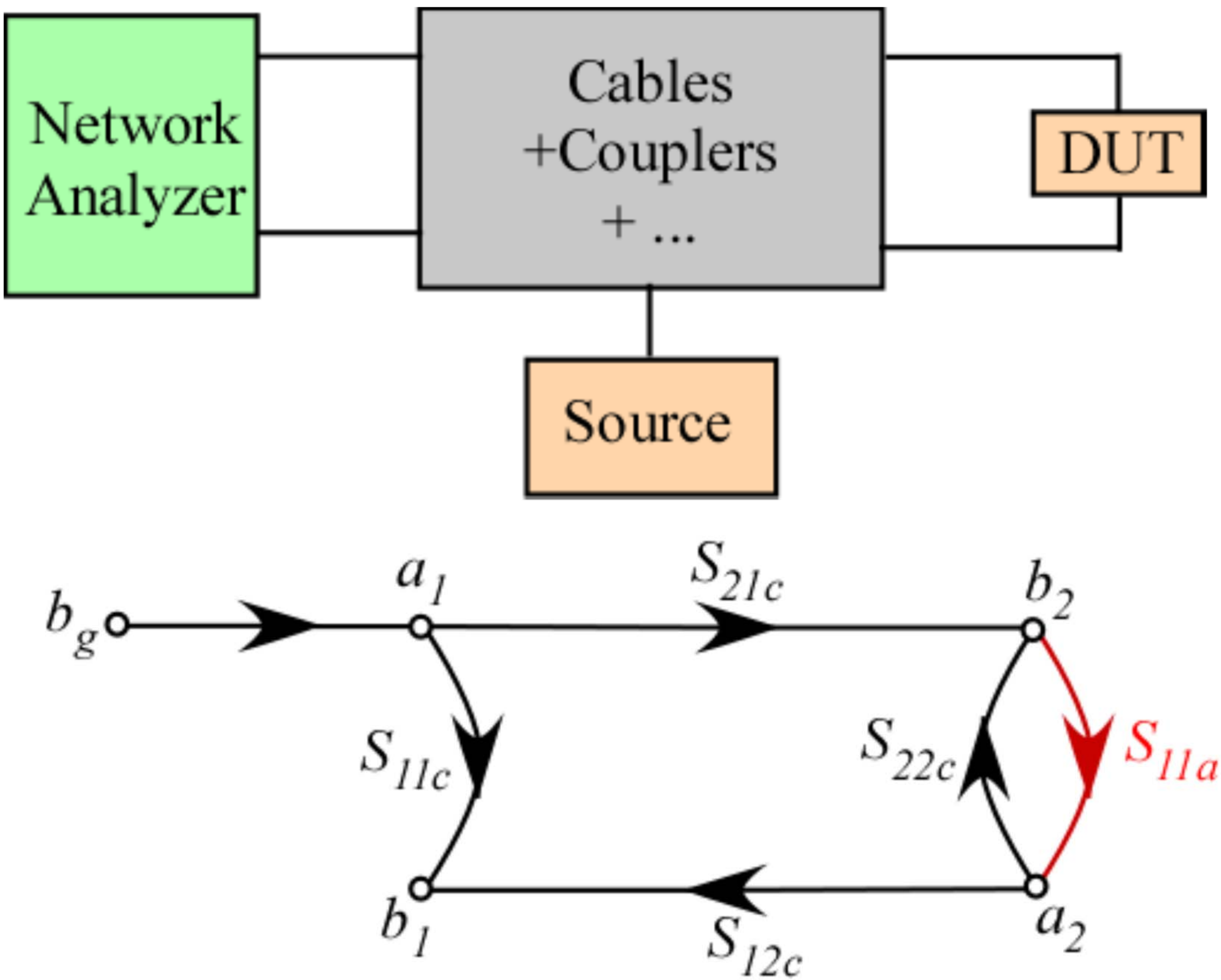
In general, the measured S_{11}, S_{12}, S_{21} and S_{22} are not the parameters of the actual DUT



Need to remove the effects of X_1 and X_2

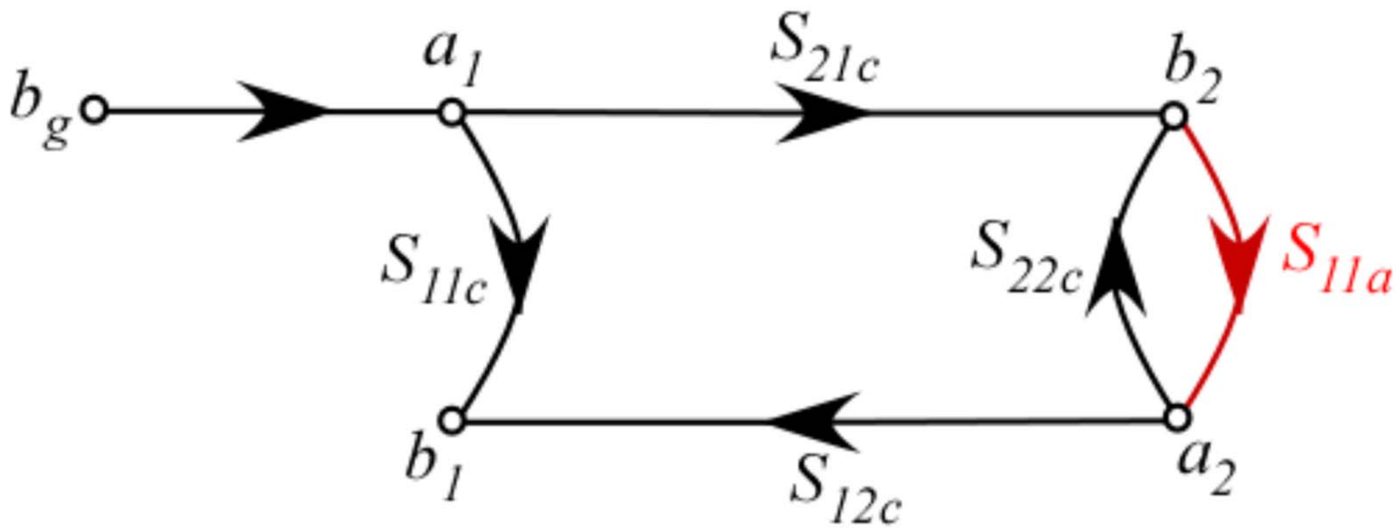
Calibration

One-Port Measurement



The system of cables, couplers, etc… represents a 2-port and must be de-embedded in order to obtain the actual S_{11} of the unknown➔S_{11a}

One-Port Measurement



Assume that the network analyzer is perfectly matched. Then,

$$S_{11m} = \frac{b_1}{a_1} = S_{11c} + \frac{S_{12c} S_{21c} S_{11a}}{1 - S_{22c} S_{11a}}$$

All quantities are complex and frequency-dependent!

