

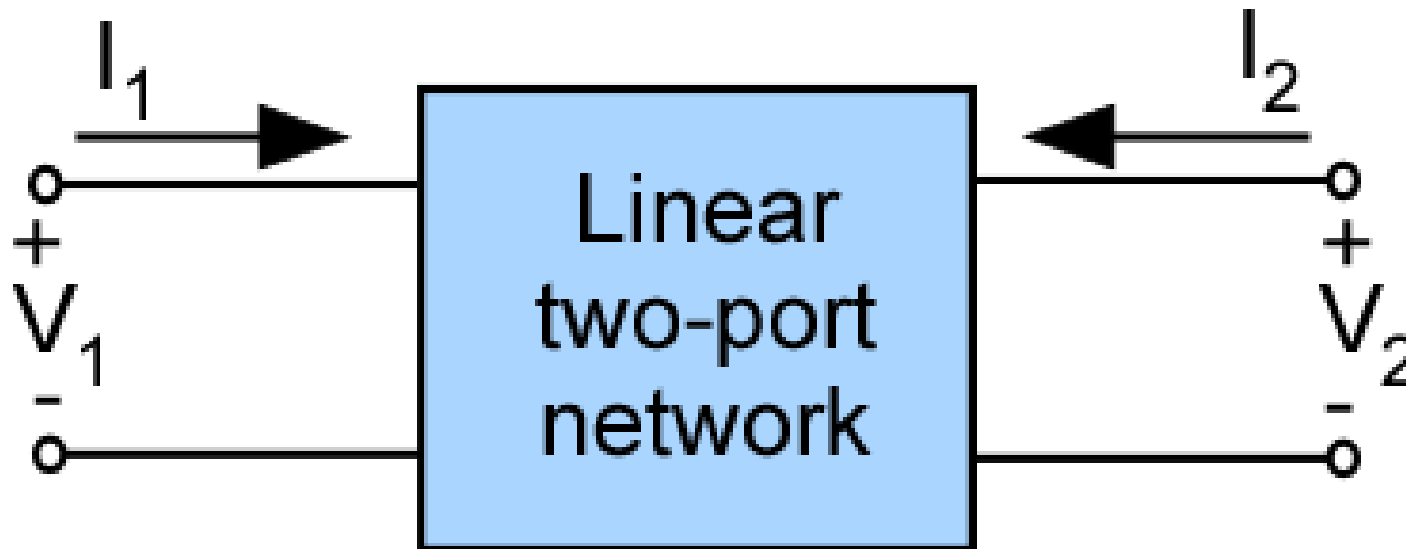
ECE 451

Advanced Microwave Measurements

S Parameters

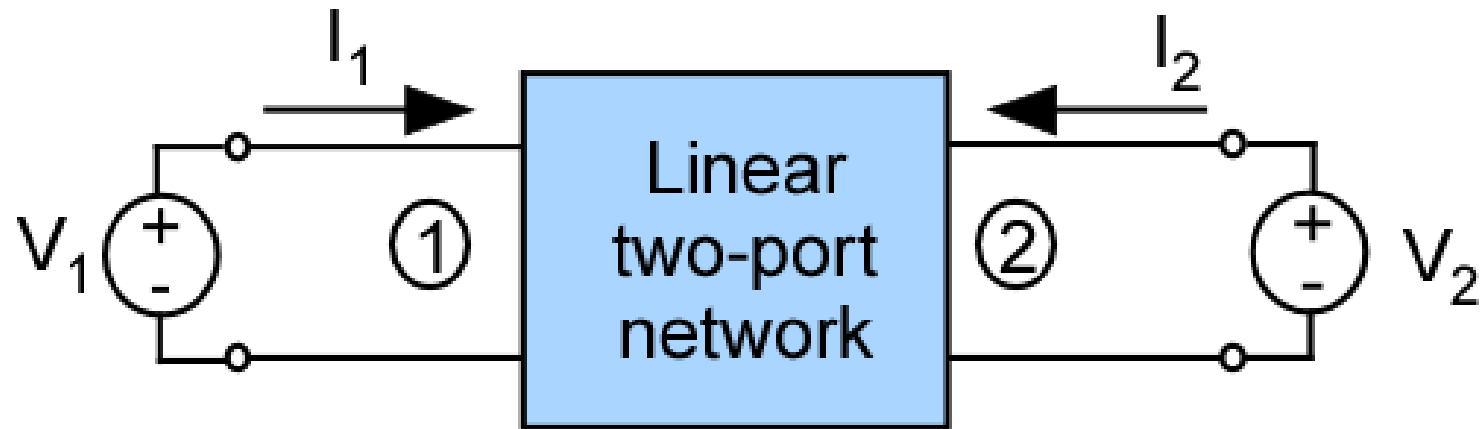
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Transfer Function Representation



