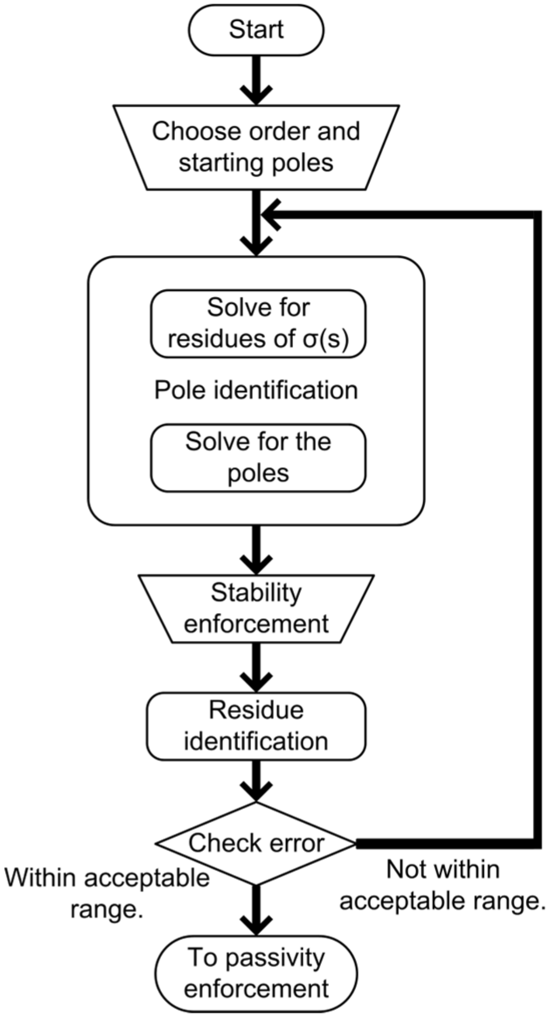
# ECE 451
# Circuit Synthesis

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jesa@illinois.edu

# MOR via Vector Fitting

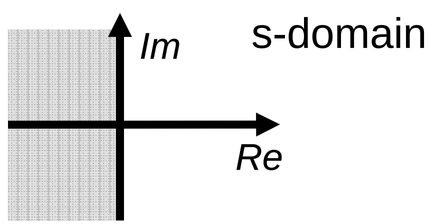



- Rational function approximation: $$f(s) \approx \sum_{n=1}^{N} \frac{c_n}{s - a_n} + d + sh$$

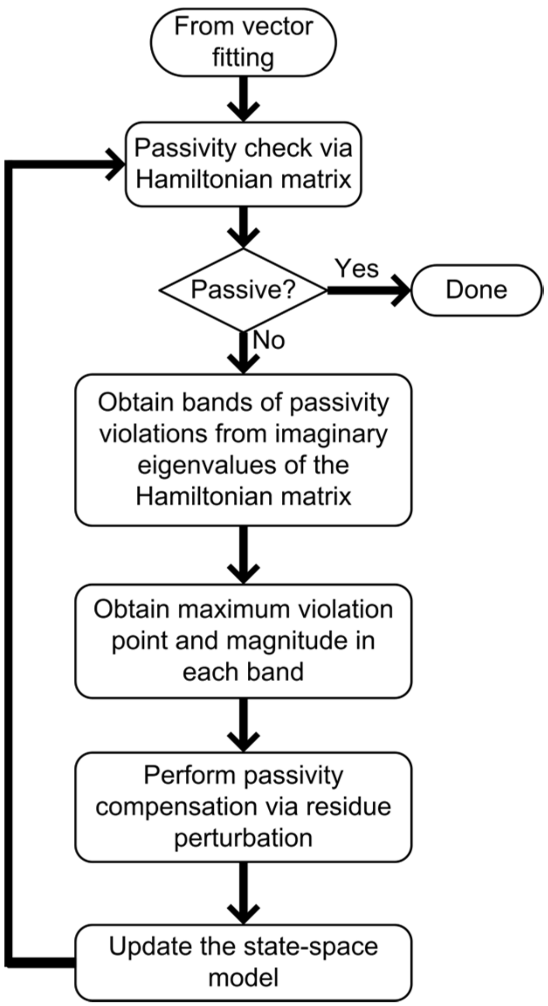
- Introduce an unknown function *σ(s)* that satisfies: $$\begin{bmatrix} \sigma(s) f(s) \\ \sigma(s) \end{bmatrix} \approx \begin{bmatrix} \sum_{n=1}^{N} \frac{c_n}{s - \tilde{a}_n} + d + sh \\ \sum_{n=1}^{N} \frac{\tilde{c}_n}{s - \tilde{a}_n} + 1 \end{bmatrix}$$

- Poles of *f(s)* = zeros of *σ(s)*: $$f(s) \approx \frac{\sum_{n=1}^{N} \frac{c_n}{s - \tilde{a}_n} + d + sh}{\sum_{n=1}^{N} \frac{\tilde{c}_n}{s - \tilde{a}_n} + 1} = \frac{\prod_{n=1}^{N+1} (s - z_n)}{\prod_{n=1}^{N} (s - \tilde{z}_n)}$$

- Flip unstable poles into the left half plane.



# Passivity Enforcement

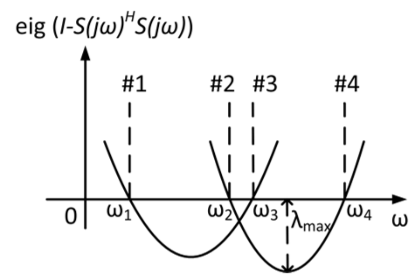



- State-space form:
$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

- Hamiltonian matrix:
$$M = \begin{bmatrix} A + BKD^T C & BKB^T \\ -C^T LC & -A^T - C^T DKB^T \end{bmatrix}$$
$$K = \left(I - D^T D\right)^{-1} \quad L = \left(I - DD^T\right)^{-1}$$

- Passive if *M* has no imaginary eigenvalues.

