ECE 453
Wireless Communication Systems

Amplifiers

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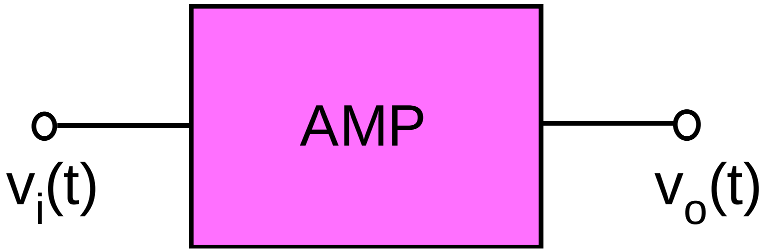
Amplifiers

- **Definitions**
  - Used to increase the amplitude of an input signal to a desired level
  - This is a fundamental signal processing function
  - Must be linear (free of distortion) – Shape of signal preserved



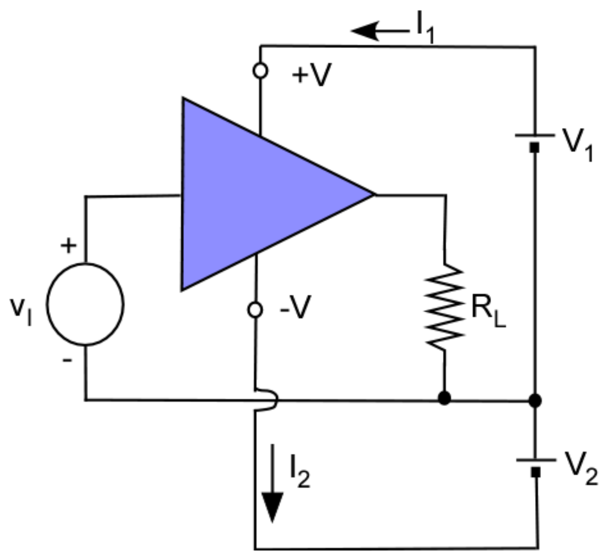
$$v_o(t) = Av_i(t),\ \ where\ A\ is\ the\ voltage\ gain$$

$$Voltage\ Gain: A_v = \frac{v_o}{v_i}$$

$$Power\ \ Gain: A_p = \frac{Load\ \ Power\ (P_L)}{Input\ \ Power\ (P_I)}$$

Amplifiers

Since output associated with the signal is larger than the input signal, power must come from DC supply