Use a two-terminal representation of system for input and output

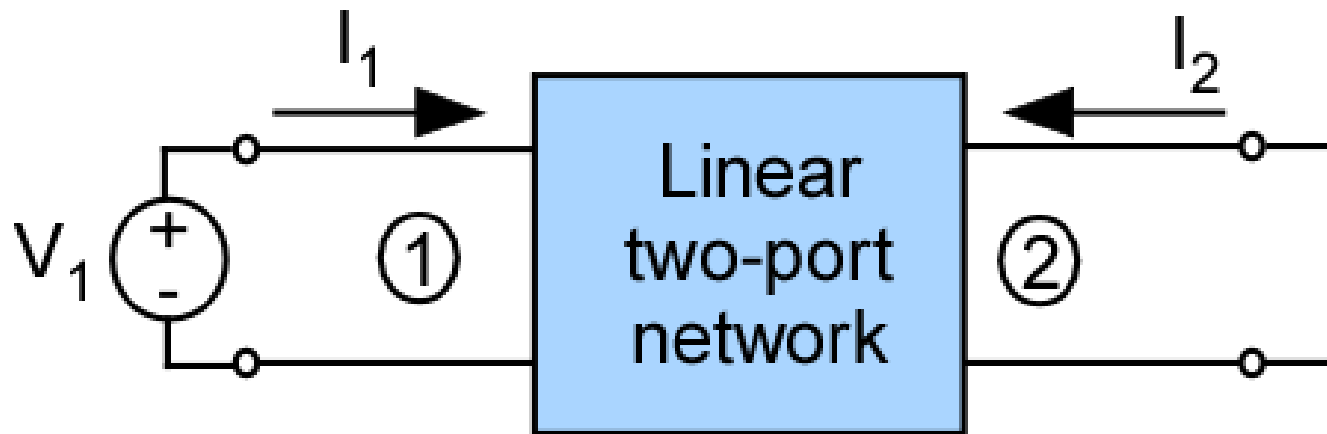
Y-parameter Representation



$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

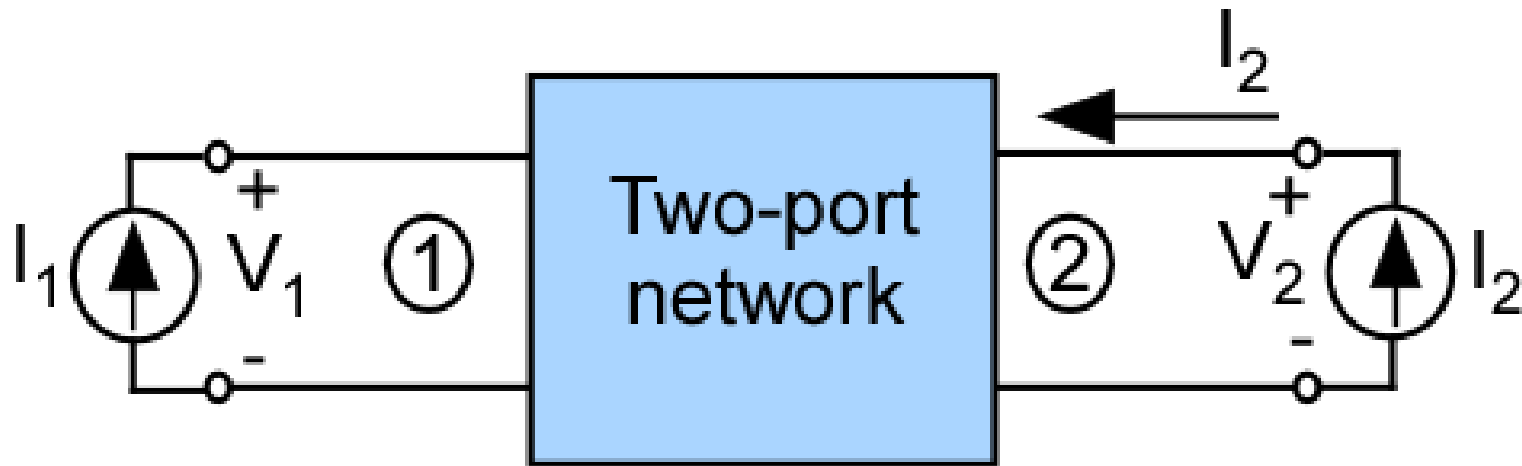
Y Parameter Calculations



$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \quad y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}$$

