ECE 451
Macromodeling

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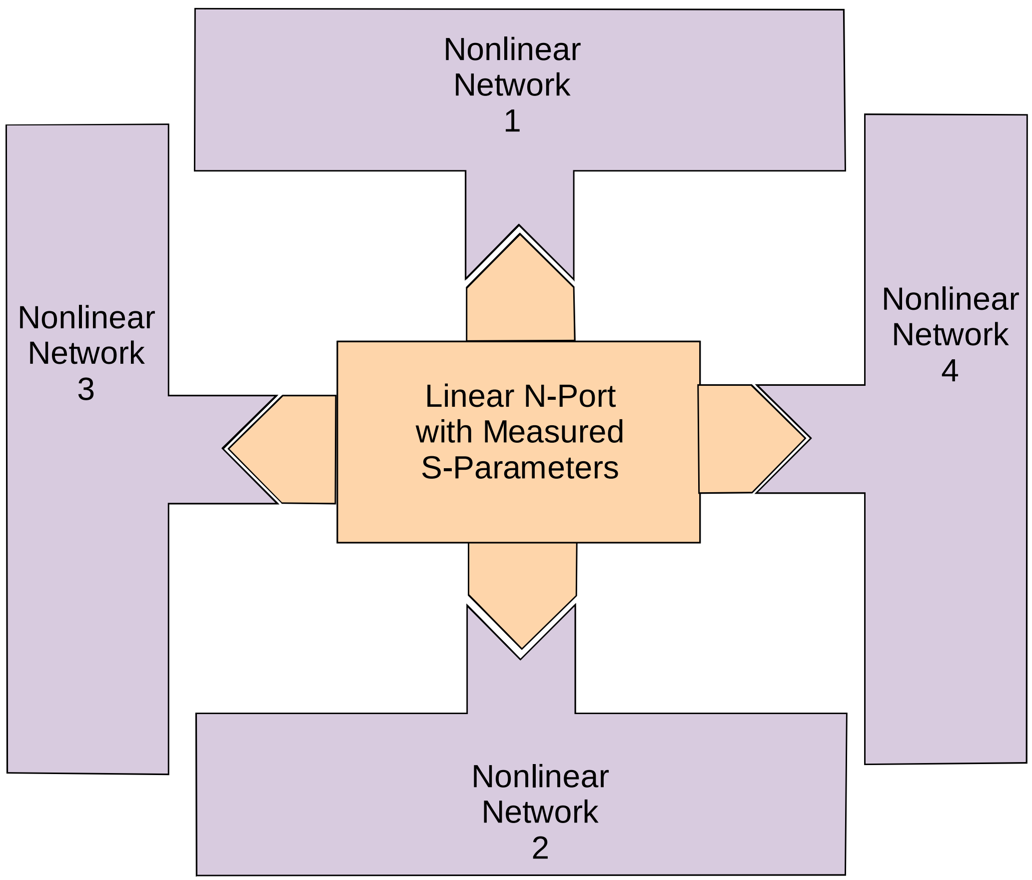
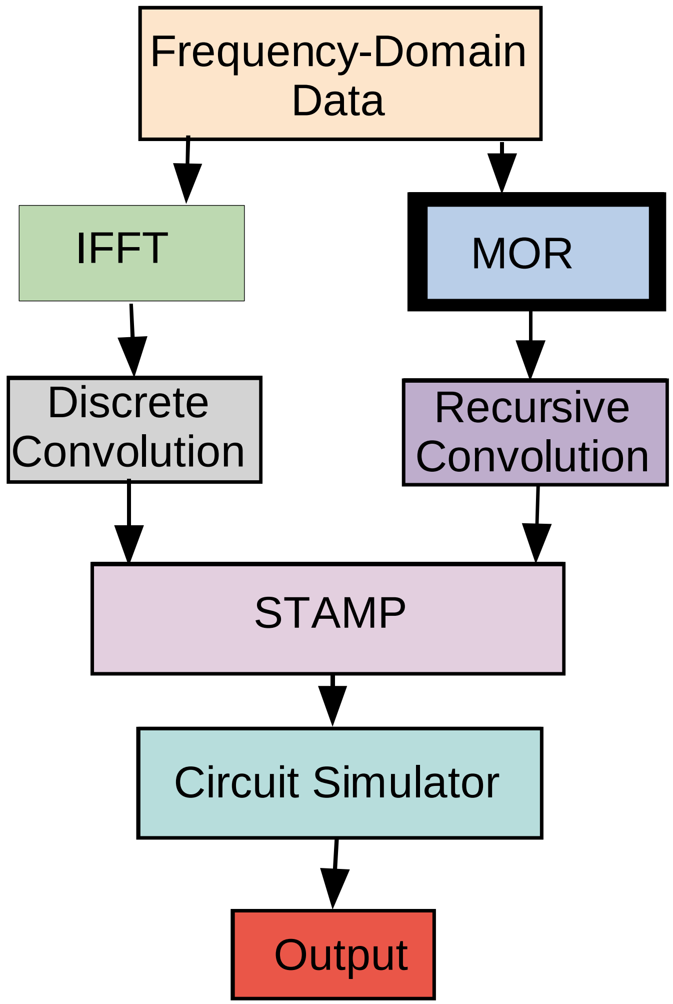
Blackbox Macromodeling



Objective: Perform time-domain simulation of composite network to determine timing waveforms, noise response or eye diagrams

Macromodel Implementation



Blackbox Synthesis

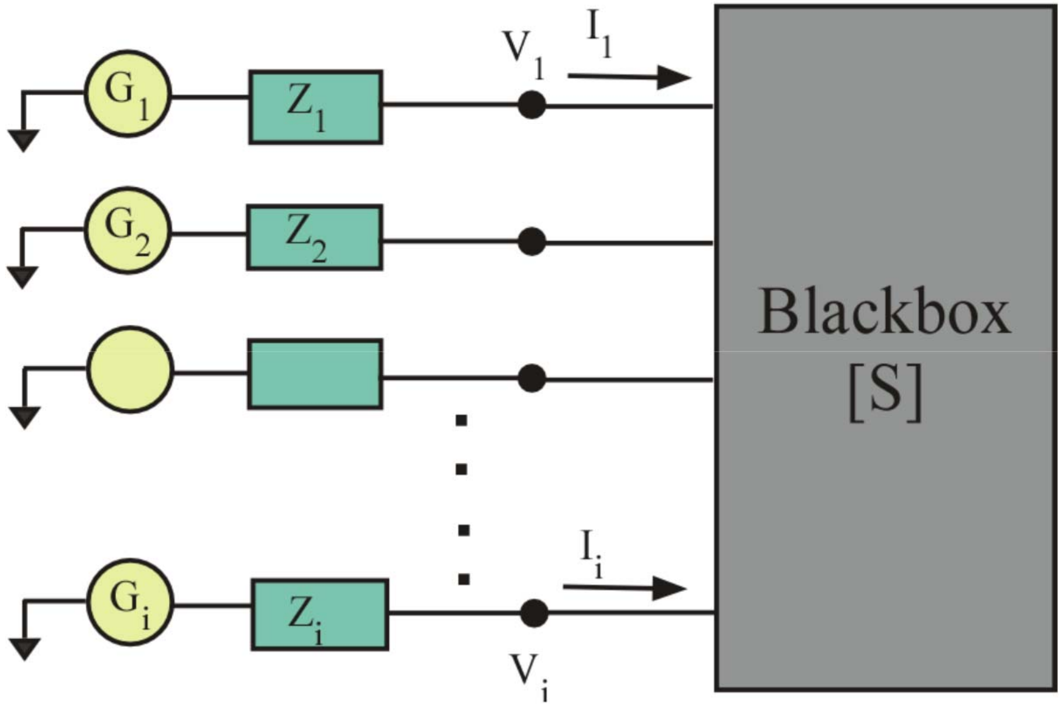
Motivations

- Only measurement data is available
- Actual circuit model is too complex

Methods

- Inverse-Transform & Convolution
  - IFFT from frequency domain data
  - Convolution in time domain
- Macromodel Approach
  - Curve fitting
  - Recursive convolution