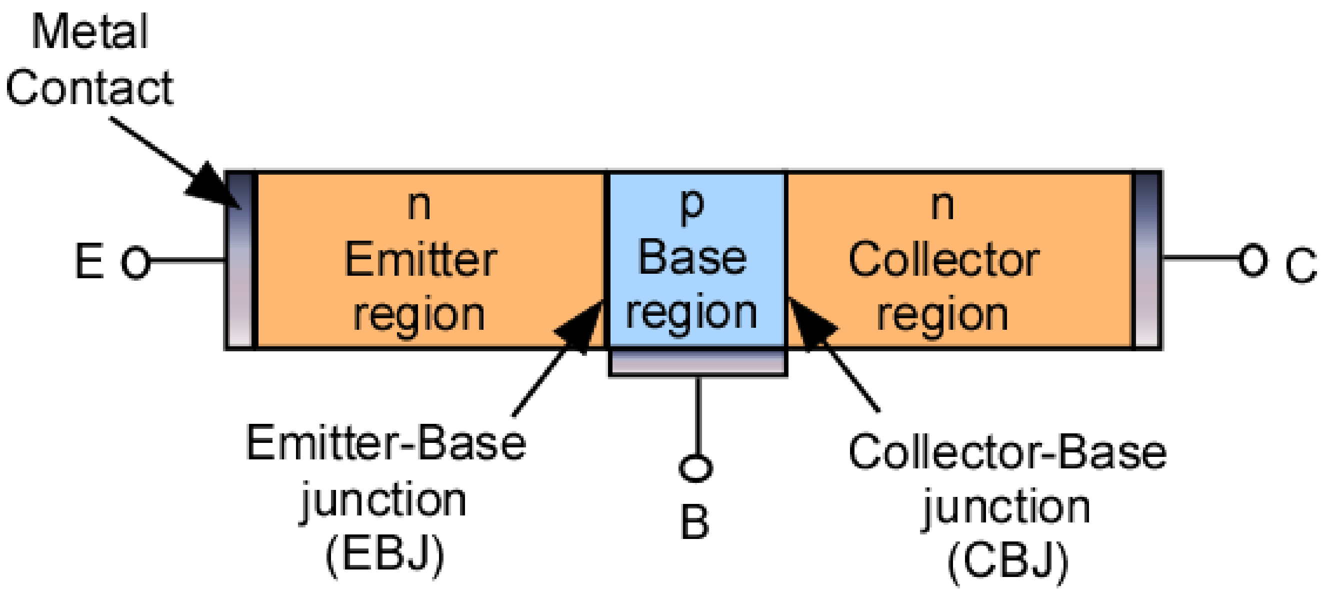
$$P_{DC} = V_1 I_1 + V_2 I_2$$

$$P_{DC} + P_I = P_L + P_{dissipated}$$

$$\eta = \frac{P_L}{P_{DC}} \times 100 = Power\ Efficiency$$

Bipolar Junction Transistor

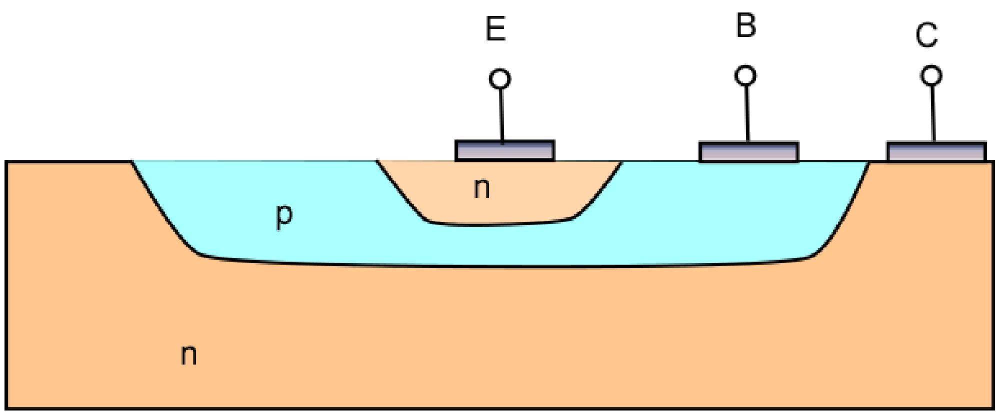
- **Bipolar Junction Transistor (BJT)**
  - First Introduced in 1948 (Bell labs)
  - Consists of 2 pn junctions
  - Has three terminals: emitter, base, collector



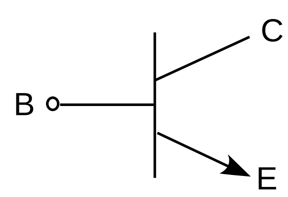
BJT – Modes of Operation

Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Forw. Active	Forward	Reverse
Rev. Active	Reverse	Forward
Saturation	Forward	Forward

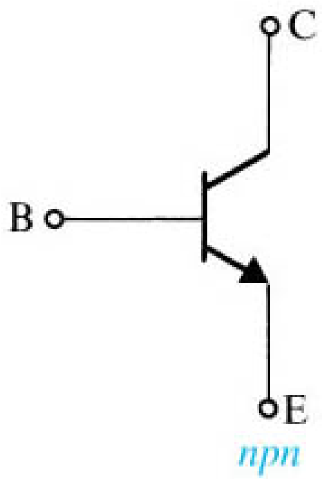
Structure of BJT's



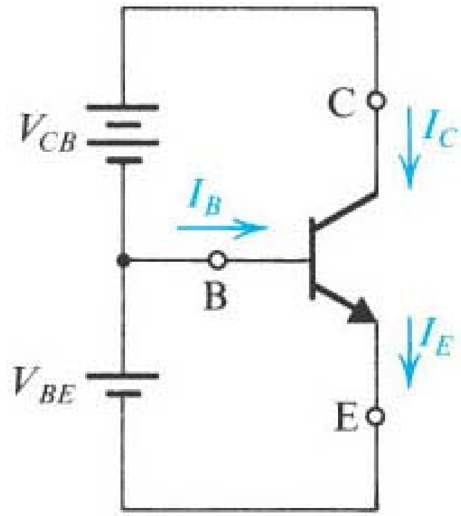
Collector surrounds emitter region ➔ electrons will be collected