- Sweep: $eig\left(I - S(j\omega)^H S(j\omega)\right)$




- Quadratic programming:
  - Minimize *(change in response)* subject to *(passivity compensation).*

$$\min\left(vec(\Delta C)^T H\, vec(\Delta C)\right) \quad \text{subject to} \quad \Delta\lambda = G \cdot vec(\Delta C).$$

# Macromodel Circuit Synthesis

## Use of Macromodel

- Time-Domain simulation using recursive convolution
- Frequency-domain circuit synthesis for SPICE netlist

# Macromodel Circuit Synthesis

**Objective: Determine equivalent circuit from macromodel representation***

## Motivation

- Circuit can be used in SPICE

## Goal

- Generate a netlist of circuit elements

*Giulio Antonini "SPICE Equivalent Circuits of Frequency-Domain Responses", IEEE Transactions on Electromagnetic Compatibility, pp 502-512, Vol. 45, No. 3, August 2003.

# Circuit Realization

Circuit realization consists of interfacing the reduced model with a general circuit simulator such as SPICE

Model order reduction gives a transfer function that can be presented in matrix form as

$$S(s) = \begin{bmatrix} s_{11}(s) & \cdot & s_{1N}(s) \\ \cdot & \cdot & \cdot \\ s_{N1}(s) & \cdot & s_{NN}(s) \end{bmatrix}$$

or

$$Y(s) = \begin{bmatrix} y_{11}(s) & \cdot & y_{1N}(s) \\ \cdot & \cdot & \cdot \\ y_{N1}(s) & \cdot & y_{NN}(s) \end{bmatrix}$$

# Y-Parameter - Circuit Realization

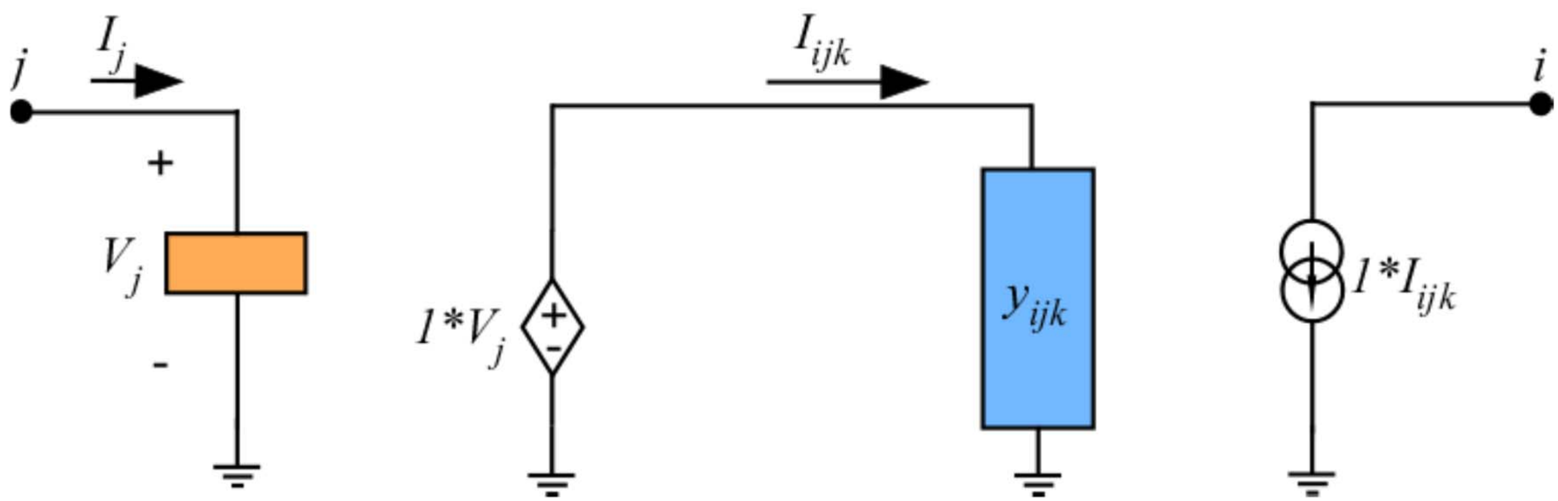
Each of the Y-parameters can be represented as

$$y_{ij}(s) = d + \sum_{k=1}^{L} \frac{a_k}{s - p_k}$$

where the $a_k$'s are the residues and the $p_k$'s are the poles. $d$ is a constant

# Y-Parameter - Circuit Realization

The realized circuit will have the following topology:




**We need to determine the circuit elements within $y_{ijk}$**

# Y-Parameter - Circuit Realization

**We try to find the circuit associated with each term:**

$$y_{ij}(s) = d + \sum_{k=1}^{L} \frac{a_k}{s - p_k}$$

**1. Constant term *d***

$$y_{ijd}(s) = d$$

**2. Each pole-residue pair**

$$y_{ijk}(s) = \frac{a_k}{s - p_k}$$

# Y-Parameter - Circuit Realization

In the pole-residue case, we must distinguish two cases

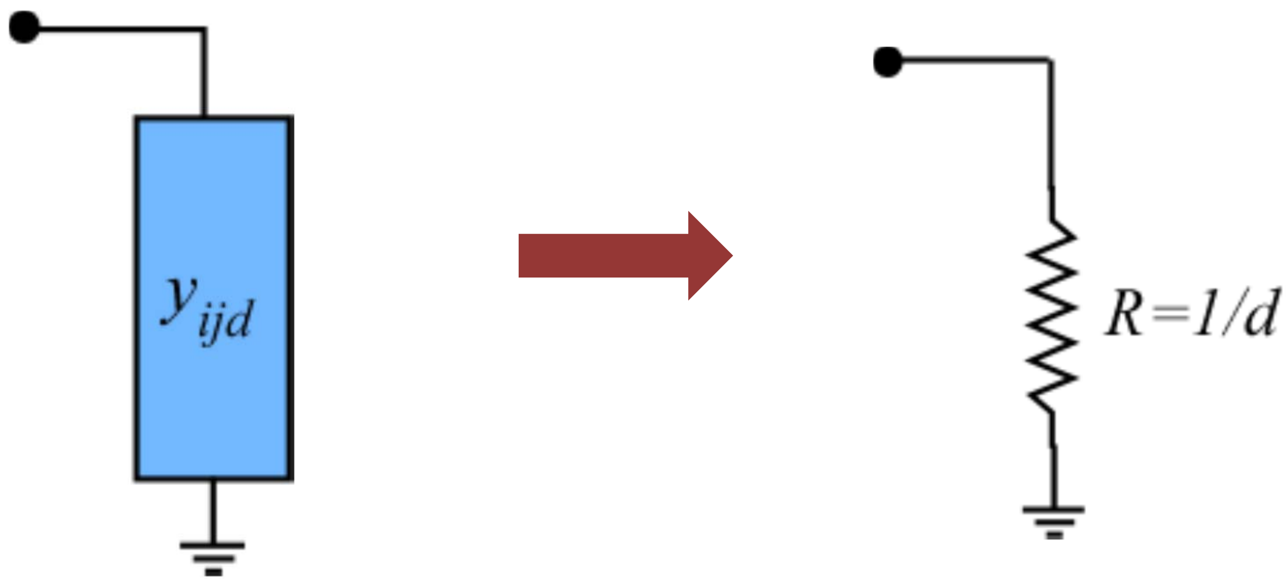
(a) Pole is real $$y_{ijk}(s) = \frac{a_k}{s - p_k}$$

(b) Complex conjugate pair of poles

$$y_{ijk}(s) = \frac{\alpha_k + j\beta_k}{s - \sigma_k - j\omega_k} + \frac{\alpha_k - j\beta_k}{s - \sigma_k + j\omega_k}$$

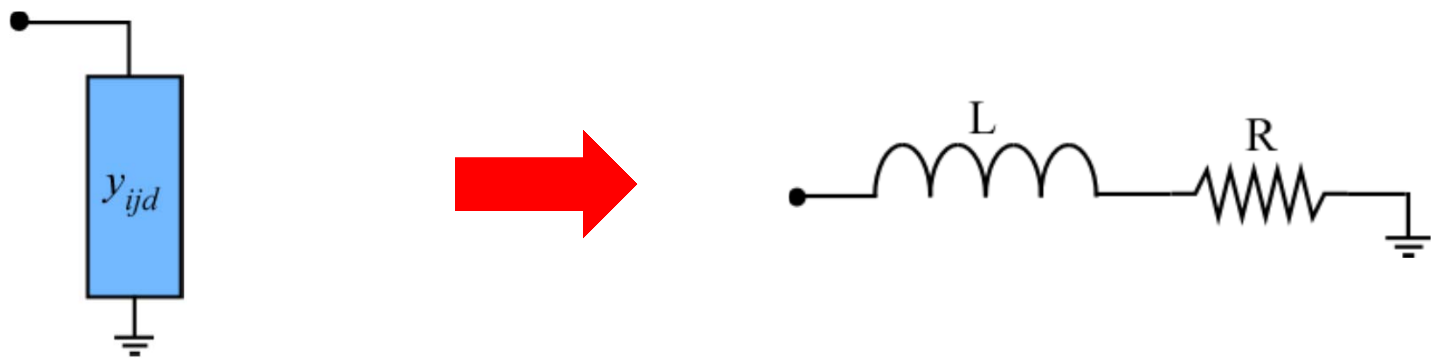
In all cases, we must find an equivalent circuit consisting of lumped elements that will exhibit the same behavior

# Circuit Realization – Constant Term




$$R = \frac{1}{d}$$

# Circuit Realization - Real Pole

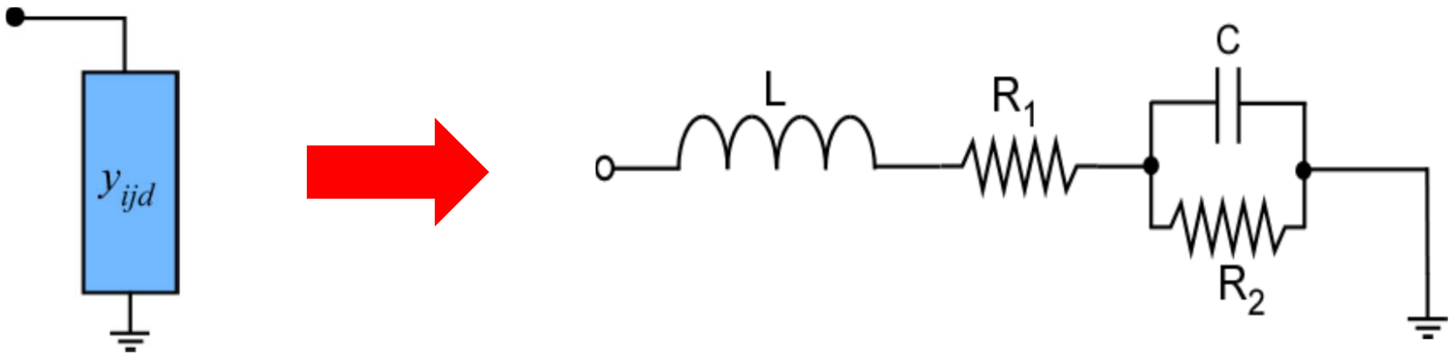



Consider the circuit shown above. The input impedance $Z$ as a function of the complex frequency $s$ can be expressed as:

$$Z = sL + R \qquad Y(s) = \frac{1/L}{s + R/L} \qquad y_{ijk}(s) = \frac{a_k}{s - p_k}$$