To make $V_2=0$, place a short at port 2

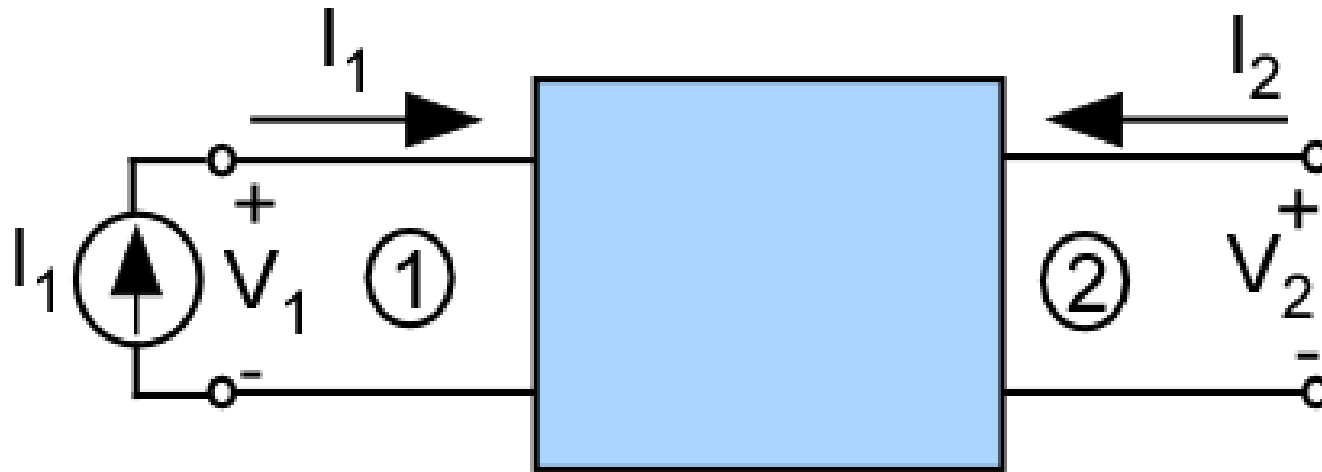
Z Parameters



$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

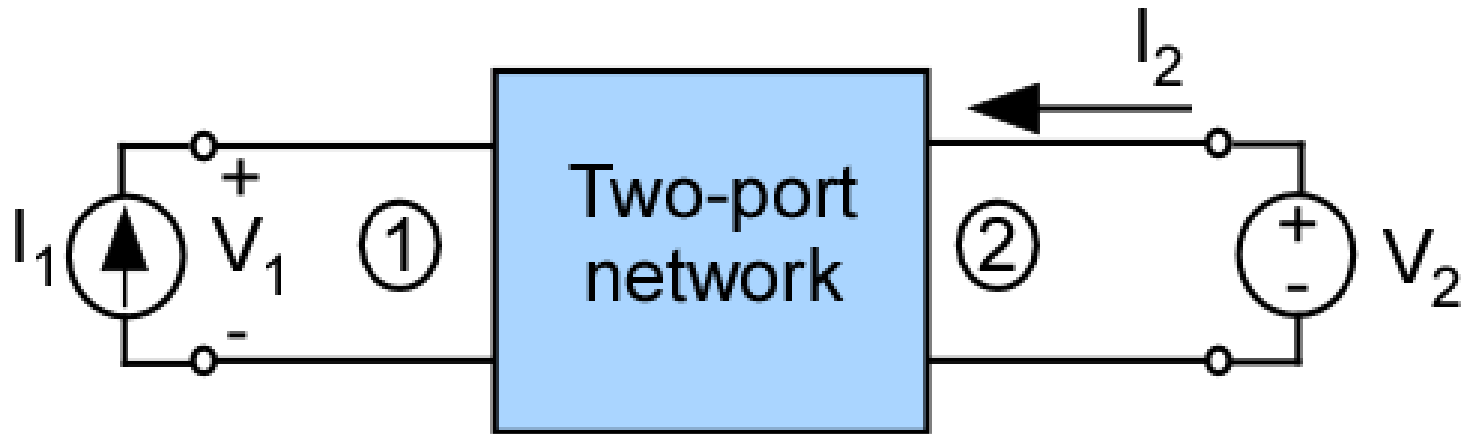
Z-parameter Calculations



$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}$$

To make $I_2=0$, place an open at port 2

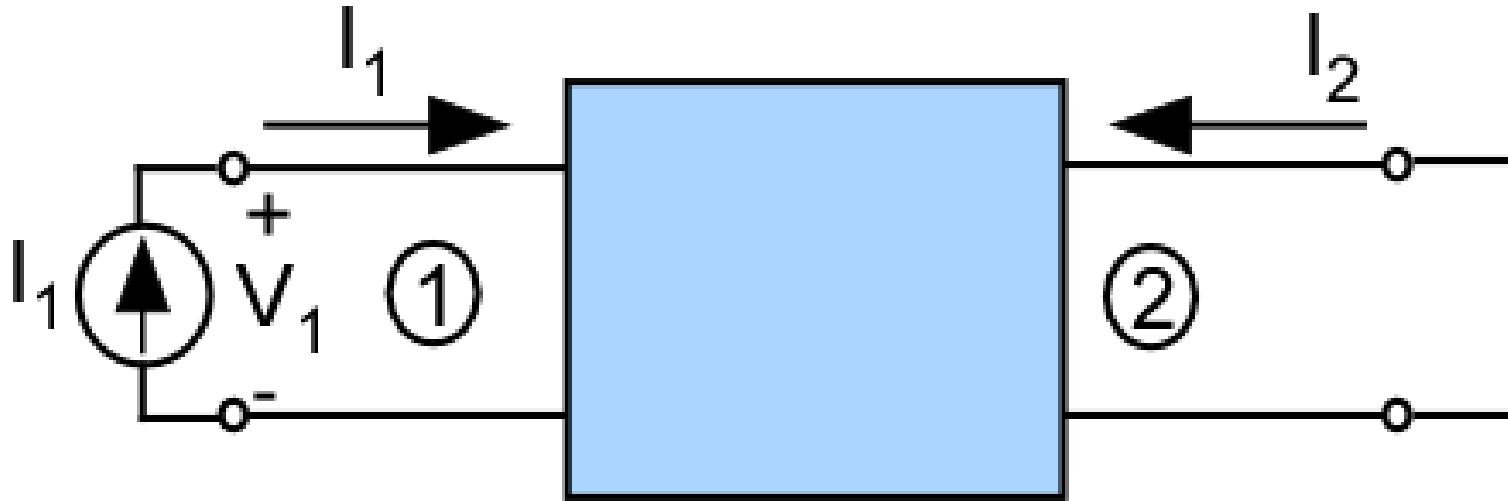
H Parameters



$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

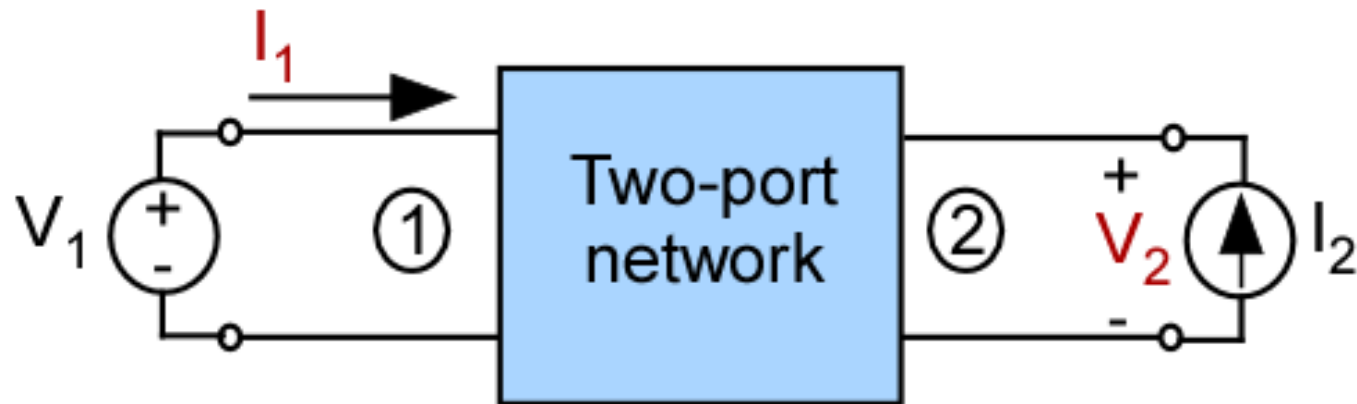
H Parameter Calculations



$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} \quad h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0}$$

To make $V_2=0$, place a short at port 2

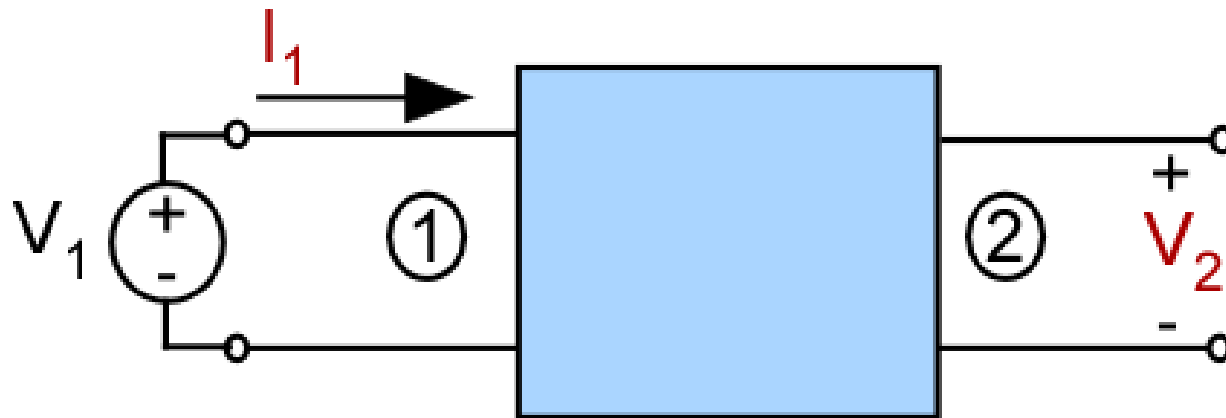
G Parameters



$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

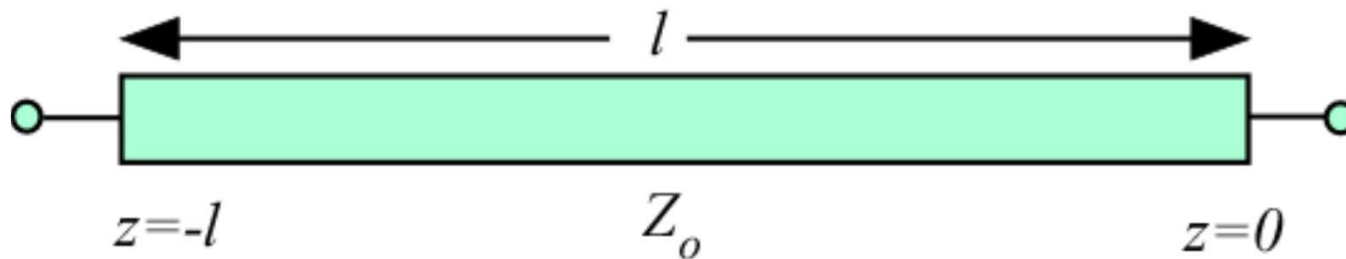
G-Parameter Calculations



$$g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0} \quad g_{21} = \left. \frac{V_2}{V_1} \right|_{I_2=0}$$