Blackbox Synthesis



Terminations are described by a source vector $G(\omega)$ **and an impedance matrix** Z

Blackbox is described by its scattering parameter matrix S

Blackbox - Method 1

Scattering Parameters $B(\omega) = S(\omega)A(\omega)$ (1)

Terminal conditions $A(\omega) = \Gamma B(\omega) + TG(\omega)$ (2)

where
$$\Gamma = -\left[U + ZZ_o^{-1}\right]^{-1}\left[U - ZZ_o^{-1}\right]$$

and
$$T = \left[U + ZZ_o^{-1}\right]^{-1}$$

U is the unit matrix, Z is the termination impedance matrix and Z_o is the reference impedance matrix

Blackbox - Method 1

Combining (1) and (2) $A(\omega)=\left[U-\Gamma S(\omega)\right]^{-1}TG(\omega)$

and $B(\omega)=S(\omega)A(\omega)=S(\omega)\left[U-\Gamma S(\omega)\right]^{-1}TG(\omega)$

$$V(\omega)=A(\omega)+B(\omega)=\left[U+S(\omega)\right]\left[U-\Gamma S(\omega)\right]^{-1}TG(\omega)$$

$$I(\omega)=Z_o^{-1}\left[A(\omega)-B(\omega)\right]=Z_o^{-1}\left[U-S(\omega)\right]\left[U-\Gamma S(\omega)\right]^{-1}TG(\omega)$$

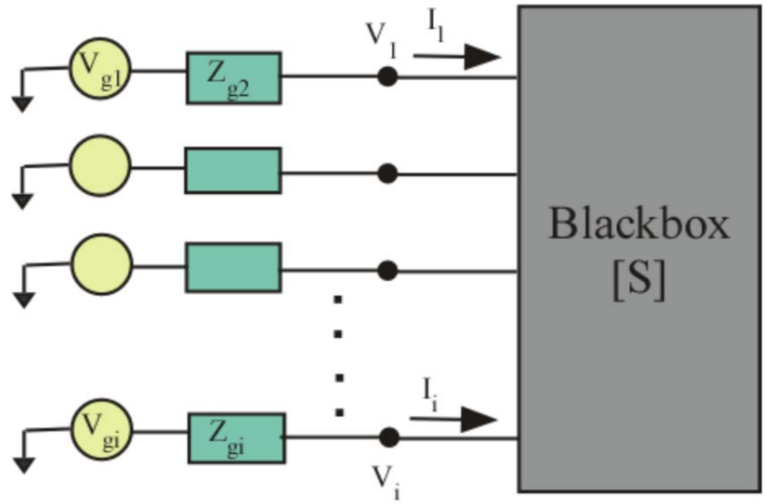
$$v(t)=IFFT\left\{V(\omega)\right\}$$

$$i(t)=IFFT\left\{I(\omega)\right\}$$

Method 1 - Limitations

- **No Frequency Dependence for Terminations**
  - Reactive terminations cannot be simulated
- **Only Linear Terminations**
  - Transistors and active nonlinear terminations cannot be described
- **Standalone**
  - This approach cannot be implemented in a simulator

Blackbox - Method 2



In frequency domain $B = SA$

In time domain $b(t) = s(t)*a(t)$

Convolution: $$s(t)*a(t) = \int_{-\infty}^{\infty} s(t-\tau)a(\tau)d\tau$$

Discrete Convolution

When time is discretized the convolution becomes

$$s(t)*a(t)=\sum_{\tau=1}^{t} s(t-\tau)a(\tau)\Delta\tau$$

Isolating *a(t)*

$$s(t)*a(t)=s(0)a(t)\Delta\tau+\sum_{\tau=1}^{t-1} s(t-\tau)a(\tau)\Delta\tau$$

Since ***a(t)*** **is known for** ***t < t*****, we have:**

$$H(t)=\sum_{\tau=1}^{t-1} s(t-\tau)a(\tau)\Delta\tau \quad : \quad History$$

Terminal Conditions

Defining $s'(0) = s(0)\Delta\tau$, we finally obtain

$$b(t) = s'(0)a(t) + H(t)$$

$$a(t) = \Gamma(t)b(t) + T(t)g(t)$$

By combining these equations, the stamp can be derived

Stamp Equation Derivation

The solutions for the incident and reflected wave vectors are given by:

$$a(t) = \left[1 - \Gamma(t)s'(0)\right]^{-1}\left[T(t)g(t) + \Gamma(t)H(t)\right]$$

$$b(t) = s'(0)a(t) + H(t)$$

The voltage wave vectors can be related to the voltage and current vectors at the terminals

$$a(t) = \frac{1}{2}\left[v(t) + Z_o i(t)\right]$$

$$b(t) = \frac{1}{2}\left[v(t) - Z_o i(t)\right]$$

Stamp Equation Derivation

From which we get

$$\frac{1}{2}\left[v(t)-Z_o i(t)\right]=\frac{s'(0)}{2}\left[v(t)+Z_o i(t)\right]+H(t)$$

or

$$Z_o i(t)+s'(0)Z_o i(t)+2H(t)=\left[1-s'(0)\right]v(t)$$

or

$$\left[1+s'(0)\right]Z_o i(t)=\left[1-s'(0)\right]v(t)-2H(t)$$

which leads to

$$i(t)=Z_o^{-1}\left[1+s'(0)\right]^{-1}\left[1-s'(0)\right]v(t)-2Z_o^{-1}\left[1+s'(0)\right]^{-1}H(t)$$