One-Term Error Model

If we assume that the cables and couplers are a perfect 50-Ω system, then $S_{11c}=S_{22c}=0$. We have a *one-term error model.*

$$S_{11m} = S_{12c}S_{21c}S_{11a} = TS_{11a}$$

where $T = S_{12c}S_{21c}$

T is not known. To determine T, we first measure a short since for a short, $S_{11a}^{(short)} = -1$

We get $S_{11m}^{(short)} = TS_{11a}^{(short)} = -T$

known

One-Term Error Model

Once T is known, we can then measure the DUT

$$S_{11m}^{(DUT)} = TS_{11a}^{(DUT)}$$

From which

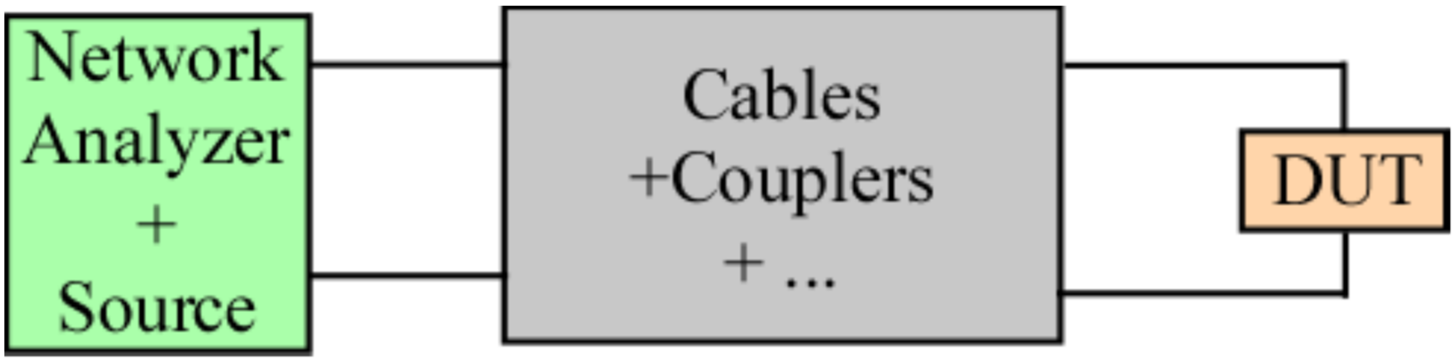
$$S_{11a}^{(DUT)} = \frac{S_{11m}^{(DUT)}}{T} = \frac{-S_{11m}^{(DUT)}}{S_{11m}^{(short)}}$$

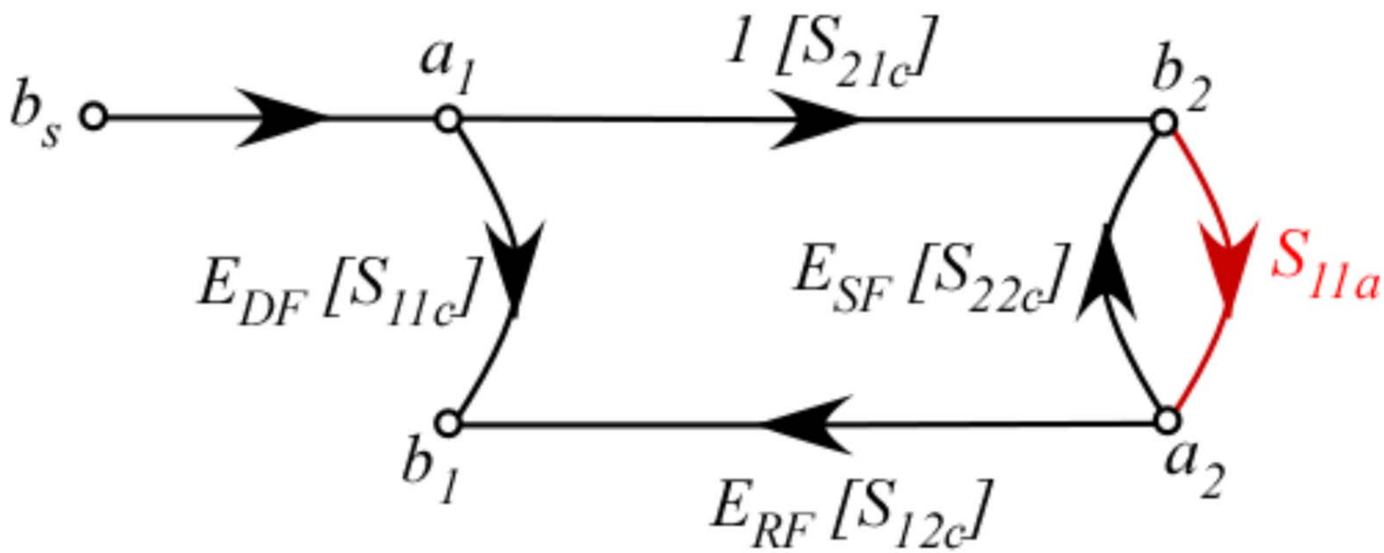
Practical Observations:

Since the correction involves a simple complex division, we can do the following

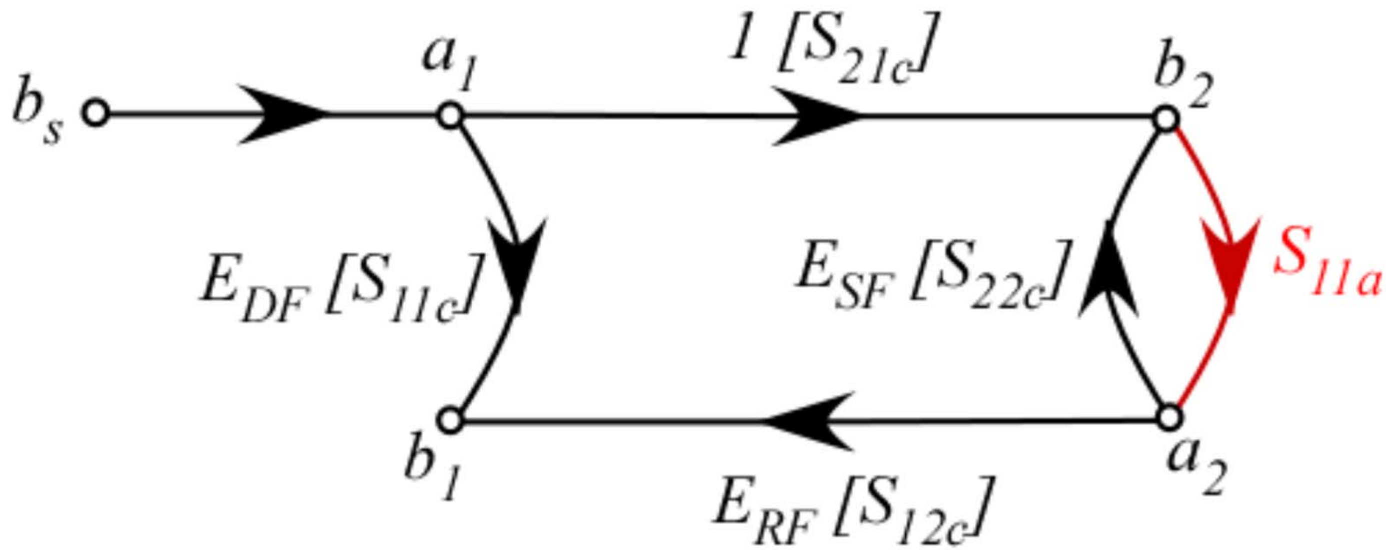
$$\left|S_{11a}^{(DUT)}\right| = \frac{\left|S_{11m}^{(DUT)}\right|}{\left|S_{11m}^{(short)}\right|} \quad \text{and} \quad \angle S_{11a}^{(DUT)} = \angle S_{11m}^{(DUT)} - \angle S_{11m}^{(short)} \pm 180°$$