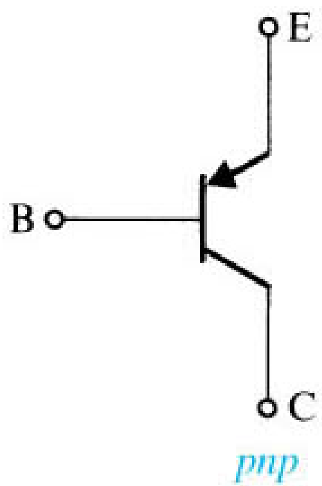


BJT Transistor Polarities

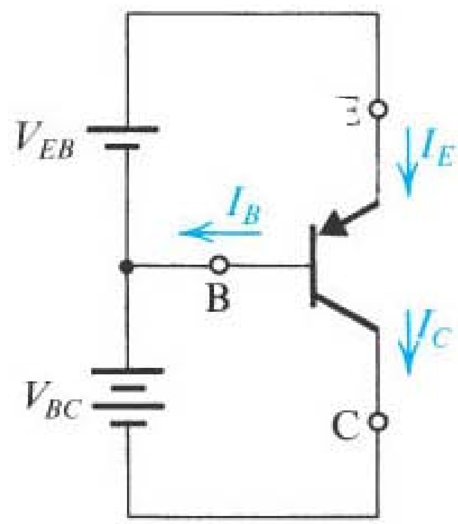


NPN



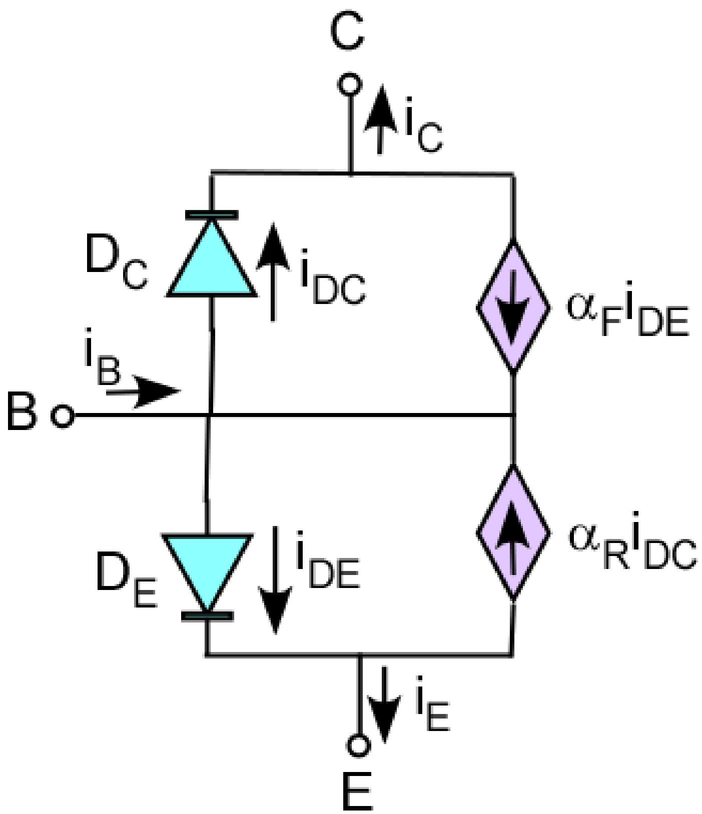


PNP



Ebers-Moll Model

NPN Transistor



$$i_E = \left(\frac{I_S}{\alpha_F}\right)\left(e^{v_{BE}/V_T}-1\right) - I_S\left(e^{v_{BC}/V_T}-1\right)$$