$$L = 1/a_k \qquad R = -p_k / a_k$$

# Circuit Realization - Complex Poles

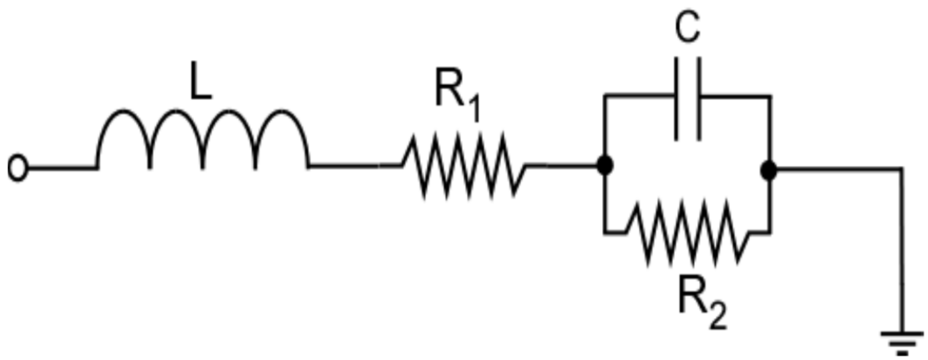



Consider the circuit shown above. The input impedance *Z* as a function of the complex frequency *s* can be expressed as:

$$Z = sL + R_1 + \frac{1}{1/R_2 + sC} = sL + R_1 + \frac{R_2}{1 + sCR_2}$$

$$Z = \frac{(R_1 + sL)(1 + sCR_2) + R_2}{1 + sCR_2}$$

# Circuit Realization - Complex Poles




$$Y = \frac{CR_2\left(s + 1/CR_2\right)}{LCR_2\left[s^2 + s\left(\frac{L + CR_1R_2}{LR_2C}\right) + \frac{\left(R_1 + R_2\right)}{LR_2C}\right]}$$

$$Y = \frac{1}{L}\frac{\left(s + 1/CR_2\right)}{\left[s^2 + s\left(\frac{L + CR_1R_2}{LR_2C}\right) + \frac{\left(R_1 + R_2\right)}{LR_2C}\right]}$$

# Circuit Realization - Complex Poles

**Each term associated with a complex pole pair in the expansion gives:**

$$\hat{Y} = \frac{r_1}{s - p_1} + \frac{r_2}{s - p_2}$$

**Where $r_1$, $r_2$, $p_1$ and $p_2$ are the complex residues and poles. They satisfy: $r_1 = r_2^*$ and $p_1 = p_2^*$**

**It can be re-arranged as:**

$$\hat{Y} = \left(r_1 + r_2\right) \frac{\left[s - \left(r_1 p_2 + r_2 p_1\right) / \left(r_1 + r_2\right)\right]}{s^2 - s\left(p_1 + p_2\right) + p_1 p_2}$$

# Circuit Realization - Complex Poles

We next compare

$$Y = \frac{1}{L}\frac{\left(s + 1/CR_2\right)}{\left[s^2 + s\left(\frac{L + CR_1R_2}{LR_2C}\right) + \frac{\left(R_1 + R_2\right)}{LR_2C}\right]}$$

and

$$\hat{Y} = \left(r_1 + r_2\right)\frac{\left[s - \left(r_1p_2 + r_2p_1\right)/\left(r_1 + r_2\right)\right]}{s^2 - s\left(p_1 + p_2\right) + p_1p_2}$$

**DEFINE**

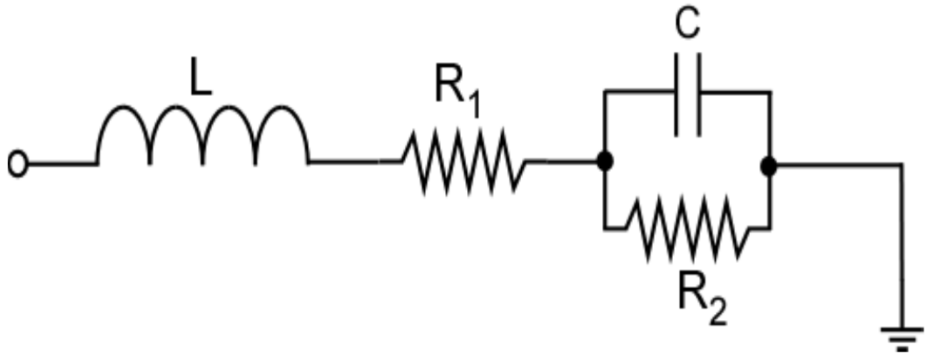
$p = p_1p_2$ product of poles

$a = r_1 + r_2$ sum of residues

$g = p_1 + p_2$ sum of poles

$x = r_1p_2 + r_2p_1$ cross product

# Circuit Realization - Complex Poles




**We can identify the circuit elements**

$$L = 1/a$$

$$R_1 = \frac{x}{a^2} - \frac{g}{a}$$

$$R_2 = -\frac{p}{x} - \frac{x}{a^2} + \frac{g}{a}$$

$$C = \frac{pa}{x^2} + \frac{1}{a} - \frac{g}{x}$$

# Circuit Realization - Complex Poles

In the circuit synthesis process, it is possible that some circuit elements come as negative. To prevent this situation, we add a contribution to the real parts of the residues of the system. In the case of a complex residue, for instance, assume that

$$\hat{Y} = \frac{r_1}{s-p_1} + \frac{r_2}{s-p_2}$$

$$\hat{Y} = \underbrace{\frac{r_1+\Delta}{s-p_1} + \frac{r_2+\Delta}{s-p_2}}_{Augmented\ Circuit} - \underbrace{\left(\frac{\Delta}{s-p_1} + \frac{\Delta}{s-p_2}\right)}_{Compensation\ Circuit}$$

Can show that both augmented and compensation circuits will have positive elements