Three-Term Error Model





Three-Term Error Model



- **E_{DF}: Directivity of couplers (leak through test ports)**
- **E_{RF}: reflection tracking error (signal path tracking error)**
- **E_{SF}: Source match error**

$$S_{11m} = \Gamma_{in} = S_{11c} + \frac{S_{12c}S_{21c}S_{11a}}{1 - S_{22c}S_{11a}} \quad \text{or} \quad S_{11m} = E_{DF} + \frac{E_{RF}S_{11a}}{1 - E_{SF}S_{11a}}$$

Three-Term Error Model

Step 1:

Use matched load (Z_o = 50 Ω) as DUT ➔ S_{11a}=0

$$S_{11m}^{(load)} = E_{DF} = A \qquad (1)$$

Step 2:

Use a perfect short as DUT ➔ S_{11a}= -1

$$S_{11m}^{(short)} = E_{DF} - \frac{E_{RF}}{1+E_{SF}} = B \qquad (2)$$

Step 3:

Use a perfect open as DUT ➔ S_{11a}= +1

$$S_{11m}^{(open)} = E_{DF} + \frac{E_{RF}}{1-E_{SF}} = C \qquad (3)$$

Three-Term Error Model

Combining (1), (2), and (3) gives E_{SF}, E_{RF} and E_{DF}

$$E_{DF} = S_{11m}^{(load)} = A$$

$$E_{SF} = \frac{B+C-2A}{C-B} = \frac{S_{11m}^{(short)} + S_{11m}^{(open)} - 2S_{11m}^{(load)}}{S_{11m}^{(open)} - S_{11m}^{(short)}}$$

$$E_{RF} = \frac{-2(B-A)(C-A)}{C-B} = \frac{-2\left(S_{11m}^{(short)} - S_{11m}^{(load)}\right)\left(S_{11m}^{(open)} - S_{11m}^{(load)}\right)}{S_{11m}^{(open)} - S_{11m}^{(short)}}$$

Step 4: Measuring the unknown DUT

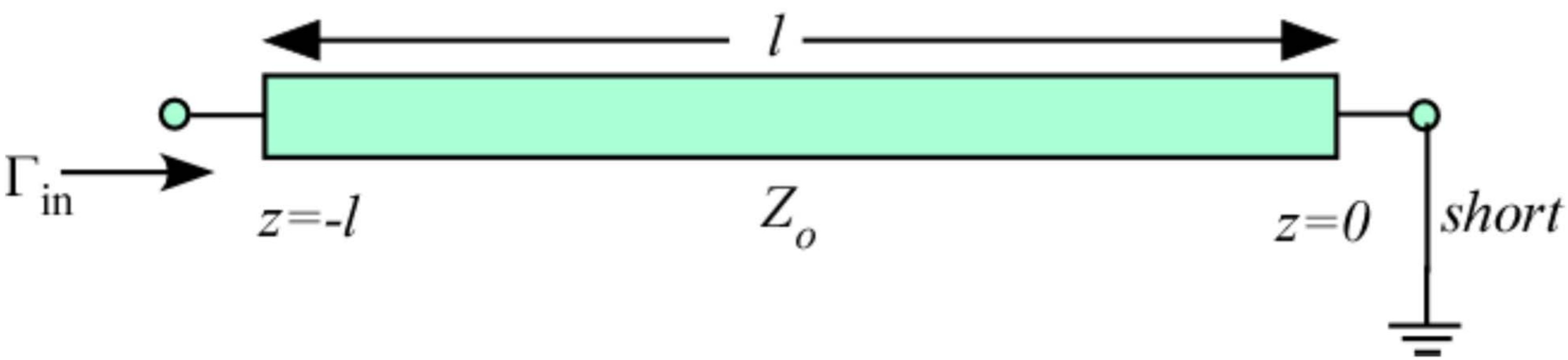
This is the actual S_{11} with corrections. →

$$S_{11a}^{(DUT)} = \frac{S_{11m}^{(DUT)} - E_{DF}}{E_{RF} + E_{SF}\left[S_{11m}^{(DUT)} - E_{DF}\right]}$$

Alternative Calibration Standards

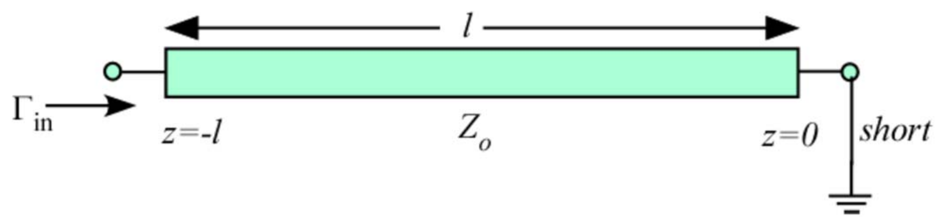
At very high frequencies, it is difficult to make a good short, open or matched termination. We need to find alternative standards for calibration.