To make $I_2=0$, place an open at port 2

Y-Parameters of TL



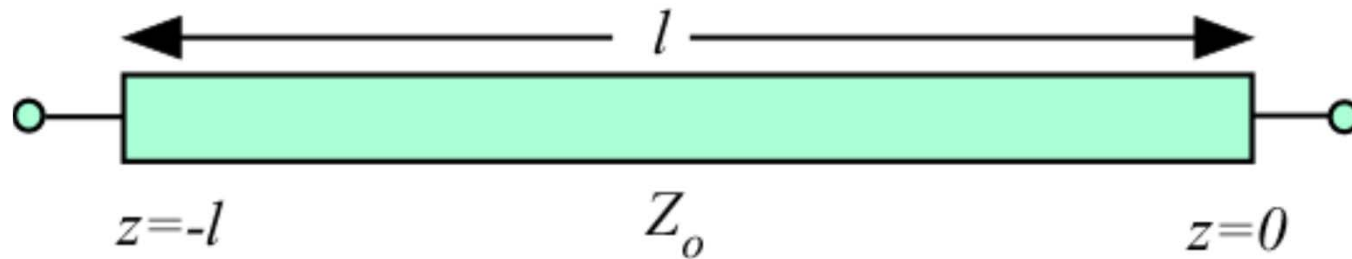
Find the Y-parameters of a lossless transmission line with propagation constant β and characteristic impedance Z_o (admittance Y_o)

$$V(z) = V_+ e^{-j\beta z} + V_- e^{+j\beta z}$$

$$I(z) = Y_o (V_+ e^{-j\beta z} - V_- e^{+j\beta z})$$

Let port 1 be at $z=-l$ and port 2 at $z=0$

Y-Parameters of TL



at port 1

$$V_1 = V_+ e^{+j\beta l} + V_- e^{-j\beta l}$$

$$I_1 = Y_o (V_+ e^{+j\beta l} - V_- e^{-j\beta l})$$

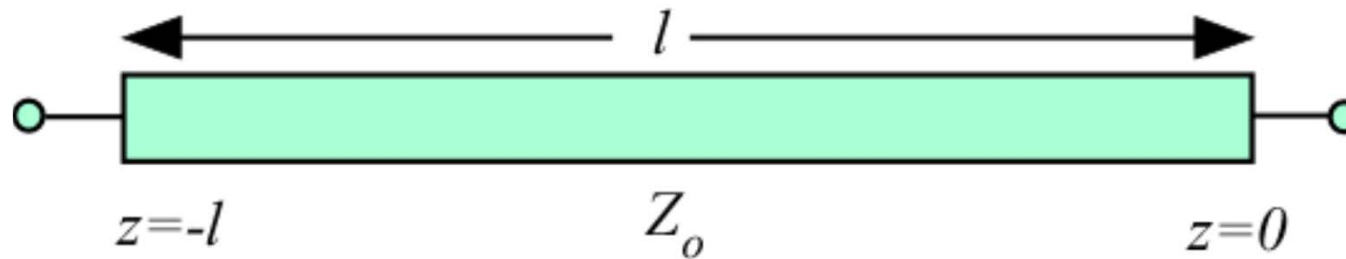
at port 2 ($z = 0$)

$$V_2 = V_+ + V_-$$

$$I_2 = -Y_o (V_+ - V_-)$$

$$V_+ = \frac{V_2 - Z_o I_2}{2} \quad \text{and} \quad V_- = \frac{V_2 + Z_o I_2}{2}$$

Y-Parameters of TL



So that

$$V_1 = \left(\frac{V_2 - Z_o I_2}{2} \right) e^{+j\beta l} + \left(\frac{V_2 + Z_o I_2}{2} \right) e^{-j\beta l}$$

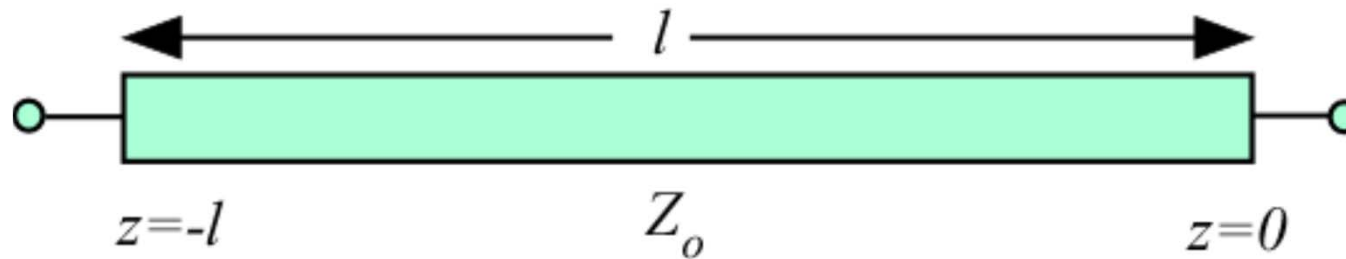
$$I_1 = Y_o \left(\frac{V_2 - Z_o I_2}{2} \right) e^{+j\beta l} - Y_o \left(\frac{V_2 + Z_o I_2}{2} \right) e^{-j\beta l}$$

and

$$V_1 = V_2 \cos \beta l - Z_o I_2 j \sin \beta l$$

$$I_1 = +Y_o V_2 j \sin \beta l - I_2 \cos \beta l$$

Y-Parameters of TL



Using definitions for Y_{11}

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \frac{-I_2 \cos \beta l}{-jZ_o I_2 \sin \beta l} = \frac{-jY_o \cos \beta l}{\sin \beta l}$$

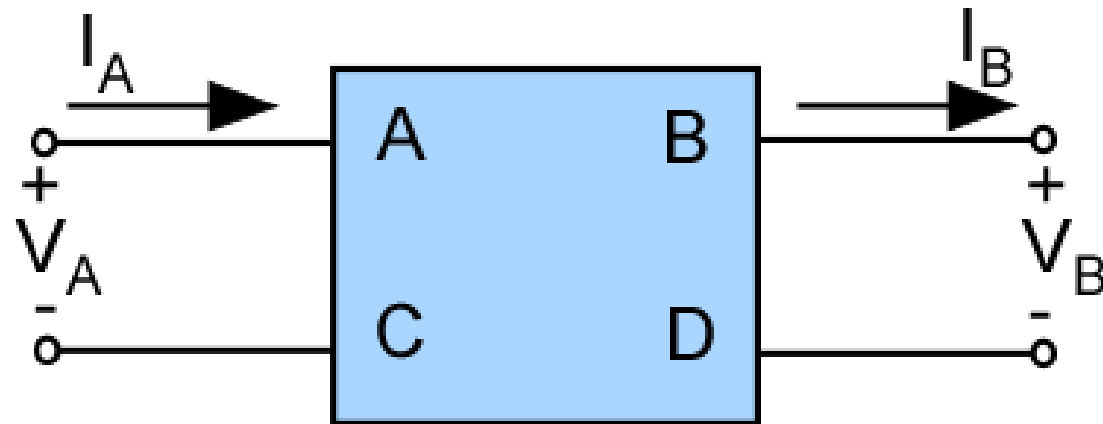
and

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = \frac{-I_2}{-jZ_o I_2 \sin \beta l} = \frac{+jY_o}{\sin \beta l}$$