# S-Parameter - Circuit Realization

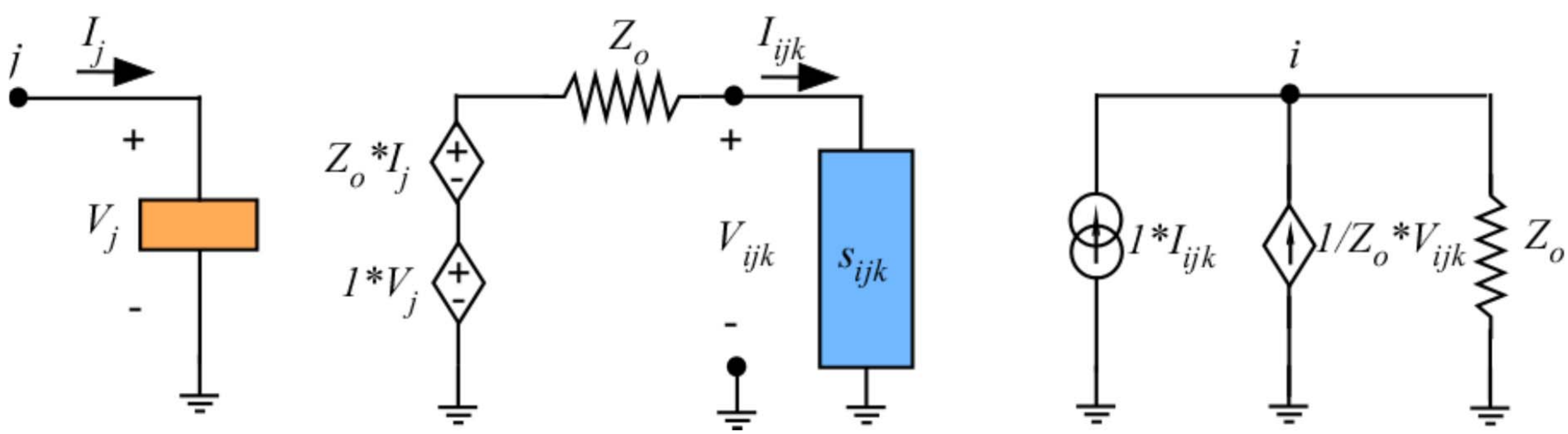
Each of the S-parameters can be represented as

$$s_{ij}(s) = d + \sum_{k=1}^{L} \frac{a_k}{s - p_k}$$

where the $a_k$'s are the residues and the $p_k$'s are the poles. $d$ is a constant

# Realization from S-Parameters

The realized circuit will have the following topology:




**We need to determine the circuit elements within $s_{ijk}$**

# S-Parameter - Circuit Realization

**We try to find the circuit associated with each term:**

$$s_{ij}(s) = d + \sum_{k=1}^{L} \frac{a_k}{s - p_k}$$

**1. Constant term *d***

$$s_{ijd}(s) = d$$

**2. Each pole and residue pair**

$$s_{ijk}(s) = \frac{a_k}{s - p_k}$$

# S-Parameter - Circuit Realization

In the pole-residue case, we must distinguish two cases

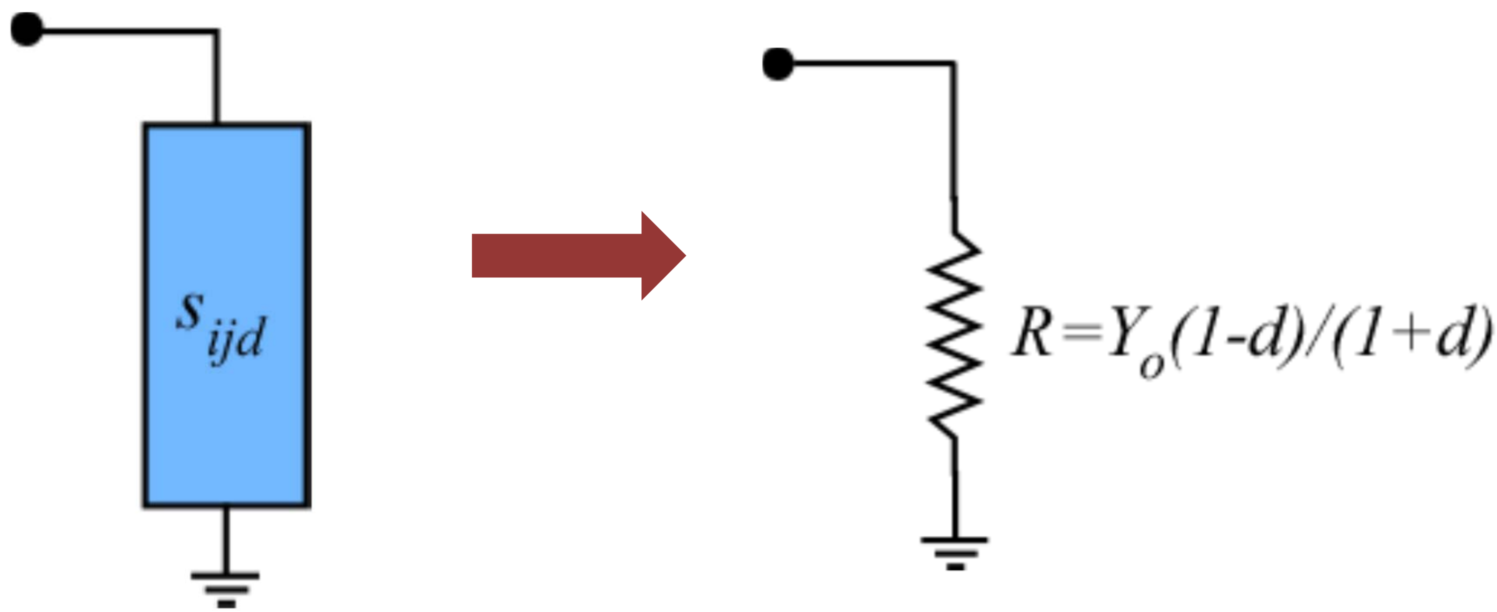
(a) Pole is real $$s_{ijk}(s) = \frac{a_k}{s - p_k}$$

(b) Complex conjugate pair of poles

$$s_{ijk}(s) = \frac{\alpha_k + j\beta_k}{s - \sigma_k - j\omega_k} + \frac{\alpha_k - j\beta_k}{s - \sigma_k + j\omega_k}$$