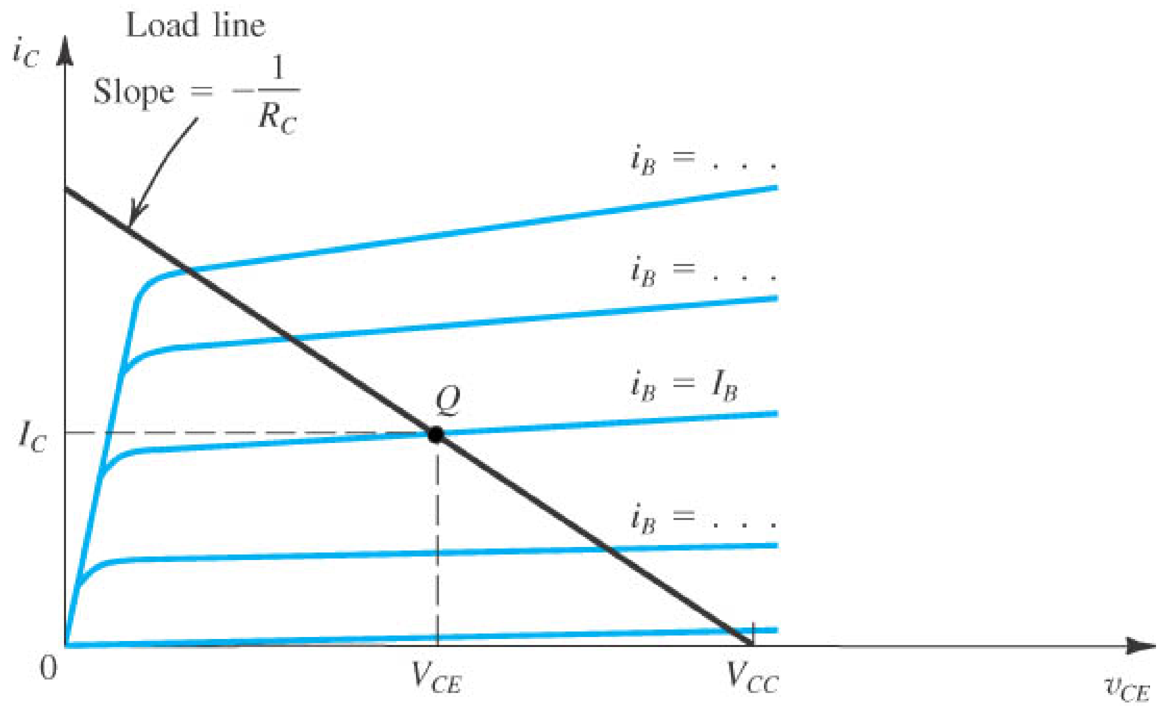
$$i_C = I_S\left(e^{v_{BE}/V_T}-1\right) - \left(\frac{I_S}{\alpha_R}\right)\left(e^{v_{BC}/V_T}-1\right)$$

$$i_B = \left(\frac{I_S}{\beta_F}\right)\left(e^{v_{BE}/V_T}-1\right) + \left(\frac{I_S}{\beta_R}\right)\left(e^{v_{BC}/V_T}-1\right)$$

$$\beta_F = \frac{\alpha_F}{1-\alpha_F} \qquad \beta_R = \frac{\alpha_R}{1-\alpha_R}$$