Offset Short



TL of length l terminated with a short

Offset Short Standard



At z=0, $\Gamma = \Gamma_L$

At z=-l, $\Gamma(-l) = \Gamma_L e^{-2j\beta l}$

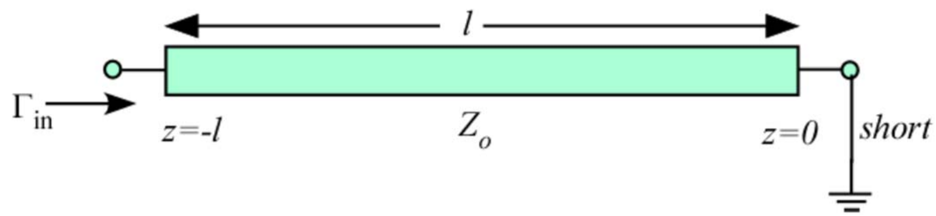
Since $\Gamma_L = -1,\ \Gamma(-l) = \Gamma_{in} = -e^{-2j\beta l} = e^{j\left(\pi - \frac{4\pi l}{\lambda}\right)}$

$$\Gamma_{in} = e^{j\theta} \quad where \quad \theta = \pi\left(1 - \frac{4l}{\lambda}\right)$$

Therefore, when calibrating with an offset short, we use: $S_{11a}^{(offset\ short)} = e^{j\theta}$

where θ is known:

Offset Short Restriction



The offset short will only work if the frequency range is such that $0 < l < \lambda / 2$

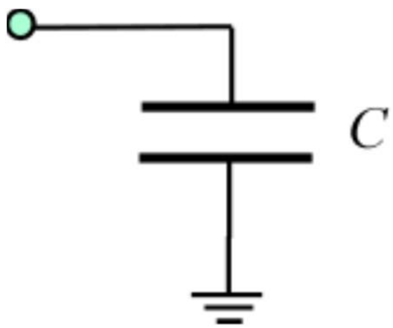
This corresponds to a frequency range of

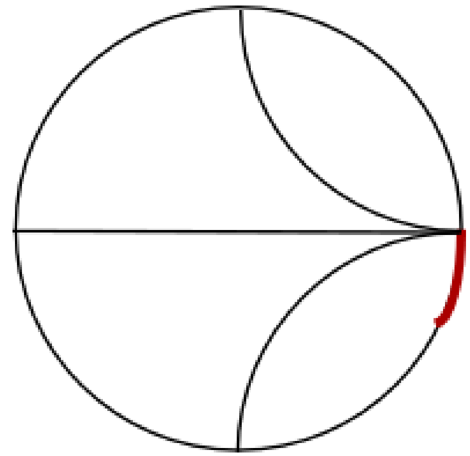
$$f < \frac{v}{2l}$$

where v is the propagation velocity in the line.

Shielded Open Standard

The shielded open can be modeled as a controlled capacitor.





For a system with reference impedance of Z_o, the associated reflection coefficient is:

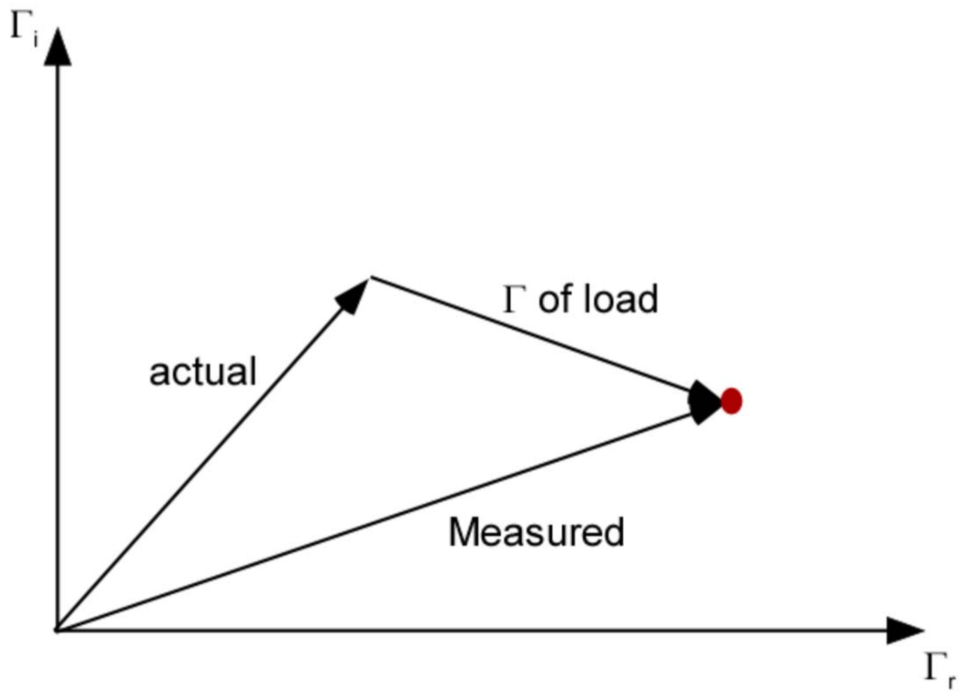
$$\Gamma_{in} = \frac{1/j\omega C - Z_o}{1/j\omega C + Z_o} = \frac{1 - j\omega C Z_o}{1 + j\omega C Z_o} = \frac{1 - ja}{1 + ja} \qquad \text{with } a = \omega C Z_o$$

$$\Gamma_{in} = e^{-2j\tan^{-1}a}$$

So, for shielded open, we use $S_{11a}^{(shielded\ open)} = e^{-2j\tan^{-1}a}$

Sliding Load

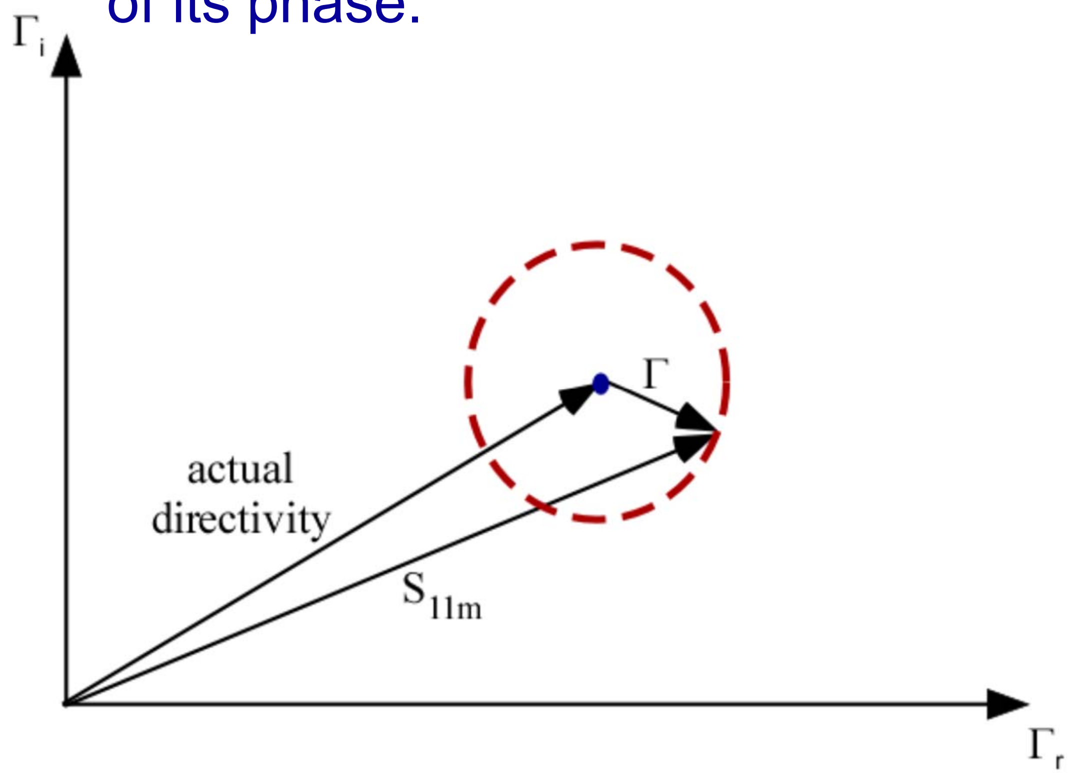
<u>Motivation:</u> Need to accurately measure the actual directivity error of the system