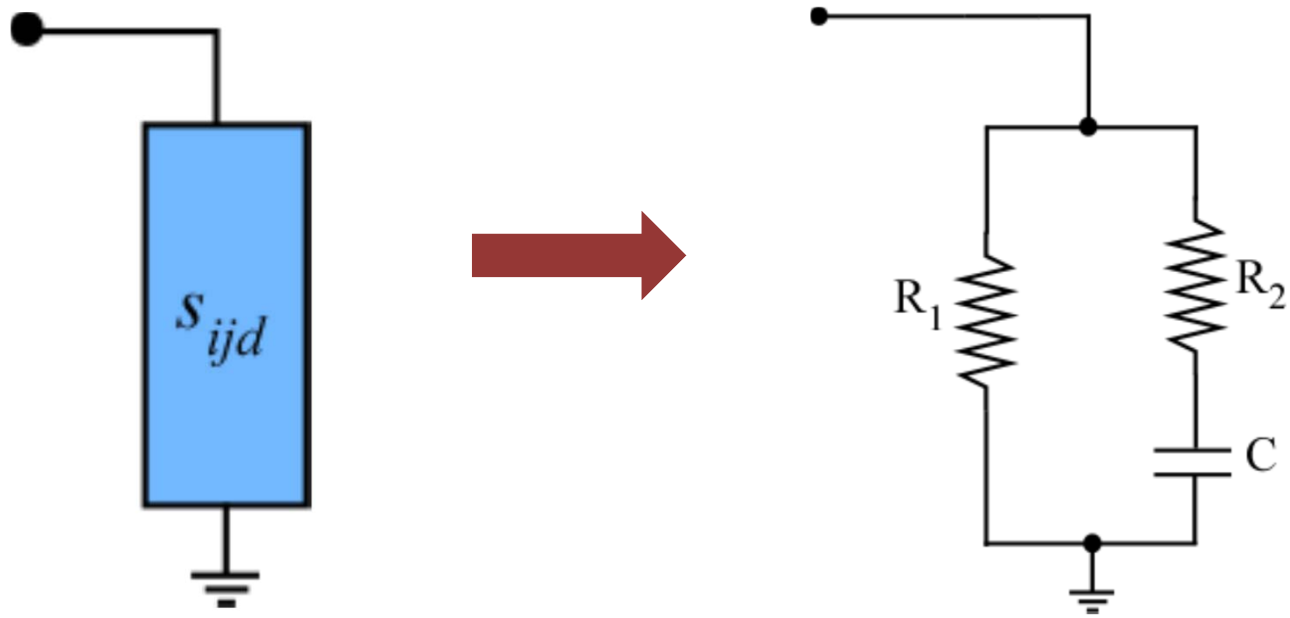
In all cases, we must find an equivalent circuit consisting of lumped elements that will exhibit the same behavior

# S- Circuit Realization – Constant Term

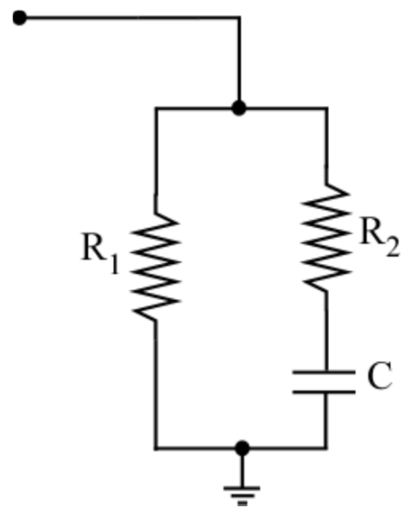



$$R = Y_o\left(\frac{1-d}{1+d}\right)$$

# S-Realization – Real Poles



# S-Realization – Real Poles



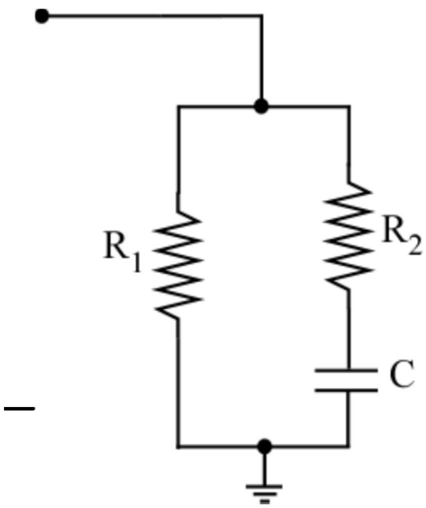

Admittance of proposed model is given by:

$$Y = \frac{\left(R_1 + R_2\right)}{R_1 R_2}\left[\frac{s + \frac{1}{\left(R_1 + R_2\right)C}}{s + \frac{1}{R_2 C}}\right]$$

# S-Realization – Real Poles

From S-parameter expansion we have:

$$s_{ijk}(s) = \frac{r_k}{s - p_k}$$




which corresponds to:

$$\hat{Y} = Y_o\left(\frac{s-a}{s-b}\right)$$ where $a = p_k + r_k$, and $b = p_k -$

from which

$$Y_o = \frac{R_1 + R_2}{R_1 R_2}$$

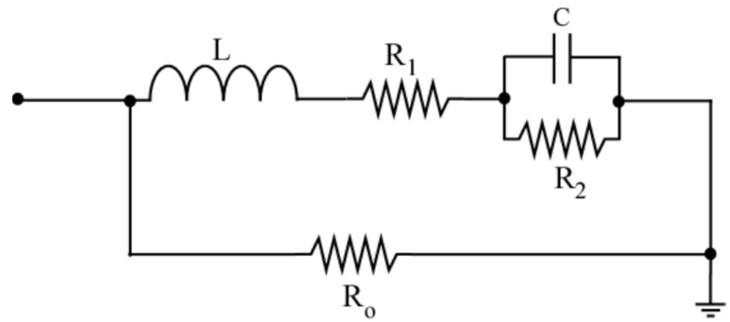
$$C = -\frac{(b-a)}{b^2 Z_o}$$

$$R_2 = \frac{-1}{bC}$$

$$R_1 = -R_2 - \frac{1}{aC}$$

# Realization – Complex Poles

## Proposed model




$$Y = Y_a + \frac{1}{R_o} = \frac{1 + sCR_2}{s^2 LCR_2 + s\left(L + R_1 R_2 C\right) + \left(R_1 + R_2\right)} + \frac{1}{R_o}$$