Stamp Equation Derivation

i(t) **can be written to take the form**

$$i(t) = Y_{stamp} v(t) - I_{stamp}$$

in which

$$Y_{stamp} = Z_o^{-1} \left[1 + s'(0) \right]^{-1} \left[1 - s'(0) \right]$$

and

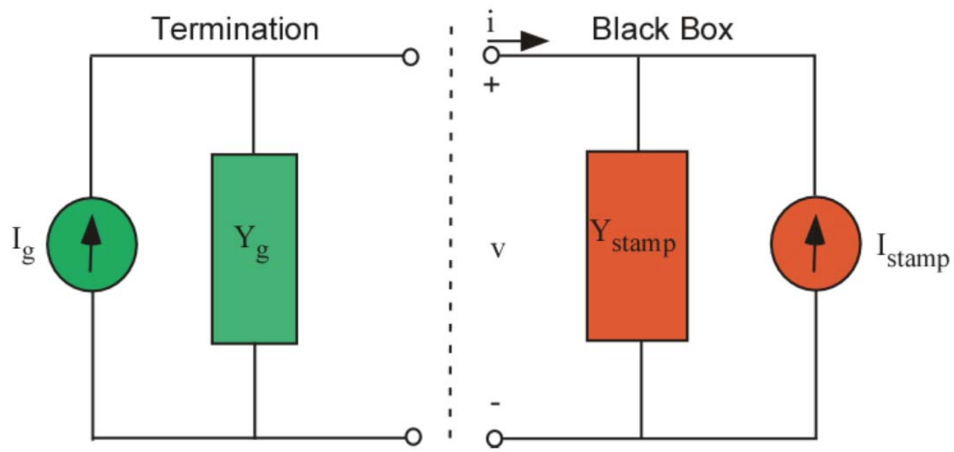
$$I_{stamp} = 2 Z_o^{-1} \left[1 + s'(0) \right]^{-1} H(t)$$

Stamp Equations

$$i(t) = Y_{stamp} v(t) - I_{stamp}$$

$$Y_{stamp} = Z_o^{-1} \left[1 + s'(0)\right]^{-1} \left[1 - s'(0)\right]$$

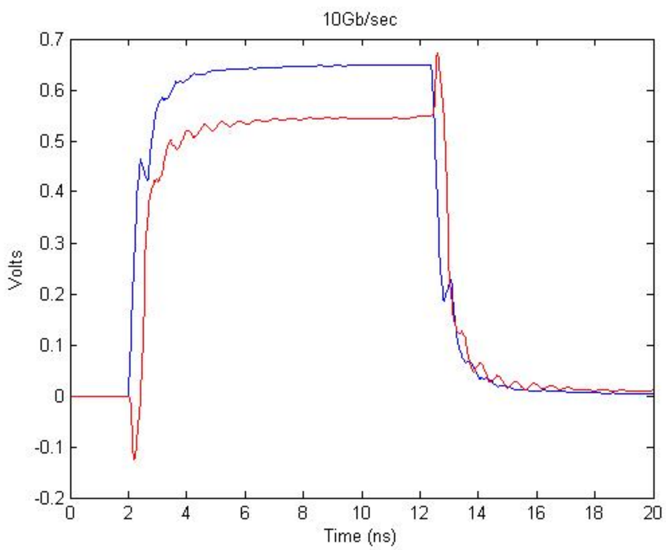
$$I_{stamp} = 2Z_o^{-1} \left[1 + s'(0)\right]^{-1} H(t)$$



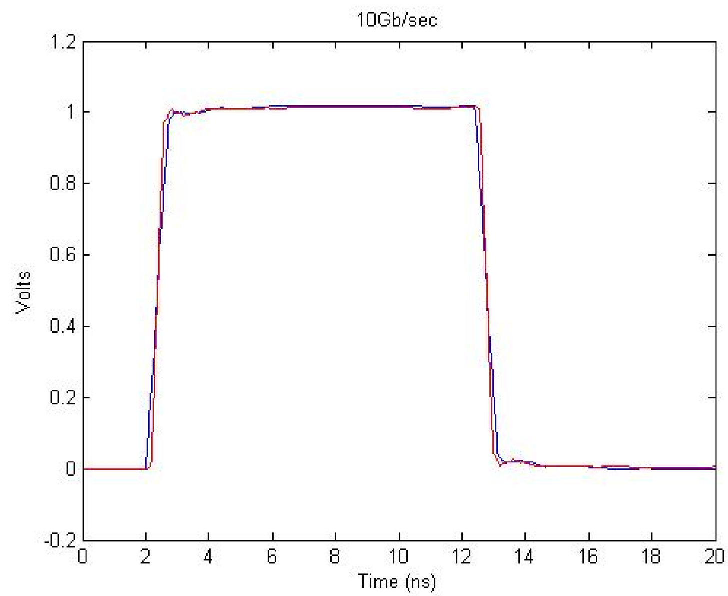
$$(Y_g + Y_{stamp}) v(t) = I_g + I_{stamp}$$

