A 2-port passive device with SMA connectors is measured:

\[
S' = \begin{bmatrix}
0.33 & 0.97 \\
0.97 & 0.54
\end{bmatrix}
\]

Connectors have different electrical delays and are known to have the following phase shift at that particular frequency:

- Phase Shift Port 1: 10°
- Phase Shift Port 2: 15°

Find the actual scattering parameters:

\[
S_{11} = S_{11}' e^{j2\Phi_1} = 0.33 \cdot 16.3° + 2 \times 10° = 33.83°
\]

\[
S_{12} = S_{21}' e^{j(\Phi_1 + \Phi_2)} = 0.97 \cdot 172° + 10° + 15° = 0.97 \cdot 192°
\]

\[
S_{22} = S_{22}' e^{j2\Phi_2} = 0.54 \cdot 25° + 2 \times 15° = 0.54 \cdot 55°
\]

\[
S = \begin{bmatrix}
0.33 & 0.97 \\
0.97 & 0.54
\end{bmatrix}
\]
A TL of length \( L = 1\ m \) has \( z_0 = 70\Omega \) and prop. velocity \( V = 0.14 \ m/\text{ns} \).

At low frequencies, this TL can be represented by a lumped circuit.

Draw a schematic of this lumped circuit and indicate the values of the circuit elements.

\[
\begin{align*}
\text{Part 1} & : \\
\text{Part 2} & \\
L_T = 500\ \mu\text{H} \\
C_T/2 & = 5\text{pF} \\
C_T/2 & = 5\text{pF} \\
L_T & = L \times L \\
20 & = \sqrt{\frac{L}{C}} \quad \text{and} \quad V = \frac{1}{\sqrt{2C}} \\
\Rightarrow L & = \frac{20}{V} = \frac{50}{0.14} = 500\ \mu\text{H}/\text{m} \\
C & = \frac{1}{2.0V} = \frac{1}{50 \times 0.14} = 10\ \text{pF}/\text{m}
\end{align*}
\]
The network is reciprocal since $S_{ij} = S_{ji}$ for all $i, j$.

(b) Write down the criteria for a network to be lossless.

The criterion is: $\sum_{j=1}^{N} |S_{ij}|^2 = 1$ for all $i, j$. 

$\begin{bmatrix}
0.2 & 1180^\circ & 0.8 & 450^\circ & 0.1 & 450^\circ \\
0.8 & 450^\circ & 0.2 & 100^\circ & 0.1 & 900^\circ \\
0.1 & 450^\circ & 0.1 & 900^\circ & 0.1 & 1800^\circ \\
\end{bmatrix}$
(c) Is the three-port lossless?

Port 1: \[ |S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 \]
\[ = 0.04 + 0.64 + 0.01 = 0.69 < 1 \]

\[ \Rightarrow \text{No, the 3 port is NOT lossless} \]

(d) Draw the Signal Flow Graph for the three-port.

(e) A 50 \( \Omega \) load is attached to port 3.

Use SFG operations to derive the SFG with just ports 1 and 2. Write down
the $S$-parameter matrix of the simplified network

With a 50-ohm load at port 3, there is no input at $a_3$. So the SFG simplifies to:

$$S = \begin{bmatrix}
-0.2 & 0.8e^{-j45^\circ} \\
0.8e^{-j45^\circ} & 0.2
\end{bmatrix}$$

A transmission line has characteristic impedance $Z_0$, propagation constant $\beta$, and length $L$. Derive an expression for $Y_{11}$ for the line.
\[
\begin{align*}
I_1 &= Y_{11} V_1 + Y_{12} V_2 \\
I_2 &= Y_{21} V + Y_{22} V_2
\end{align*}
\]

\(Y_{11}\) is the admittance seen at port 1 with a short at port 2.

\[
\Gamma_{in} = -e^{-2j\beta l} \quad \rightarrow \quad \Gamma_{in}
\]

\[
Y_{in} = \frac{1}{\pi_0} \left( \frac{1 - \Gamma_{in}}{1 + \Gamma_{in}} \right)
\]

\[
Y_{in} = Y_{11} = \frac{1 + e^{-2j\beta l}}{1 - e^{-2j\beta l}} \cdot \frac{1}{\pi_0}
\]

A 75-Ω microstrip line is a half wavelength at a specific frequency. What are \(S_{11}\), \(S_{12}\), \(S_{21}\), and \(S_{22}\) at that frequency as measured on a 50-Ω network analyzer?

\[
S_{11} = S_{22} = \frac{(1 - x^2)\Gamma}{1 - \Gamma^2 x^2}
\]

\[
\beta = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \beta l = \frac{2\pi}{\lambda} \cdot l
\]
\[ S_{12} = S_{21} = \frac{(1 - \rho^2)X}{1 - \rho^2 x^2} \]

\[ X = e^{-\hat{\beta} \ell} \]

\[ \Gamma = \frac{\varepsilon_c - \varepsilon_0}{\varepsilon_c + \varepsilon_0} \]

\[ \varepsilon_0 = 50 \Omega \]

\[ \Rightarrow \Gamma = \frac{75 - 50}{75 + 50} = 0.2 \]

at \( \lambda_2 \), \( X = e^{-\hat{\beta} \pi} = -1 \)

\[ X^2 = 1 \]

So, \( S_{11} = S_{22} = \frac{(1 - X^2) \Gamma}{1 - X^2 \rho^2} = \frac{(1 - 1) 0.2}{1 - (0.2)^2} \)

\[ S_{11} = S_{22} = 0 \]

\[ S_{12} = S_{21} = -\frac{(1 - \rho^2)}{1 - \rho^2} = -1 \]

\[ S_{12} = S_{21} = -1 \]
What is the magnitude and phase of the load current $I_R$?

$$V_+ = \frac{T_5 V_s e^{-j \beta l}}{1 - \Gamma_R \Gamma_s e^{-2j \beta l}}$$

$$T_5 = \frac{Z_0}{Z_5 + Z_0} = \frac{50}{50 + 25} = \frac{2}{3}$$

$$\Gamma_s = \frac{Z_5 - Z_0}{Z_5 + Z_0} = \frac{25 - 50}{25 + 50} = -\frac{1}{3}$$

$$\Gamma_R = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{75 - 50}{75 + 50} = \frac{1}{3}$$

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \Rightarrow e^{-j \beta l} = -j$$

$$V_+ = \frac{(\frac{2}{3})(1)(-j)}{(-(-\frac{1}{3})(\frac{1}{3})(-1))} = -\frac{j^2 \sqrt{3}}{1 - \frac{1}{15}} = -j \frac{\sqrt{5}}{2}$$
\[ V_+ = -j \times 0.714285 \, V \]

\[ I(\beta) = \frac{V_+}{Z_0} \left[ e^{-j \beta z} - e^{+j \beta z} \right] \]

At \( z = 0 \), \( I(0) = I_R \)

\[ I_R = \frac{V_+}{Z_0} \left[ 1 - R \right] = \]

\[ I_R = -j \times 0.714285 \times (1 - 0.2) \]

\[ I_R = -j \times 0.714285 \times 0.8 \]

\[ I_R = -j \times 0.014285 \, A \]

\[ I_R = 0.014285 \angle -90^\circ \, A \]