## which can be re-arranged as:

$$Y = \frac{1}{R_o}\left[\frac{s^2 + s\left(\frac{L + R_1 R_2 C + R_o R_2 C}{LCR_2}\right) + \frac{R_o + R_1 + R_2}{LCR_2}}{s^2 + s\left(\frac{L + R_1 R_2 C}{LCR_2}\right)_o + \frac{R_1 + R_2}{LCR_2}}\right]$$

# Realization – Complex Poles

From the S-parameter expansion, the complex pole pair gives:

$$\hat{S} = \frac{r_1}{s-p_1} - \frac{r_2}{s-p_2} = \frac{s(r_1+r_2)-(r_1p_2+r_2p_1)}{s^2-s(p_1+p_2)+p_1p_2}$$

which corresponds to an admittance of:

$$\hat{Y} = Y_o\left(\frac{1-\hat{S}}{1+\hat{S}}\right) = \left(\frac{1-\dfrac{sa-x}{s^2-sg+p}}{1+\dfrac{sa-x}{s^2-sg+p}}\right)Y_o$$

# Realization – Complex Poles

The admittance expression can be re-arranged as

$$\hat{Y} = \left( \frac{s^2 - sg + p - sa + x}{s^2 - sg + p + sa - x} \right) Y_o = \left( \frac{s^2 - s(g+a) + p + x}{s^2 - s(g-a) + p - x} \right) Y_o$$

**WE HAD DEFINED**

$p = p_1 p_2$ product of poles

$g = p_1 + p_2$ sum of poles

$a = r_1 + r_2$ sum of residues

$x = r_1 p_2 + r_2 p_1$ cross product

# Realization – Complex Poles

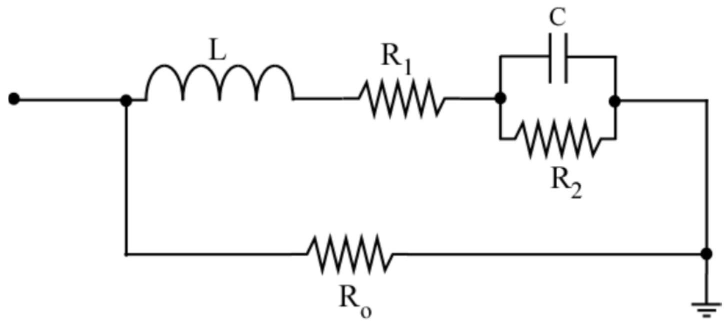
## Matching the terms with like coefficients gives

$$R_o = \frac{1}{Y_o}$$

$$p + x = \frac{R_o + R_1 + R_2}{LCR_2} \qquad p - x = \frac{R_1 + R_2}{LCR_2}$$

$$2p = \frac{R_o + 2R_1 + 2R_2}{LCR_2} \qquad 2x = \frac{R_o}{LCR_2}$$

# Realization from S-Parameters




## Solving gives

$$L = -\frac{R_o}{2a}$$

$$R_2 = \frac{R_o\left(\frac{p}{x}-1\right)}{2} - R_1$$

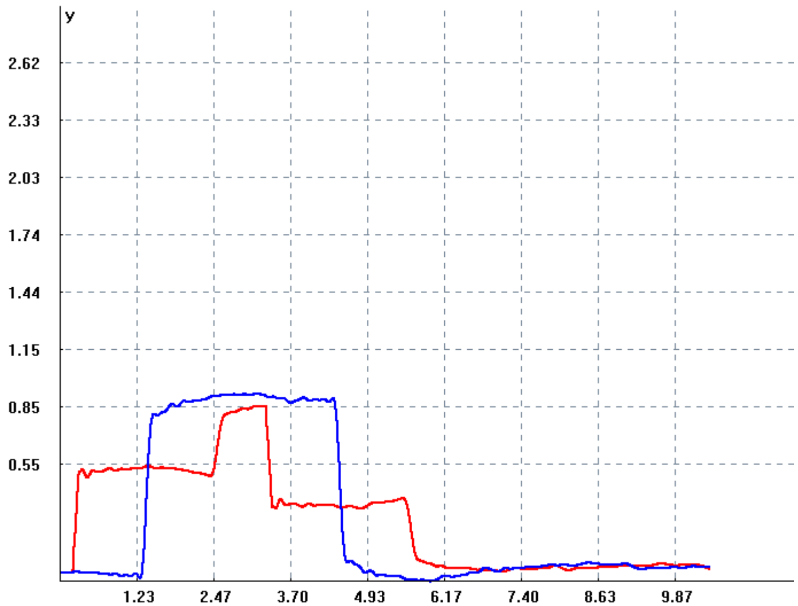
$$R_1 = \frac{1}{2}\left(\frac{gR_o}{a} + \frac{2Lx}{a} - R_o\right)$$