Effects of DC Data

No DC Data Point

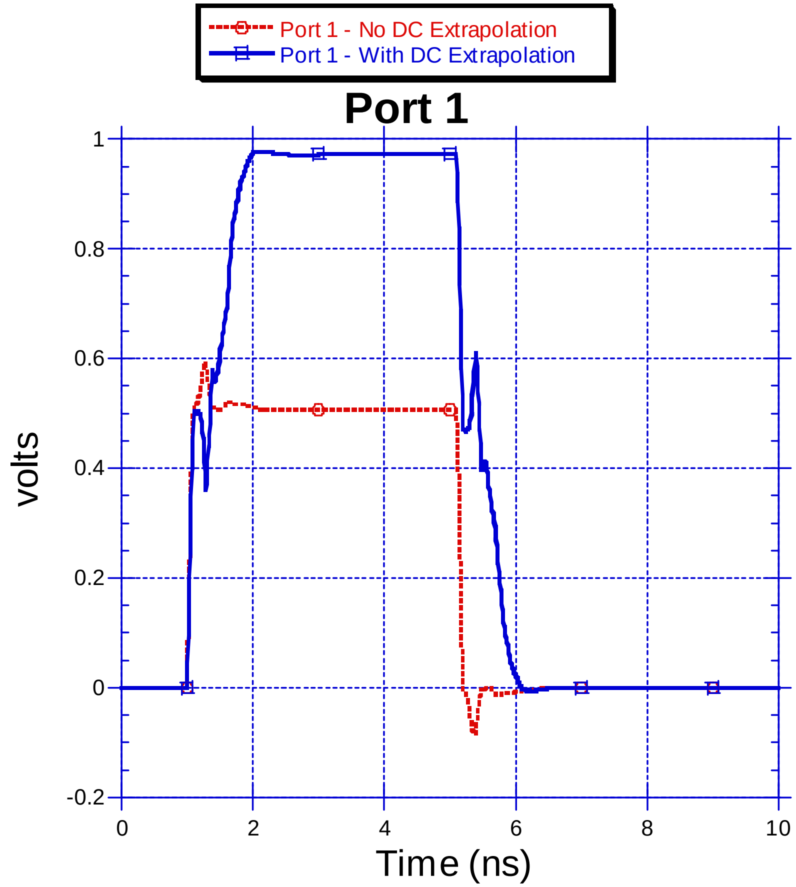


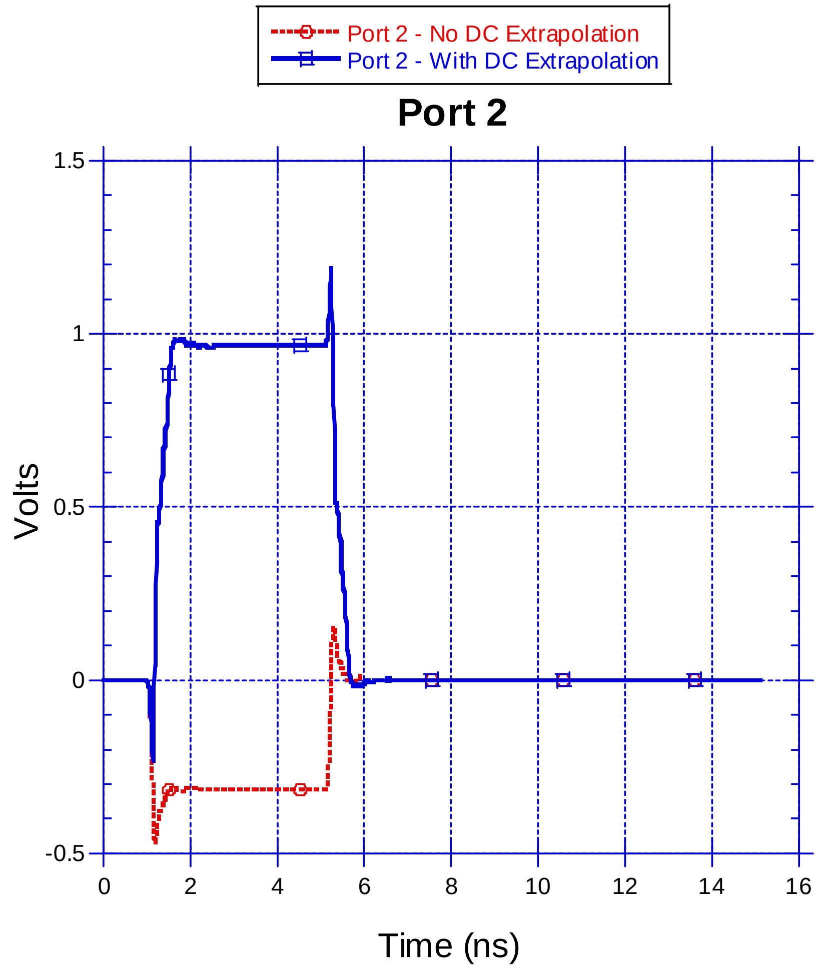
With DC Data Point



If low-frequency data points are not available, extrapolation must be performed down to DC.

Effect of Low-Frequency Data





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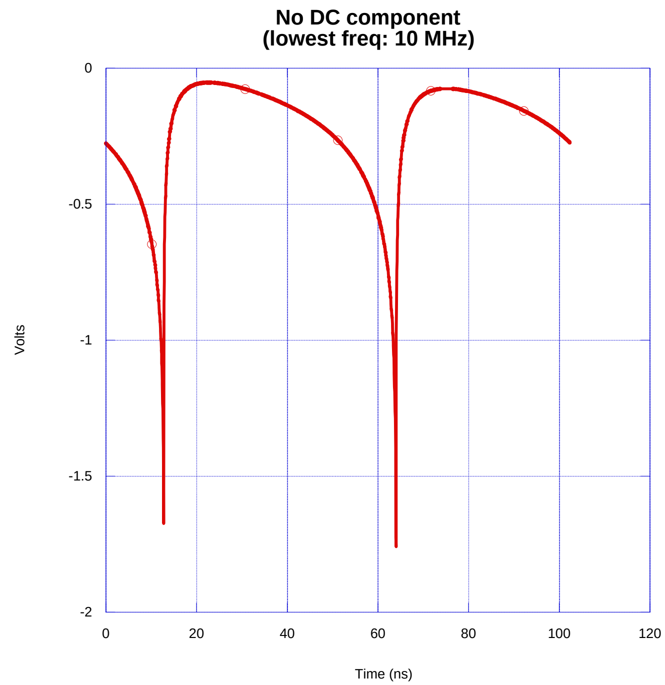
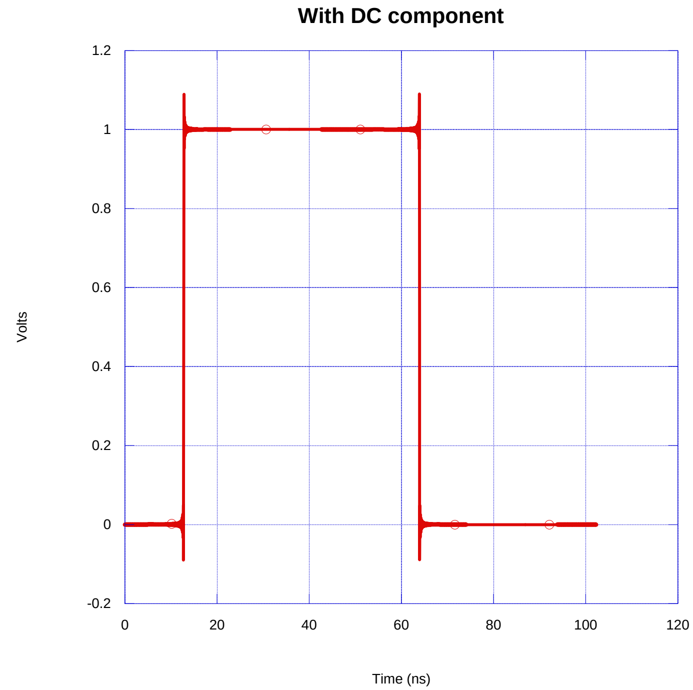
Effect of Low-Frequency Data

Calculating inverse Fourier Transform of: $$V(f) = \frac{2\sin(2\pi ft)}{2\pi ft}$$





Left: IFFT of a sinc pulse sampled from 10 MHz to 10 GHz. Right: IFFT of the same sinc pulse with frequency data ranging from 0-10 GHz. In both cases 1000 points are used

Convolution Limitations

Frequency-Domain Formulation $$Y(\omega) = H(\omega)X(\omega)$$

Time-Domain Formulation $$y(t) = h(t) * x(t)$$

Convolution $$y(t) = h(t) * y(t) = \int_0^t h(t-\tau)y(\tau)d\tau$$

Discrete Convolution $$h(t) * x(t) = \sum_{\tau=1}^{t} h(t-\tau)x(\tau)\Delta\tau$$

$$H(t) = \sum_{\tau=1}^{t-1} h(t-\tau)x(\tau)\Delta\tau \quad : \quad History$$

Computing History is computationally expensive ➔ Use FD rational approximation and TD recursive convolution

Frequency and Time Domains

1. For negative frequencies use conjugate relation $V(-\omega) = V^*(\omega)$

2. DC value: use lower frequency measurement

3. Rise time is determined by frequency range or bandwidth

4. Time step is determined by frequency range

5. Duration of simulation is determined by frequency step

Problems and Issues

- **Discretization:** (not a continuous spectrum)

- **Truncation:** frequency range is band limited

F: frequency range
N: number of points
$\Delta f = F/N$: frequency step
Δt = time step

Problems and Issues

Problems & Limitations (in frequency domain)	Consequences (in time domain)	Solution
Discretization	Time-domain response will repeat itself periodically (Fourier series) Aliasing effects	Take small frequency steps. Minimum sampling rate must be the Nyquist rate
Truncation in Frequency	Time-domain response will have finite time resolution (Gibbs effect)	Take maximum frequency as high as possible
No negative frequency values	Time-domain response will be complex	Define negative-frequency values and use $V(-f)=V^*(f)$ which forces $v(t)$ to be real
No DC value	Offset in time-domain response, ringing in base line	Use measurement at the lowest frequency as the DC value

Complex Plane

– An arbitrary network's transfer function can be described in terms of its s-domain representation

– s is a complex number $s = \sigma + j\omega$

– The impedance (or admittance) or transfer function of networks can be described in the s domain as

$$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}$$

Transfer Functions

$$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}$$

The coefficients *a* and *b* are real and the order *m* of the numerator is smaller than or equal to the order *n* of the denominator

A stable system is one that does not generate signal on its own.