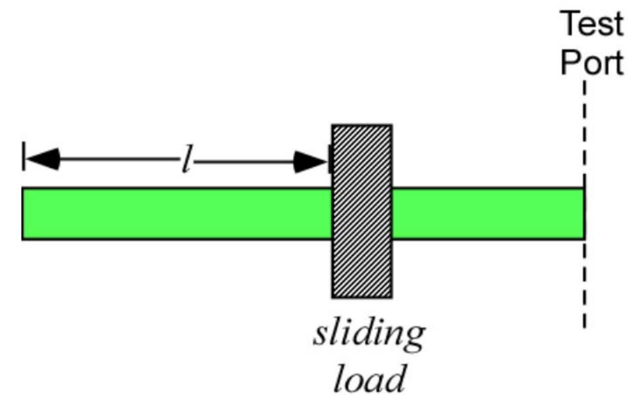


<u>Observation:</u> If termination is imperfect, then the measured directivity is the vector sum of the actual directivity and the reflection coefficient of the load.

Sliding Load

With the sliding load, a small Γ is willfully introduced and varied in terms of its phase.





By sliding the load at a given frequency point, a circle is defined about the tip of the directivity vector.

We find the best circle that fits the measured S_{11}. The center of that circle is the tip of the actual (desired) directivity vector.

Alternate Combinations

- Matched Load
- Offset Short
- Short

- Matched Load
- Short
- Shielded open

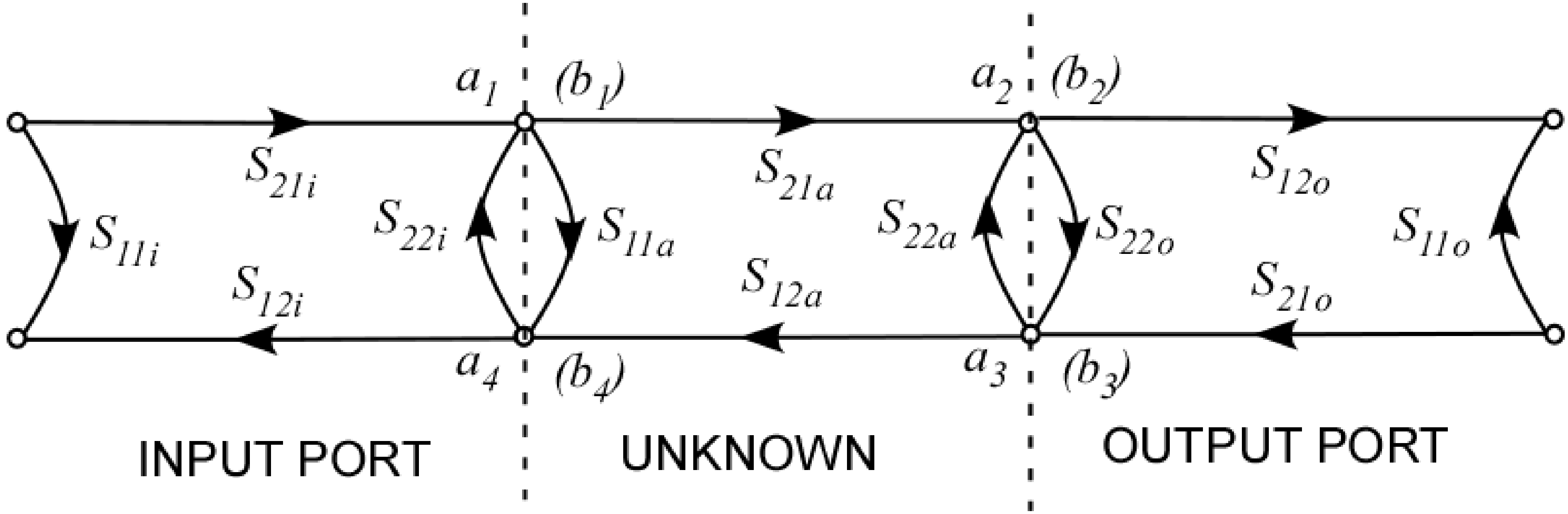
- Matched Load
- Offset Short
- Shielded open

- Sliding Load
- Offset Short
- Short

- Sliding Load
- Short
- Shielded open

- Sliding Load
- Offset Short
- Shielded open

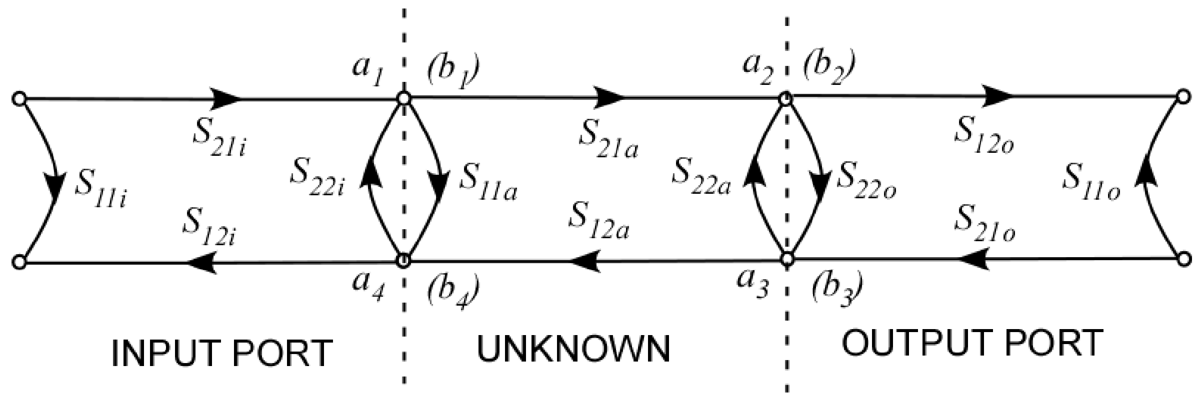
8-Term Error Model



Calibration consists of determining the *i* and *o* terms by placing known standards as the *a* terms.