$$Y_{22} = Y_{11} \text{ by symmetry}$$

$$Y_{12} = Y_{21} \text{ by reciprocity}$$

ABCD -Parameters



$$V_A = AV_B + BI_B$$

$$I_A = CV_B + DI_B$$

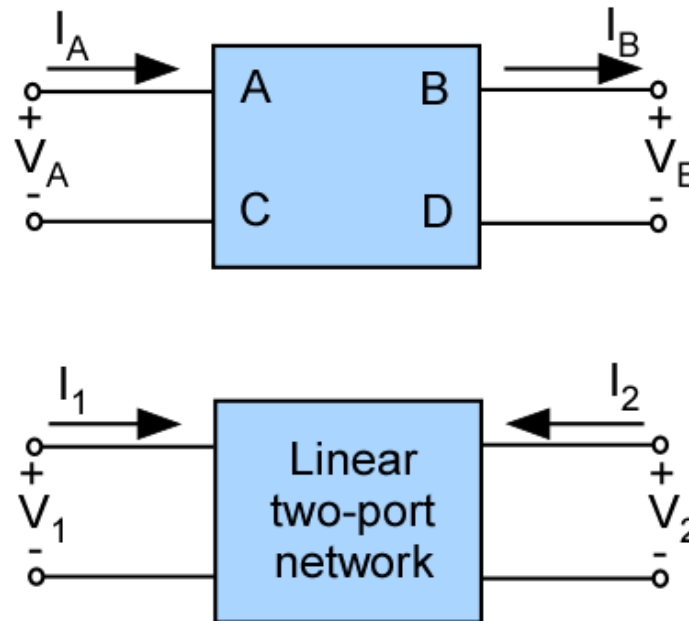
ABCD -Parameters

$$V_A = V_1$$

$$V_B = V_2$$

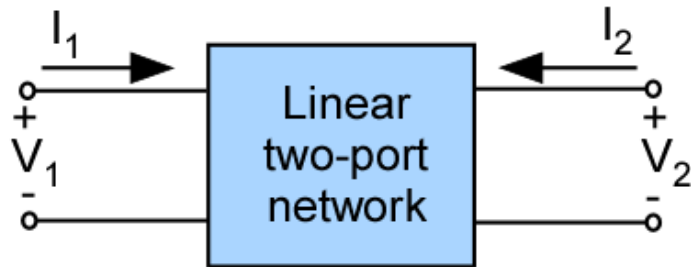
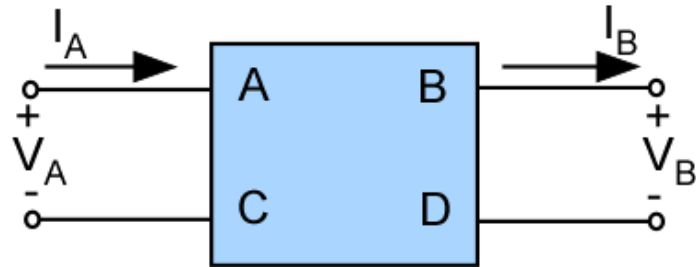
$$I_A = I_1$$

$$I_B = -I_2$$



Relationship with Z parameters is obtained by first expressing ABCD parameters in terms of Z parameters

ABCD -Parameters



From

$$V_A = Z_{11}I_A - Z_{12}I_B$$

$$V_B = Z_{21}I_A - Z_{22}I_B$$

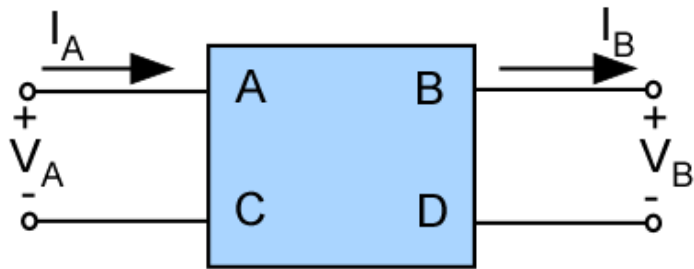
We get

$$A = \frac{Z_{11}}{Z_{21}} \quad B = \frac{\Delta}{Z_{21}}$$

$$C = \frac{1}{Z_{21}} \quad D = \frac{Z_{22}}{Z_{21}}$$

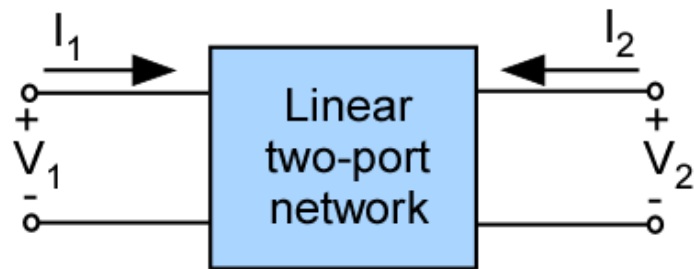
$$\Delta = Z_{11}Z_{22} - Z_{12}Z_{21}$$