For a stable network, the roots of the denominator should have negative real parts

Transfer Functions

The transfer function can also be written in the form

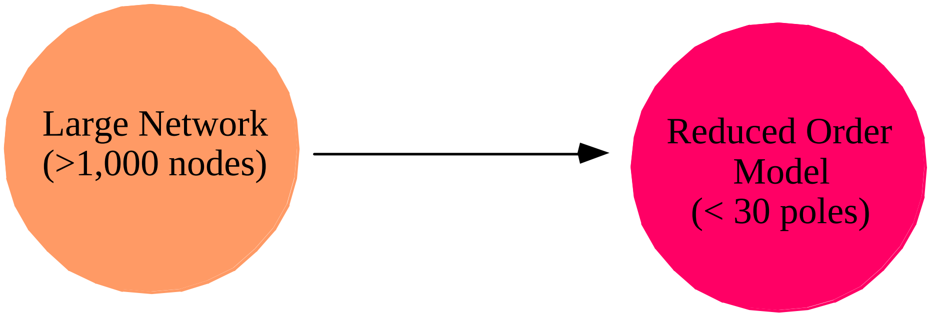
$$T(s) = a_m \frac{(s - Z_1)(s - Z_2)\ldots(s - Z_m)}{(s - P_1)(s - P_2)\ldots(s - P_m)}$$

$Z_1, Z_2, \ldots Z_m$ are the **zeros** of the transfer function

$P_1, P_2, \ldots P_m$ are the **poles** of the transfer function

For a stable network, the poles should lie on the left half of the complex plane

Model Order Reduction



SPICE
$Y(t)\, v(t) = i(t)$

$$\tilde{Y}(\omega) = \left[A_1 + \sum_{i=1}^{L} \frac{a_{1i}}{1 + j\omega / \omega_{c1i}} \right]$$

$Y(\omega)\, V(\omega) = I(\omega)$

↓

Order Reduction

↓

$Y(\omega) \cong \tilde{Y}(\omega)$

↓

Recursive Convolution

↓

$\tilde{Y}(t)\, v(t) = i(t)$

Model Order Reduction

Objective: Approximate frequency-domain transfer function to take the form:

$$H(\omega) = \left[A_1 + \sum_{i=1}^{L} \frac{a_{1i}}{1 + j\omega/\omega_{c1i}} \right]$$

Methods

- AWE – Pade
- Pade via Lanczos (Krylov methods)
- Rational Function
- Chebyshev-Rational function
- Vector Fitting Method

Model Order Reduction (MOR)

Question: Why use a rational function approximation?

Answer: because the frequency-domain relation

$$Y(\omega)=H(\omega)X(\omega)=\left[d+\sum_{k=1}^{L}\frac{c_k}{1+j\omega/\omega_{ck}}\right]X(\omega)$$

will lead to a time-domain *recursive* convolution:

$$y(t)=dx(t-T)+\sum_{k=1}^{L}y_{pk}(t)$$

where

$$y_{pk}(t)=a_k x(t-T)\left(1-e^{-\omega_{ck}T}\right)+e^{-\omega_{ck}T}y_{pk}(t-T)$$

which is very fast!

Model Order Reduction

Transfer function is approximated as

$$H(\omega) = d + \sum_{k=1}^{L} \frac{c_k}{1 + j\omega / \omega_{ck}}$$

In order to convert data into rational function form, we need a curve fitting scheme ➔ Use Vector Fitting

History of Vector Fitting (VF)

- **1998 - Original VF formulated by Bjorn Gustavsen and Adam Semlyen***
- **2003 - Time-domain VF (TDVF) by S. Grivet-Talocia.**
- **2005 - Orthonormal VF (OVF) by Dirk Deschrijver, Tom Dhaene, et al.**
- **2006 - Relaxed VF by Bjorn Gustavsen.**
- **2006 - VF re-formulated as Sanathanan-Koerner (SK) iteration by W. Hendrickx, Dirk Deschrijver and Tom Dhaene, et al.**

* B. Gustavsen and A. Semlyen, "Rational approximation of frequency responses by vector fitting," IEEE Trans. Power Del., vol. 14, no. 3, pp 1052–1061, Jul. 1999

Vector Fitting (VF)

Vector fitting algorithm

$$\begin{bmatrix} \sigma(s) f(s) \\ \sigma(s) \end{bmatrix} \approx \begin{bmatrix} \sum_{n=1}^{N} \frac{c_n}{s - \tilde{a}_n} + d + sh \\ \sum_{n=1}^{N} \frac{\tilde{c}_n}{s - \tilde{a}_n} + 1 \end{bmatrix}$$

$$\left(\sum_{n=1}^{N} \frac{c_n}{s - \tilde{a}_n} + d + sh \right) - \left(\sum_{n=1}^{N} \frac{\tilde{c}_n}{s - \tilde{a}_n} \right) f(s) \approx f(s).$$

Solve for $c_n, \tilde{c}_n, d, h$

Avoid ill-conditioned matrix

Guarantee stability

Converge, accurate

With Good Initial Poles

Can show* that the zeros of $\sigma(s)$ are the poles of $f(s)$ for the next iteration

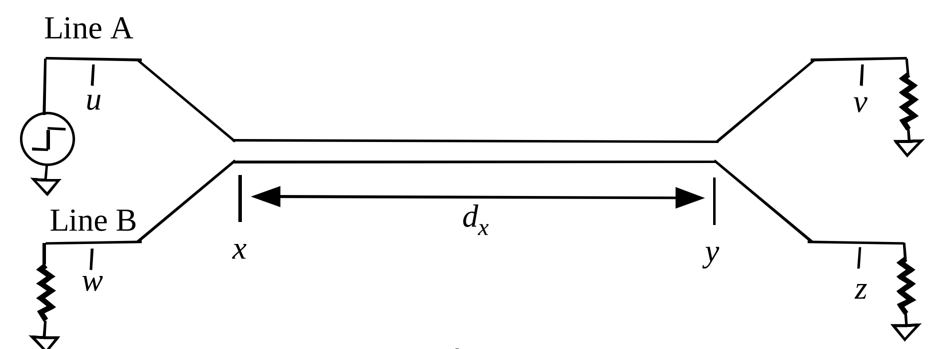
* B. Gustavsen and A. Semlyen, "Rational approximation of frequency responses by vector fitting," IEEE Trans. Power Del., vol. 14, no. 3, pp 1052–1061, Jul. 1999