8-Term Error Model

Calibration Stage: Reflection/Port 1

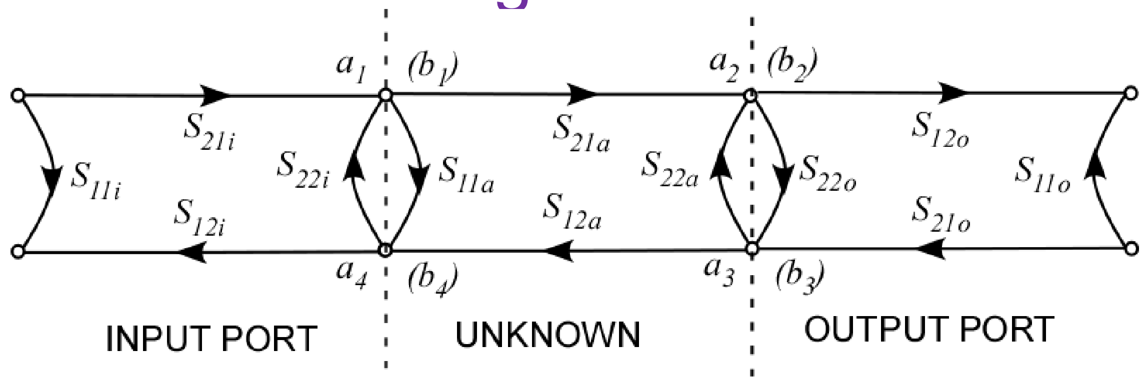


Placing open, short and load as standards (S_{11a}) in port 1 yields

$$S_{11m}^{(op)} = S_{11i} + \frac{S_{21i}S_{12i}}{e^{j\beta} - S_{21i}} \qquad S_{11m}^{(sh)} = S_{11i} - \frac{S_{21i}S_{12i}}{1 + S_{21i}} \qquad S_{11m}^{(ld)} = S_{11i}$$

8-Term Error Model

Calibration Stage: Reflection/Port 1



The system is simultaneously solved to give

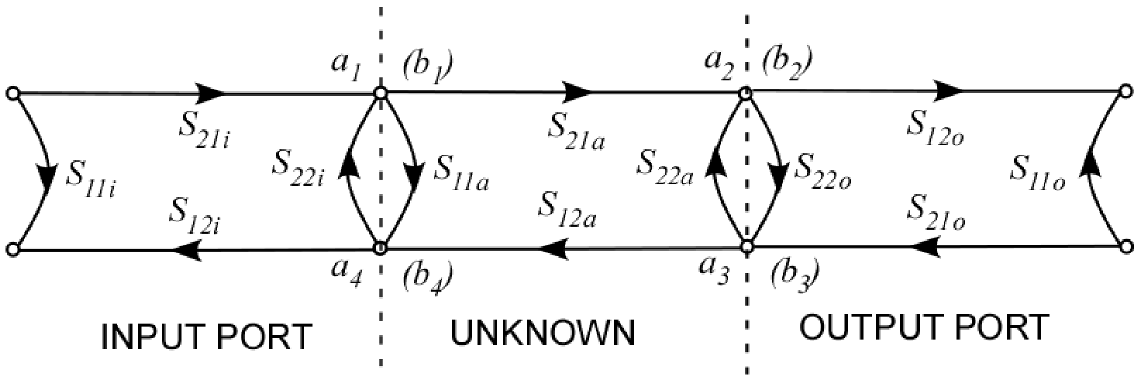
$$S_{11i} = S_{11m}^{(ld)}$$

$$S_{22i} = \frac{e^{j\beta}\left[S_{11m}^{(op)} - S_{11m}^{(ld)}\right] - \left[S_{11m}^{(ld)} - S_{11m}^{(sh)}\right]}{S_{11m}^{(op)} - S_{11m}^{(sh)}}$$

$$S_{12i}S_{21i} = \frac{\left(1+e^{j\beta}\right)\left[S_{11m}^{(op)} - S_{11m}^{(ld)}\right]\left[S_{11m}^{(sh)} - S_{11m}^{(ld)}\right]}{S_{11m}^{(sh)} - S_{11m}^{(op)}}$$

8-Term Error Model

Calibration Stage: Reflection/Port 2



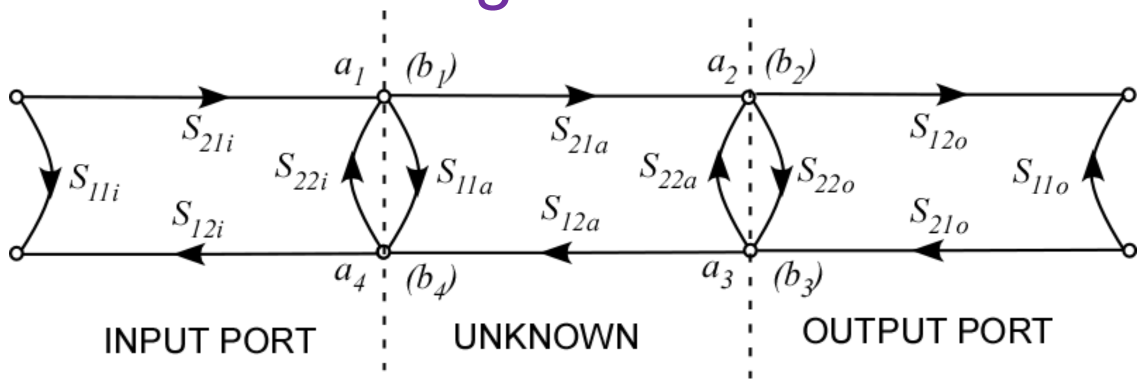
Same principle can be applied to port 2 to give

$$S_{11o} = S_{22m}^{(ld)} \qquad S_{22o} = \frac{e^{j\beta}\left[S_{22m}^{(op)} - S_{22m}^{(ld)}\right] - \left[S_{22m}^{(ld)} - S_{22m}^{(sh)}\right]}{S_{22m}^{(op)} - S_{22m}^{(sh)}}$$

$$S_{12o}S_{21o} = \frac{\left(1+e^{j\beta}\right)\left[S_{22m}^{(op)} - S_{22m}^{(ld)}\right]\left[S_{22m}^{(sh)} - S_{22m}^{(ld)}\right]}{S_{22m}^{(sh)} - S_{22m}^{(op)}}$$