ABCD -Parameters



$$Z_{11} = \frac{A}{C}$$

$$Z_{11} = \frac{(AD - BC)}{C}$$



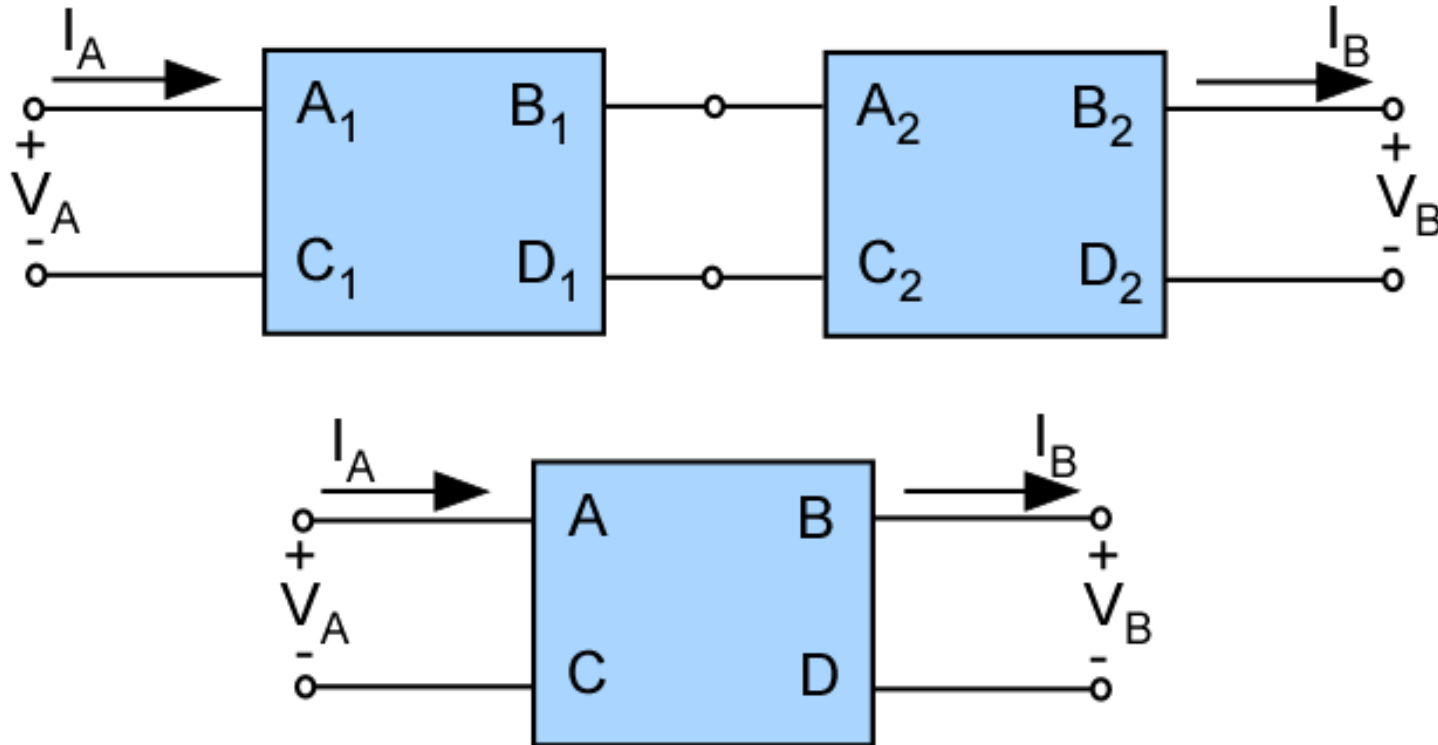
$$Z_{21} = \frac{1}{C}$$

$$Z_{22} = \frac{1}{C}$$

For a reciprocal network, $Z_{21} = Z_{12}$, therefore

$$AD - BC = 1 \quad \leftarrow \text{Reciprocity condition for ABCD parameters}$$

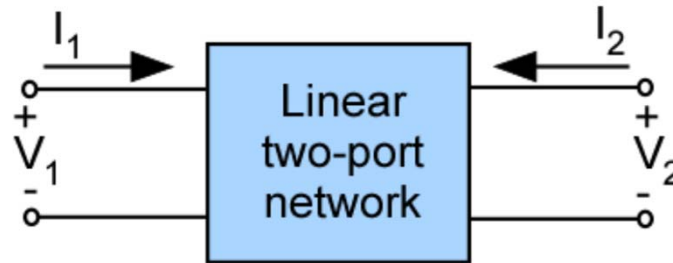
ABCD -Parameters



When cascading two-ports, it is best to use ABCD parameters. Put voltage and currents in cascadable form with the input variables in terms of the output variables

$$ABCD = (ABCD)_1 \cdot (ABCD)_2$$

TWO-PORT NETWORK REPRESENTATION



Z Parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

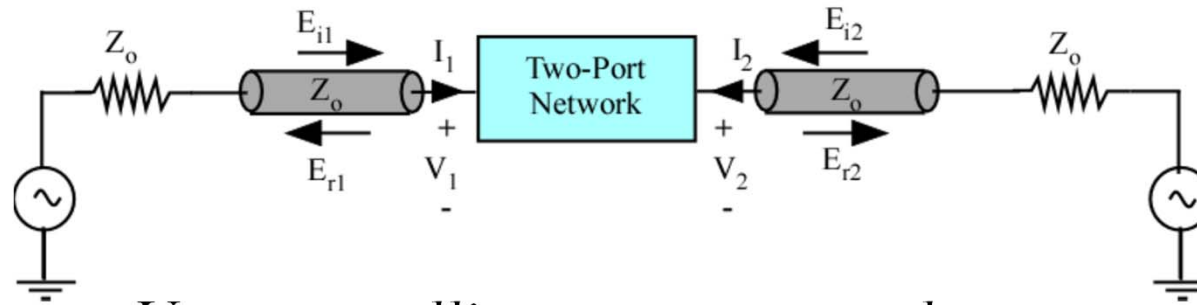
Y Parameters

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

- **At microwave frequencies, it is more difficult to measure total voltages and currents.**
- **Short and open circuits are difficult to achieve at high frequencies.**
- **Most active devices are not short- or open-circuit stable.**

Wave Approach



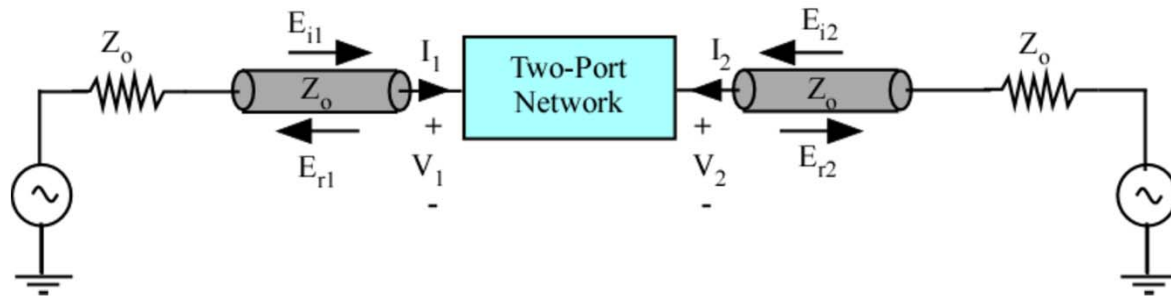
Use a travelling wave approach

$$V_1 = E_{i1} + E_{r1} \quad V_2 = E_{i2} + E_{r2}$$

$$I_1 = \frac{E_{i1} - E_{r1}}{Z_o} \quad I_2 = \frac{E_{i2} - E_{r2}}{Z_o}$$

- Total voltage and current are made up of sums of forward and backward traveling waves.
- Traveling waves can be determined from standing-wave ratio.