Examples

1.- DISC: Transmission line with discontinuities

Length = 7 inches

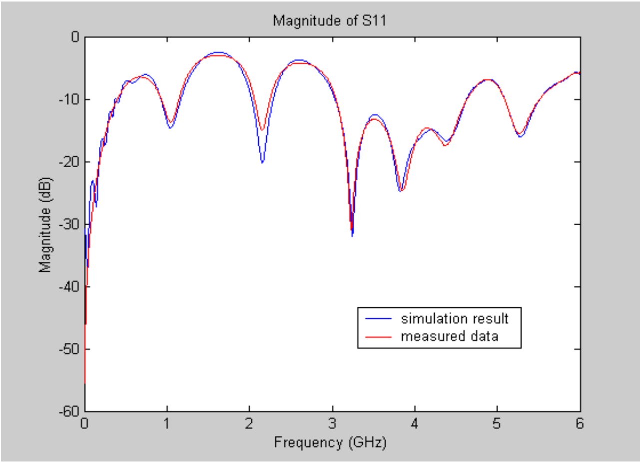
2.- COUP: Coupled transmission line2

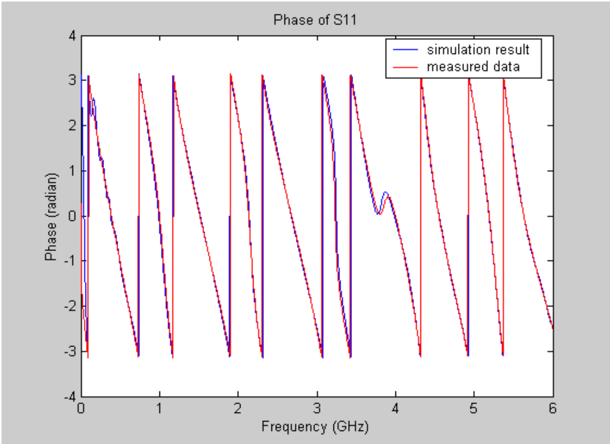


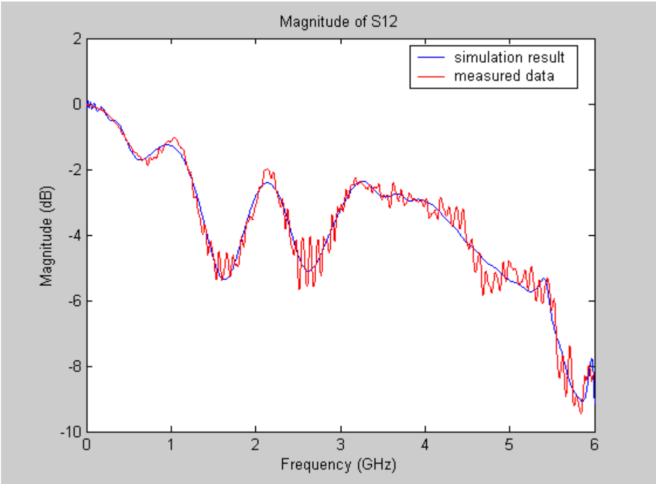
$d_x = 5^{1/2}$ inches

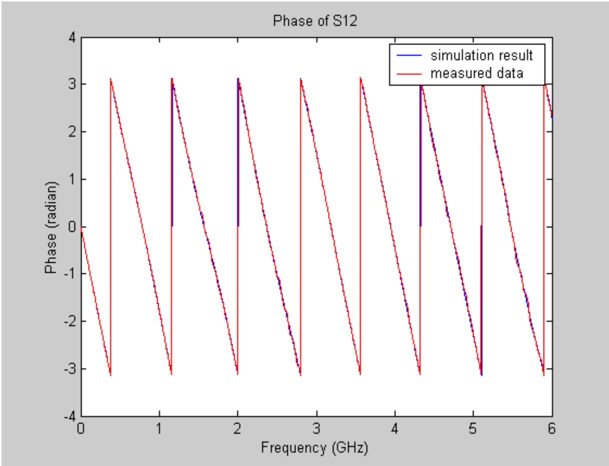
Frequency sweep: 300 KHz – 6 GHz

DISC: Approximation Results





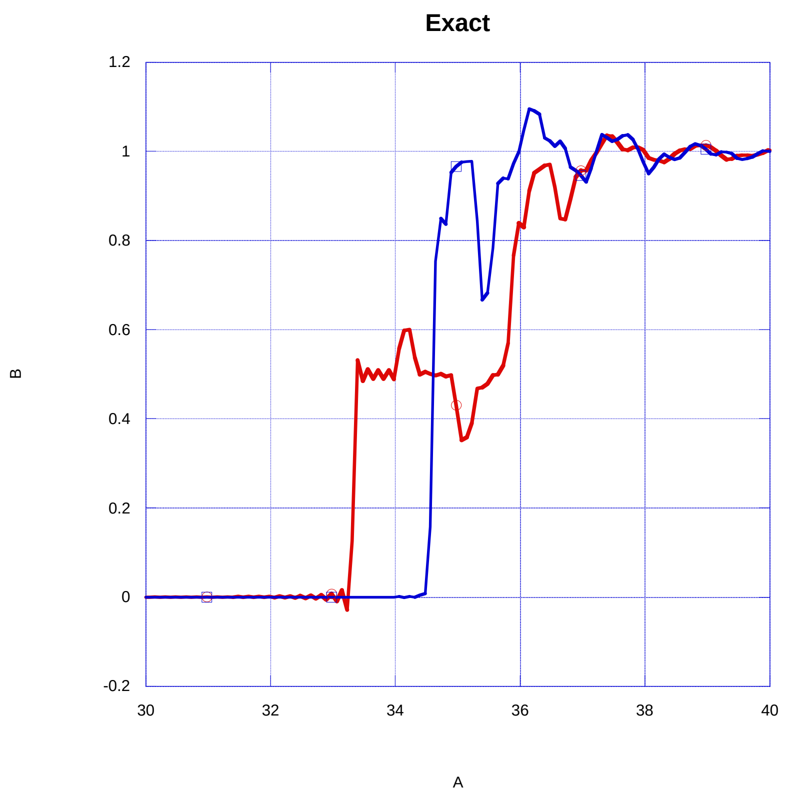


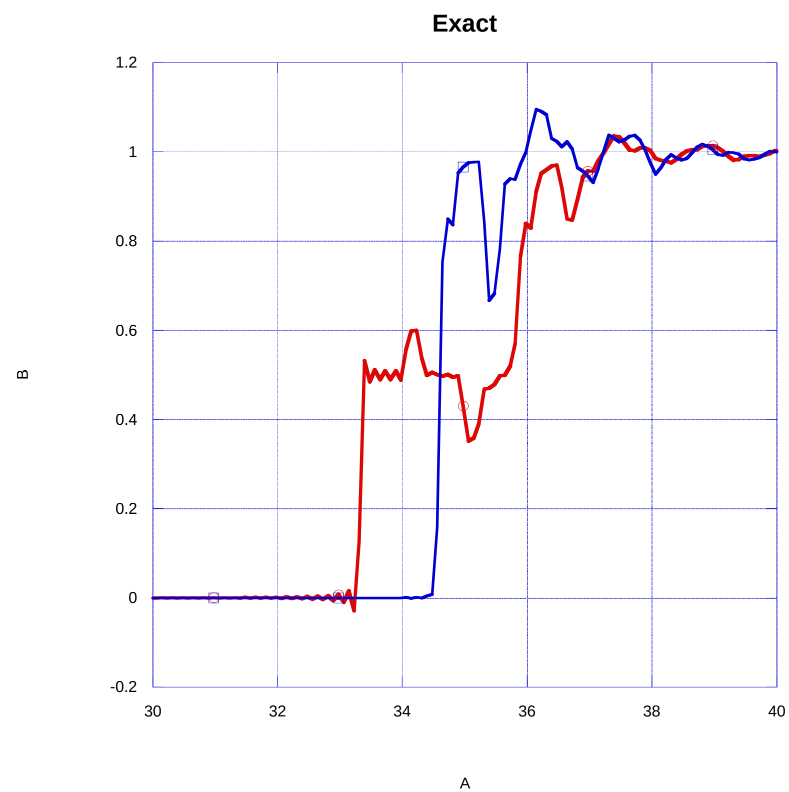


DISC: Approximation order 90

DISC: Simulations

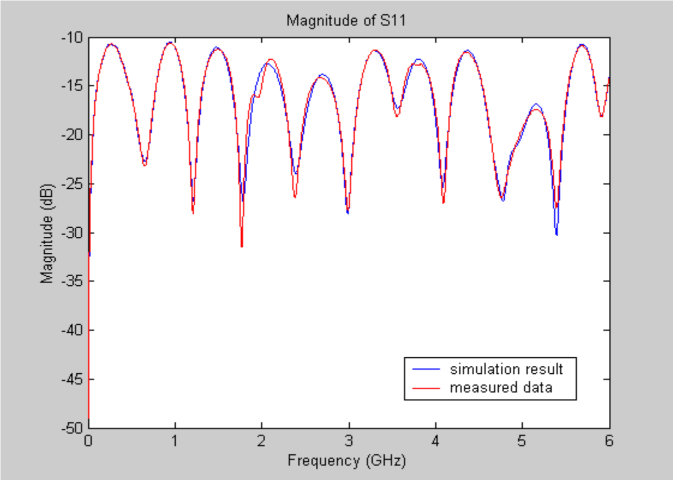
Microstrip line with discontinuities
Data from 300 KHz to 6 GHz