8-Term Error Model

Calibration Stage: Transmission



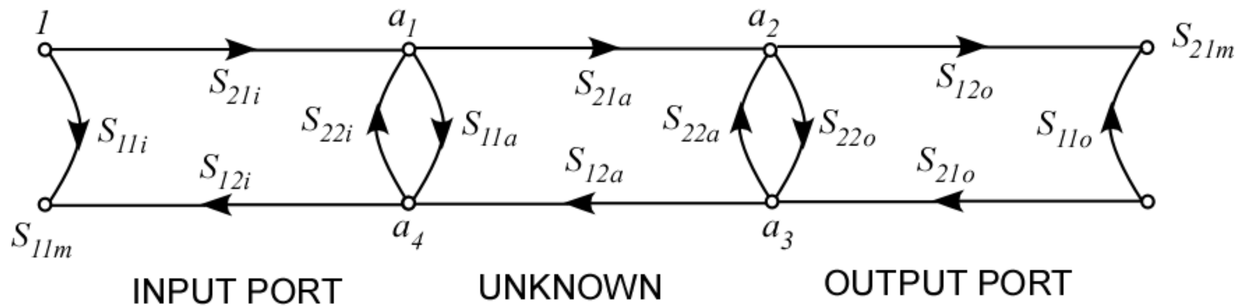
Next, connect the two ports together for transmission calibration $S_{21a}=S_{12a}=1$

$$S_{21m}^{(thr)} = \frac{S_{21i}S_{12o}}{1 - S_{22i}S_{22o}}$$

$$S_{12m}^{(thr)} = \frac{S_{12i}S_{21o}}{1 - S_{22i}S_{22o}}$$

8-Term Error Model

Measurement Stage



Insert unknown and provide unit reference signal at input

$$a_1 = S_{21i} + S_{22i}a_4 \qquad a_4 = S_{11a}a_1 + S_{12a}a_3$$

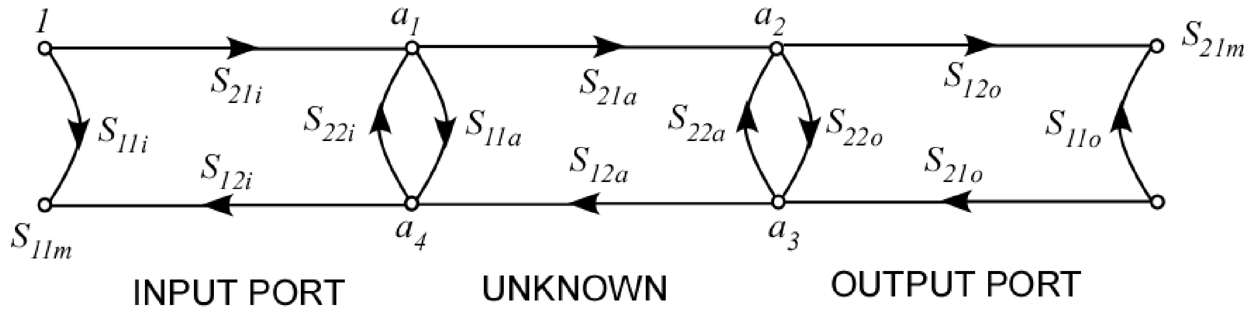
$$a_3 = S_{22o}a_2 \qquad a_2 = S_{21a}a_1 + S_{22a}a_3$$

$$S_{11m} = S_{11i} + S_{12i}a_4$$

$$S_{21m} = S_{12o}a_2$$

8-Term Error Model

Measurement Stage



Solve for the *a*'s

$$a_1 = S_{21i} + S_{22i}\left[\frac{S_{11m} - S_{11i}}{S_{21i}}\right]$$

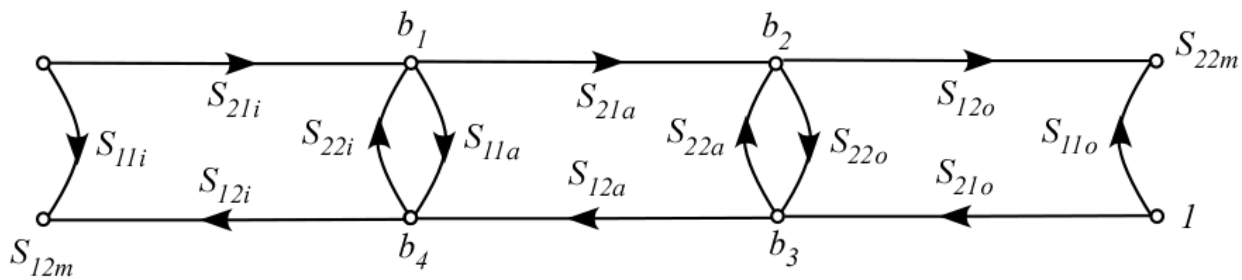
$$a_2 = \frac{S_{21m}}{S_{12o}}$$

$$a_3 = \frac{S_{22o}S_{21m}}{S_{12o}}$$

$$a_4 = \frac{S_{11m}S_{11i}}{S_{12i}}$$

8-Term Error Model

Measurement Stage



Insert unknown and provide unit reference signal at port 2

$$b_1 = S_{22i} b_4$$

$$b_4 = S_{11a} b_1 + S_{12a} b_3$$

$$b_3 = S_{22o} b_2 + S_{12o}$$

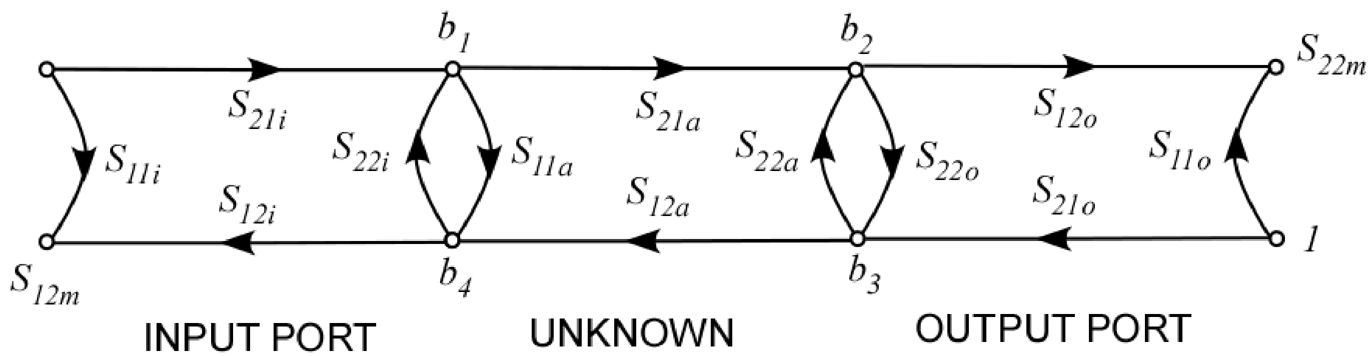
$$b_2 = S_{21a} b_1 + S_{22a} b_3$$

$$S_{22m} = S_{11o} + S_{12o} b_2$$

$$S_{12m} = S_{12i} b_4$$

8-Term Error Model

Measurement Stage



Solve for the b's

$$b_1 = \frac{S_{22i}S_{12m}}{S_{12i}}$$

$$b_3 = S_{22o}\left[\frac{S_{22m} - S_{11o}}{S_{12o}}\right] + S_{21o}$$

$$b_2 = \frac{S_{22m} - S_{11o}}{S_{12o}}$$

$$b_4 = \frac{S_{21m}}{S_{12i}}$$

8-Term Error Model - Solution

$$S_{11a} = \frac{\dfrac{S_{11m} - S_{11i}}{S_{21i}S_{12i}}\left[1 + \dfrac{S_{22o}\left(S_{22m} - S_{11o}\right)}{S_{12o}S_{21o}}\right] - \dfrac{S_{22o}S_{21m}}{S_{21i}S_{12o}} \times \dfrac{S_{12m}}{S_{21o}S_{12i}}}{D}$$