Wave Approach



$$a_1 = \frac{E_{i1}}{\sqrt{Z_o}}$$

$$a_2 = \frac{E_{i2}}{\sqrt{Z_o}}$$

$$b_1 = \frac{E_{r1}}{\sqrt{Z_o}}$$

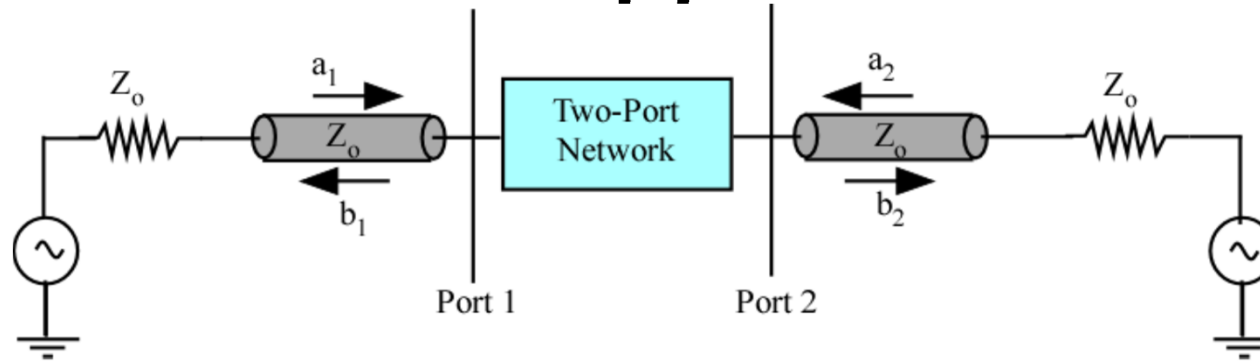
$$b_2 = \frac{E_{r2}}{\sqrt{Z_o}}$$

Z_o is the reference impedance of the system

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

Wave Approach



$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}$$

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$

To make $a_i = 0$

- 1) Provide no excitation at port i
- 2) Match port i to the characteristic impedance of the reference lines.

CAUTION : a_i and b_i are the traveling waves in the reference lines.

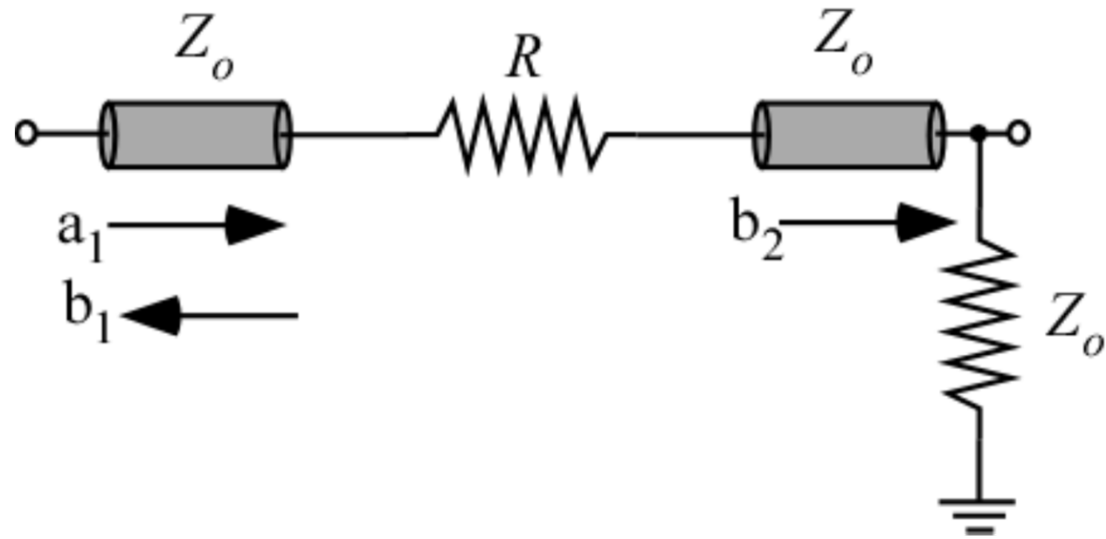
S-Parameters of Resistor



Determine S-Parameter of 2-port resistance

- Insert R between two reference TL
- Provide excitation at port 1 for S_{11} and S_{21}
- Provide excitation at port 2 for S_{12} and S_{22}
- Can use symmetry and reciprocity

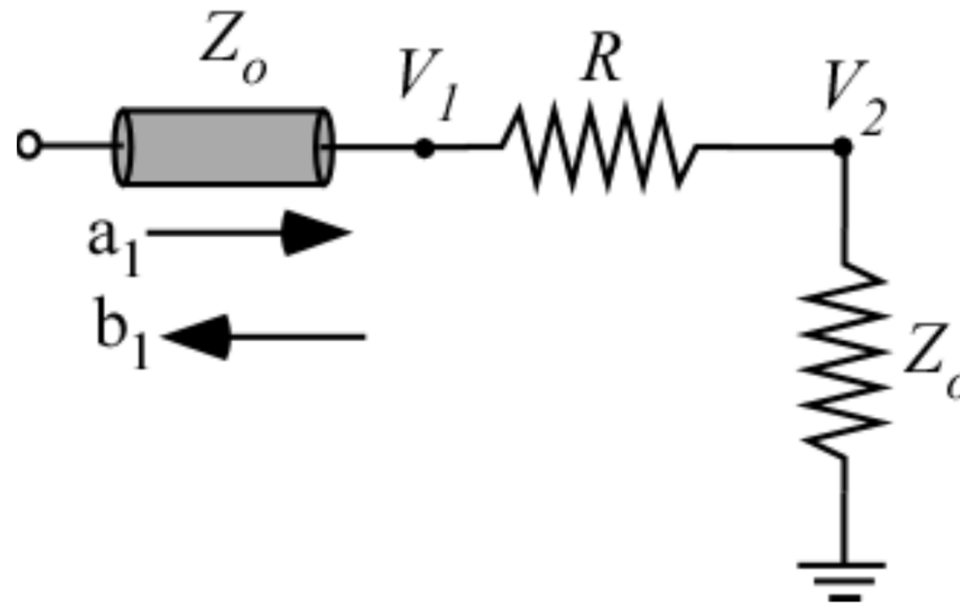
S-Parameters of Resistor



$$S_{11} = \frac{b_1}{a_1} = \Gamma = \frac{(R + Z_o) - Z_o}{(R + Z_o) + Z_o} = \frac{R}{R + 2Z_o}$$

$$S_{11} = \frac{R}{R + 2Z_o} \quad \text{and by symmetry,} \quad S_{22} = \frac{R}{R + 2Z_o}$$

Calculating S_{21} of Resistor



Since $a_2=0$, the total voltage in port 2 is: $V_2 = b_2\sqrt{Z_o}$

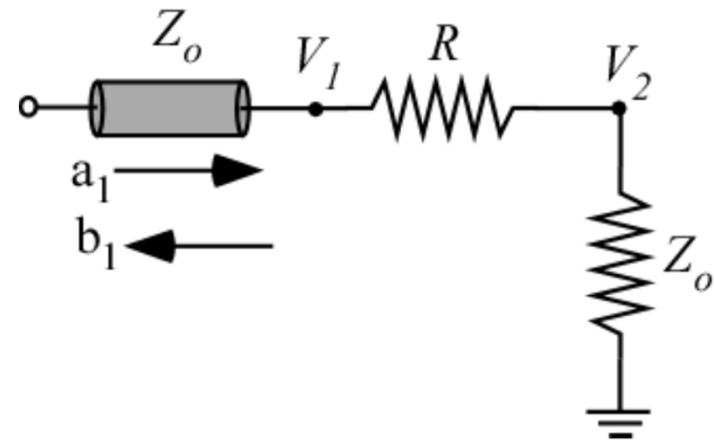
$$V_2 = \frac{V_1 Z_o}{R + Z_o} = \frac{\sqrt{Z_o} (a_1 + b_1) Z_o}{R + Z_o} = \frac{\sqrt{Z_o} (a_1 + S_{11} a_1) Z_o}{R + Z_o}$$