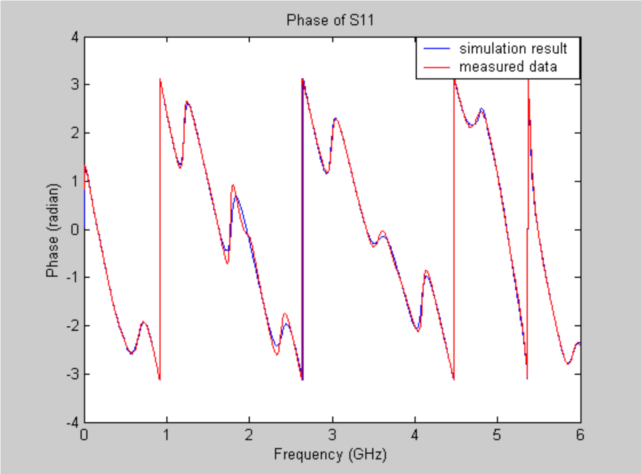


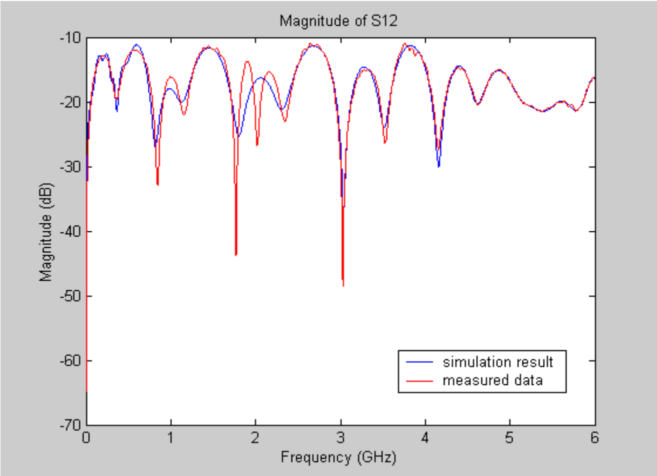


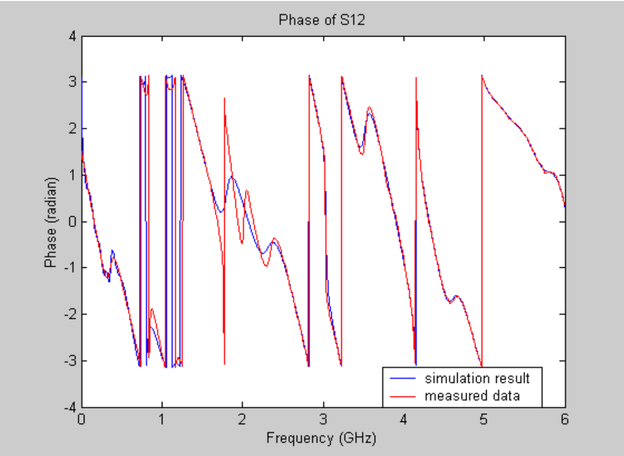
Observation: Good agreement

COUP: Approximation Results





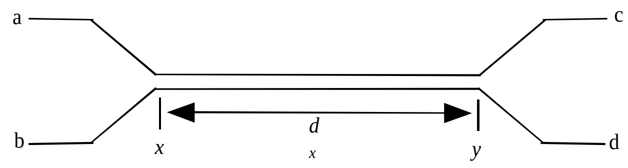


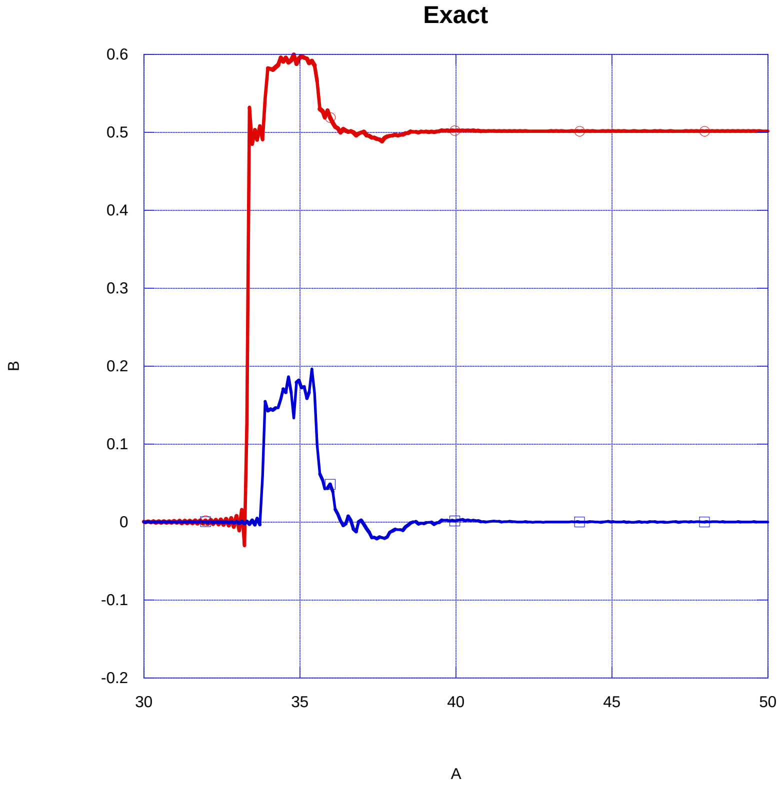


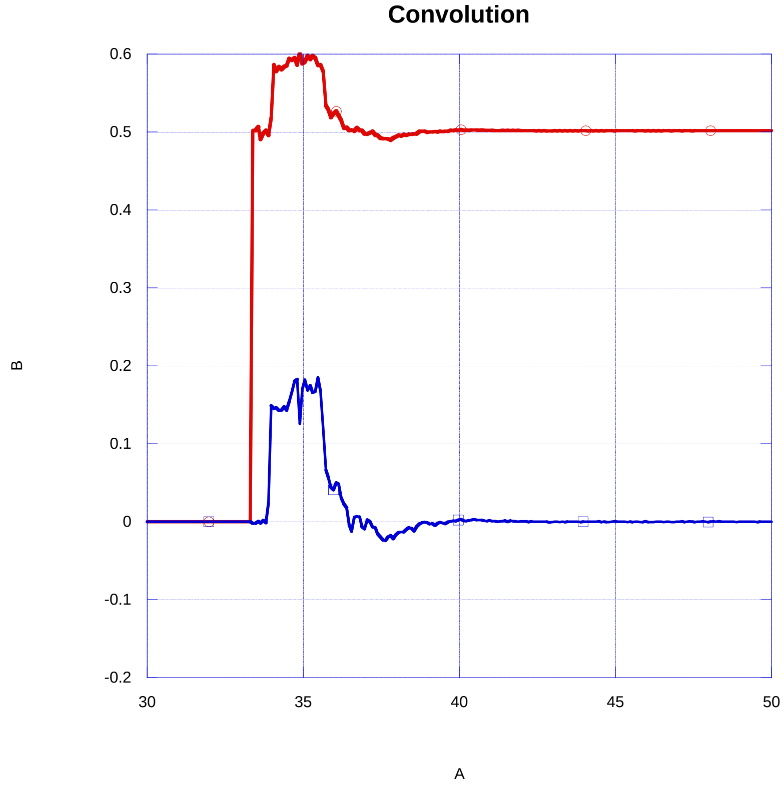
COUP: Approximation order 75 – Before Passivity Enforcement

COUP: Simulations

Port 1: a – Port 2: d
Data from 300 KHz to 6 GHz

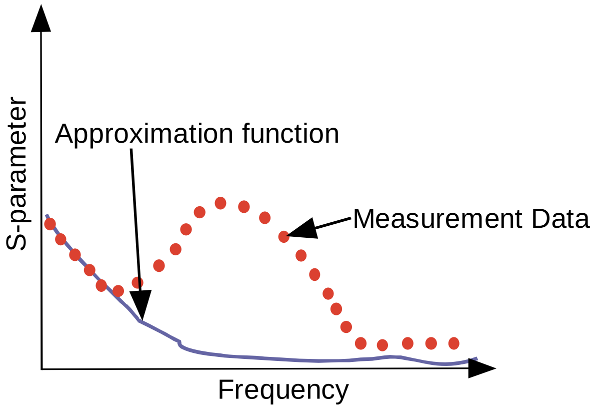


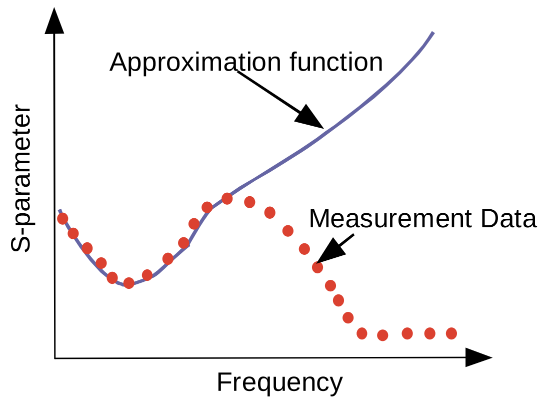


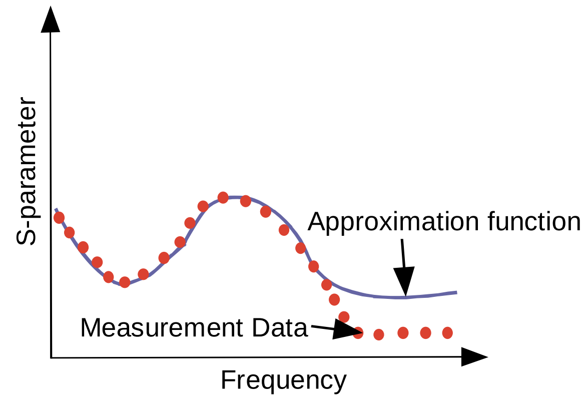


Observation: Good agreement

Orders of Approximation



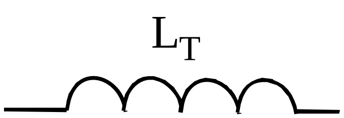


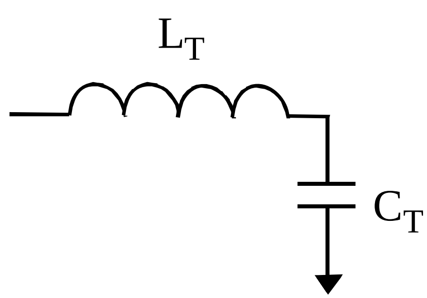


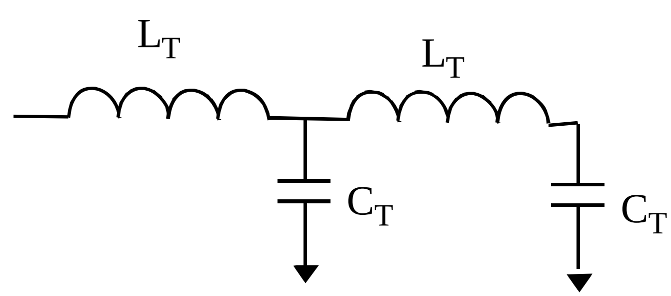
Low order

Medium order

Higher order







MOR Attributes

- Accurate:- over wide frequency range.
- Stable:- All poles must be in the left-hand side in s-plane or inside in the unit-circle in z-plane.
- Causal:- Hilbert transform needs to be satisfied.
- Passive:- H(s) is analytic

$$h[n] = h_e[n] + h_o[n] \Leftrightarrow H(j\omega) = H_R(j\omega) + jH_I(j\omega)$$

$$H^*(s) = H(s^*),$$

$$z^{*T}[H^T(s^*) + H(s)]z \geq 0, \quad \mathbb{R}[s] > 0$$,for Y or Z-parameters.

$$I - H^T(s^*)H(s) \geq 0, \quad \mathbb{R}[s] > 0$$,for S-parameters.

MOR Issues

- **Bandwidth**
  - Low-frequency data must be added
- **Passivity**
  - Passivity enforcement
- **Causality**
  - Causality enforcement
- **High Order of Approximation**
  - Orders > 800 for some serial links
  - Delay need to be extracted