$$S_{12a} = \frac{\dfrac{S_{12m}}{S_{21o}S_{12i}}}{D} \qquad S_{21a} = \frac{\dfrac{S_{21m}}{S_{21i}S_{12o}}}{D}$$

$$S_{22a} = \frac{\dfrac{S_{22m} - S_{11o}}{S_{21o}S_{12o}}\left[1 + \dfrac{S_{22i}\left(S_{11m} - S_{11i}\right)}{S_{21i}S_{12i}}\right] - \dfrac{S_{22i}S_{21m}}{S_{21i}S_{12o}} \times \dfrac{S_{12m}}{S_{21o}S_{12i}}}{D}$$

8-Term Error Model - Solution

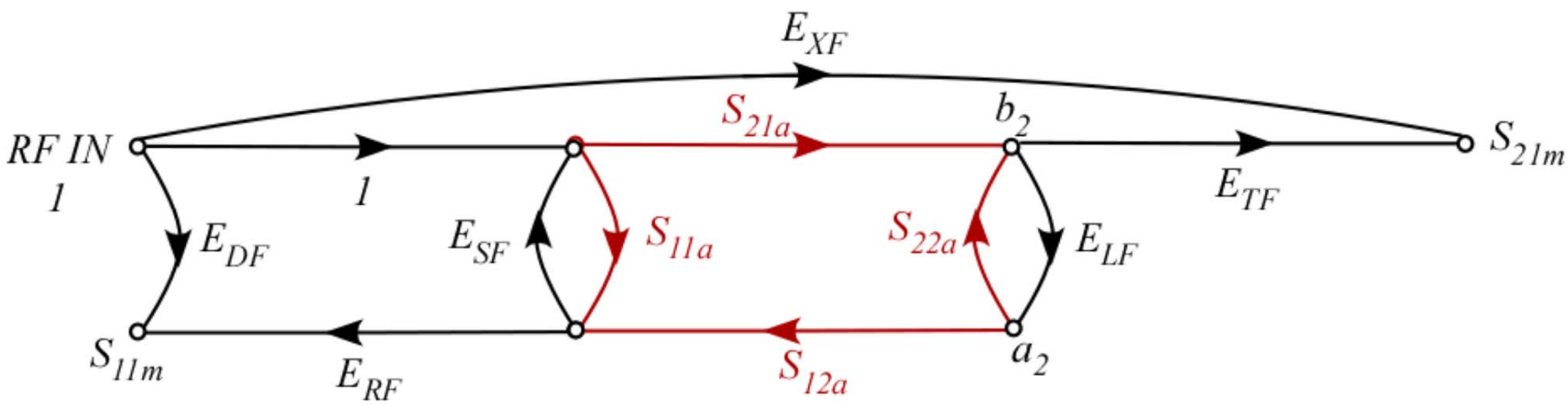
$$D = \left[1 + \frac{S_{22i}\left(S_{11m} - S_{11i}\right)}{S_{21i}S_{12o}}\right]\left[1 + \frac{S_{22o}\left(S_{22m} - S_{11o}\right)}{S_{12o}S_{21o}}\right] - S_{22i}S_{22o}\frac{S_{21m}}{S_{21i}S_{12o}}\frac{S_{12m}}{S_{21o}S_{12i}}$$

Reference

[1]. W. Kruppa and K. F. Sodomsky, “An Explicit Solution for the Scattering Parameters of a Linear Two-Port Measured with an Imperfect Test Set,” IEEE Transactions on Microwave Theory and Techniques, vol. 19, pp. 122-123, January 1971

12-Term Error Model

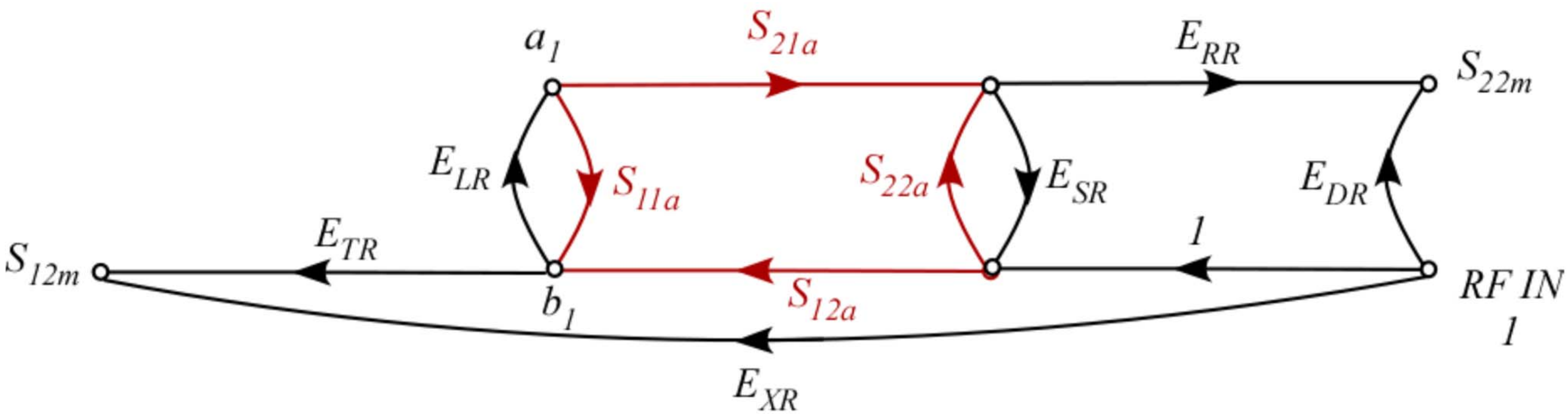
Forward Mode



Reflection terms	ESF EDF ERF	Open Short Load	3 measurements of S_{11m}
Transmission terms	ELF ETF	Thru	Measure S_{11m} and S_{21m}
Isolation term	EXF	Loads	Measure S_{21m}

12-Term Error Model

Reverse Mode



Reflection terms	ESR EDR ERR	Open Short Load	3 measurements of S_{22m}
Transmission terms	ELR ETR	Thru	Measure S_{22m} and S_{12m}
Isolation term	EXR	Loads	Measure S_{12m}