S-Parameters of Resistor

$$V_2 = \frac{Z_o \sqrt{Z_o} (1 + S_{11}) a_1}{R + Z_o} = \frac{2Z_o a_1 \sqrt{Z_o}}{R + 2Z_o}$$

$$S_{21} = \frac{b_2}{a_1} = \frac{V_2}{\sqrt{Z_o}} \frac{1}{a_1} = \frac{2Z_o}{R + 2Z_o}$$

$$S_{21} = \frac{2Z_o}{R + 2Z_o} \quad \text{and by reciprocity,} \quad S_{12} = \frac{2Z_o}{R + 2Z_o}$$



S parameters of
resistor R

$$S = \begin{bmatrix} \frac{R}{R + 2Z_o} & \frac{2Z_o}{R + 2Z_o} \\ \frac{2Z_o}{R + 2Z_o} & \frac{R}{R + 2Z_o} \end{bmatrix}$$

N-Port S Parameters

$$\begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdot & \cdot \\ S_{21} & S_{22} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ a_n \end{bmatrix}$$

$$\mathbf{b} = \mathbf{S}\mathbf{a}$$

If $b_i = 0$, then no reflected wave on port $i \rightarrow$ port is matched

$$a_i = \frac{V_i^+}{\sqrt{Z_{oi}}}$$

V_i^+ : incident voltage wave in port i

V_i^- : reflected voltage wave in port i

$$b_i = \frac{V_i^-}{\sqrt{Z_{oi}}}$$

Z_{oi} : impedance in port i

N-Port S Parameters

$$\mathbf{v} = \sqrt{Z_o}(\mathbf{a} + \mathbf{b}) \quad (1) \quad \mathbf{i} = \frac{1}{\sqrt{Z_o}}(\mathbf{a} - \mathbf{b}) \quad (2) \quad \mathbf{v} = \mathbf{Z}\mathbf{i} \quad (3)$$

Substitute (1) and (2) into (3)

$$\sqrt{Z_o}(\mathbf{a} + \mathbf{b}) = \mathbf{Z} \frac{1}{\sqrt{Z_o}}(\mathbf{a} - \mathbf{b})$$

Defining \mathbf{S} such that $\mathbf{b} = \mathbf{S}\mathbf{a}$ and substituting for \mathbf{b}

$$Z_o(\mathbf{U} + \mathbf{S})\mathbf{a} = Z_o(\mathbf{U} - \mathbf{S})\mathbf{a}$$

\mathbf{U} : unit matrix

$\mathbf{S} \rightarrow \mathbf{Z}$

$$\mathbf{Z} = Z_o(\mathbf{U} + \mathbf{S})(\mathbf{U} - \mathbf{S})^{-1}$$

$\mathbf{Z} \rightarrow \mathbf{S}$

$$\mathbf{S} = (\mathbf{Z} + Z_o\mathbf{U})^{-1}(\mathbf{Z} - Z_o\mathbf{U})$$

N-Port S Parameters

If the port reference impedances are different, we define \mathbf{k} as

$$\mathbf{k} = \begin{bmatrix} \sqrt{Z_{o1}} & & & \\ & \sqrt{Z_{o2}} & & \\ & & \ddots & \\ & & & \sqrt{Z_{on}} \end{bmatrix}$$

$$\mathbf{v} = \mathbf{k}(\mathbf{a} + \mathbf{b}) \quad \text{and} \quad \mathbf{i} = \mathbf{k}^{-1}(\mathbf{a} - \mathbf{b}) \quad \text{and} \quad \mathbf{k}(\mathbf{a} + \mathbf{b}) = \mathbf{Z}\mathbf{k}^{-1}(\mathbf{a} - \mathbf{b})$$

$\mathbf{Z} \rightarrow \mathbf{S}$

$\mathbf{S} \rightarrow \mathbf{Z}$

$$\mathbf{S} = (\mathbf{Z}\mathbf{k}^{-1} + \mathbf{k})(\mathbf{Z}\mathbf{k}^{-1} - \mathbf{k})$$

$$\mathbf{Z} = \mathbf{k}(\mathbf{U} + \mathbf{S})(\mathbf{U} - \mathbf{S})^{-1}\mathbf{k}$$

Normalization

Assume original S parameters as S_1 with system k_1 . Then the representation S_2 on system k_2 is given by

Transformation Equation

$$S_2 = \left[k_1(U + S_1)(U - S_1)^{-1}k_1k_2 + k_2 \right]^{-1} \left[k_1(U + S_1)(U - S_1)^{-1}k_1k_2 - k_2 \right]$$

If Z is symmetric, S is also symmetric

Dissipated Power

$$P_d = \frac{1}{2} \mathbf{a}^T (\mathbf{U} - \mathbf{S}^T \mathbf{S}^*) \mathbf{a}^*$$

The dissipation matrix \mathbf{D} is given by:

$$\mathbf{D} = \mathbf{U} - \mathbf{S}^T \mathbf{S}^*$$

Passivity insures that the system will always be stable provided that it is connected to another passive network

For passivity

- (1) the determinant of \mathbf{D} must be ≥ 0
- (2) the determinant of the principal minors must be ≥ 0

Dissipated Power

When the dissipation matrix is 0, we have a lossless network →

$$\mathbf{S}^T \mathbf{S}^* = \mathbf{U}$$

The \mathbf{S} matrix is unitary.

For a lossless two-port:

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

$$|S_{22}|^2 + |S_{12}|^2 = 1$$

If in addition the network is reciprocal, then

$$S_{12} = S_{21} \quad \text{and} \quad |S_{11}| = |S_{22}| = \sqrt{1 - |S_{12}|^2}$$

Scattering Transfer Parameters

In T-Parameters, traveling waves at the input are related to those at the output

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_1 = T_{11}a_2 + T_{12}b_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$a_1 = T_{21}a_2 + T_{22}b_2$$

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} T_{12}T_{22}^{-1} & T_{11} - T_{12}T_{21}T_{22}^{-1} \\ T_{22}^{-1} & -T_{21}T_{22}^{-1} \end{pmatrix}$$

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} S_{12} - S_{11}S_{22}^{-1}S_{21} & S_{11}S_{21}^{-1} \\ -S_{22}^{-1}S_{21} & S_{21}^{-1} \end{pmatrix}$$

T parameters can be cascaded $\mathbf{T} = \mathbf{T}_A \cdot \mathbf{T}_B$