$$C = \frac{-a}{R_2 x}$$

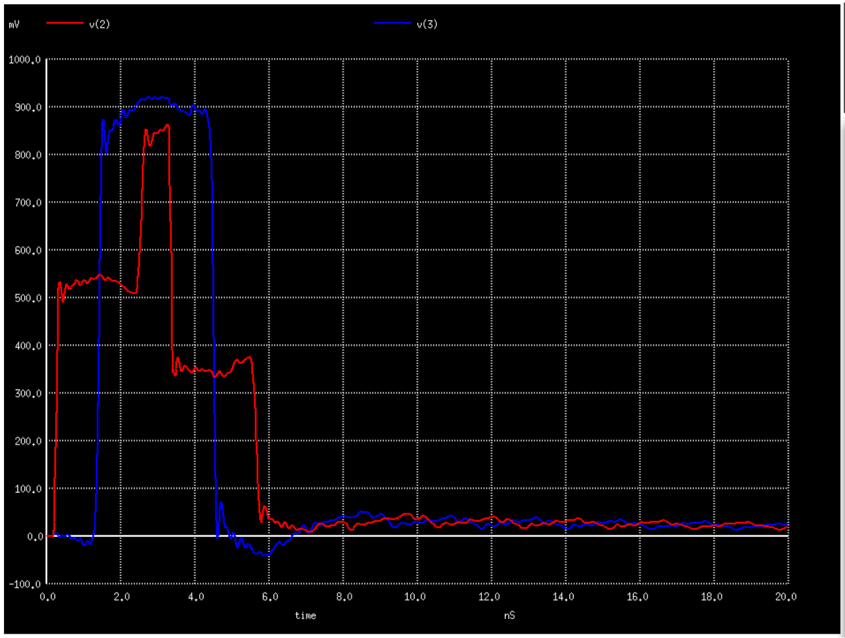
# Typical SPICE Netlist

```
* 32 -pole approximation
*This subcircuit has  16 pairs of complex poles and   0 real poles
.subckt sample   8000   9000
vsens8001  8000  8001 0.0
vsens9001  9000  9001 0.0
*subcircuit for s[1][1]
*complex residue-pole pairs for k=  1 residue: -6.4662e-002 8.1147e-002 pole: -4.4593e-001 -2.4048e+001
elc1  1 0 8001 0 1.0
hc2  2  1 vsens8001 50.0
rtersc3  2  3 50.0
vp4  3  4 0.0
l1cd5  4  5 1.933e-007
rocd5  4  0 5.000e+001
r1cd6  5  6 5.895e+003
c1cd6  6  0 3.474e-015
r2cd6  6  0 -9.682e+003
:
*constant term 2 2 -6.192e-003
edee397 397 0 9001 0 1.0e+000
hdee398 398 397 vsens9001 50.0
rterdee399 398 399 50.0
vp400 399 400 0.0
rdee400 400 0 49.4
*current sources
fs4    0 8001 vp4 -1.0
gs4    0 8001 4 0 0.020
fs10    0 8001 vp10 -1.0
gs10    0 8001 10 0 0.020
fs16    0 8001 vp16 -1.0
gs16    0 8001 16 0 0.020
fs22    0 8001 vp22 -1.0
gs22    0 8001 22 0 0.020
fs28    0 8001 vp28 -1.0
gs28    0 8001 28 0 0.020
```

# Realization from Y-Parameters

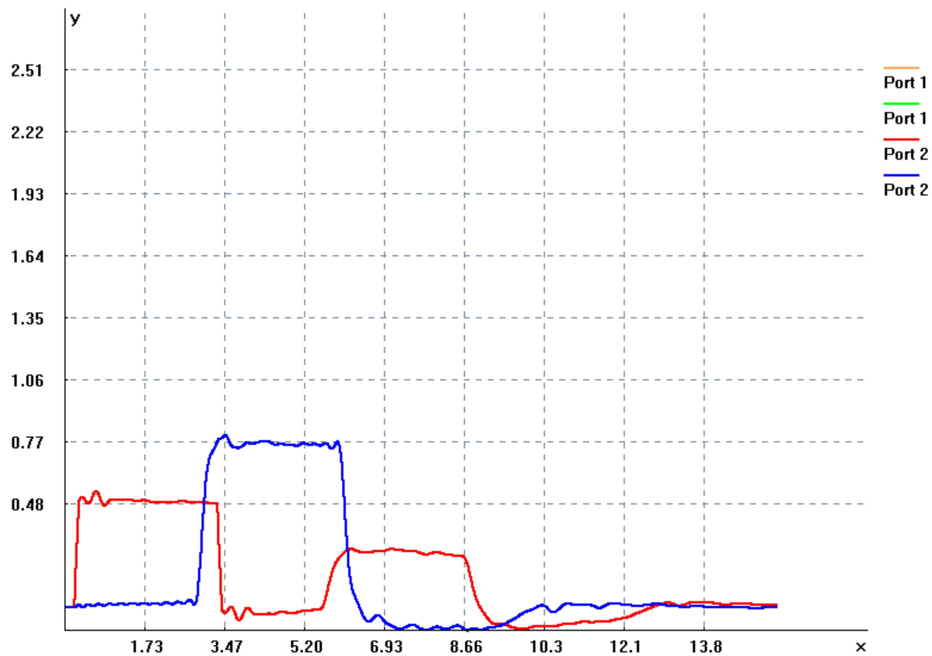



Recursive convolution

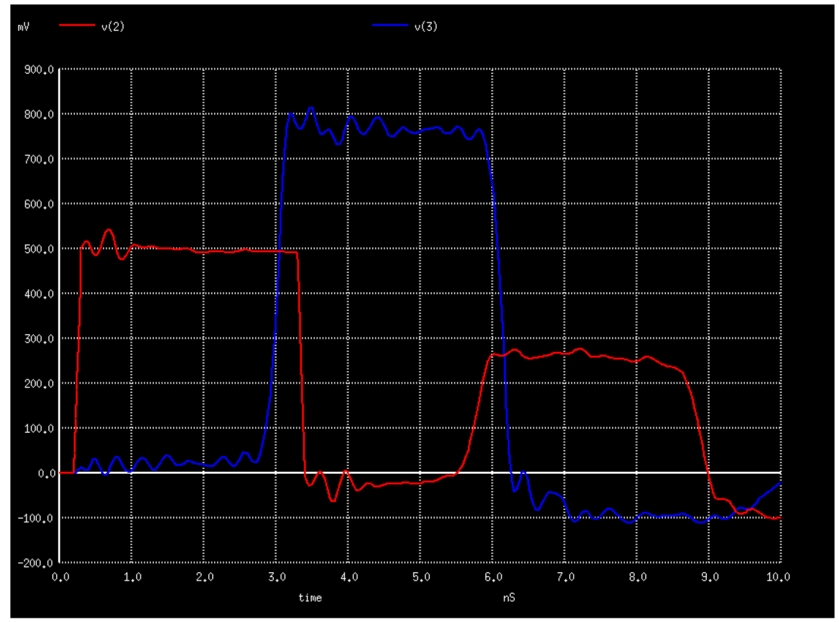



SPICE realization

# Realization from S-Parameters




Recursive convolution




SPICE realization