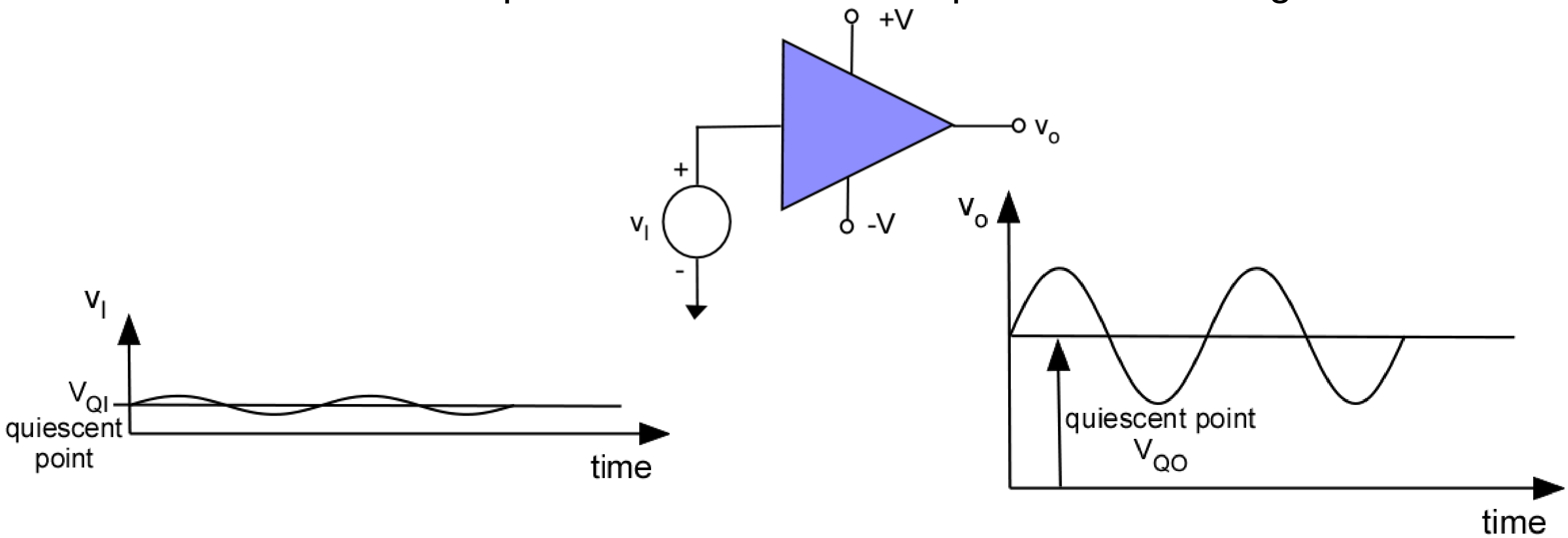
Describes BJT operation in <u>all</u> of its possible modes

Biasing Bipolar Transistors



Biasing of Amp

Bias will provide quiescent points for input and output about which variations will take place. Bias maintain amplifier in active region.



$$V_I(t) = V_{QI} + v_I(t)$$

$$V_o(t) = V_{QO} + v_o(t)$$

$$v_o(t) = A_v v_I(t)$$

$$A_v = \left.\frac{dv_o}{dv_I}\right|_{at\ Q}$$

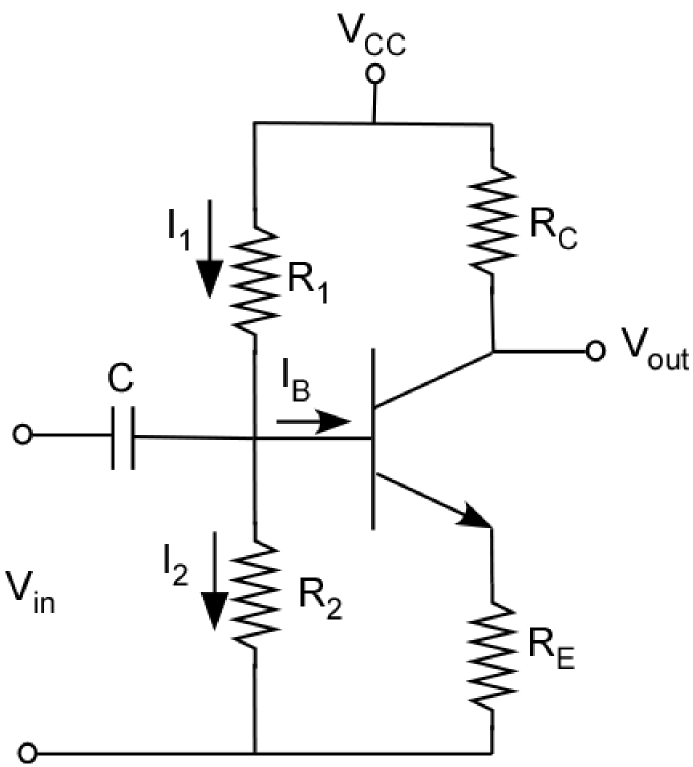
Amplifier characteristics are determined by bias point

Small-Signal Model

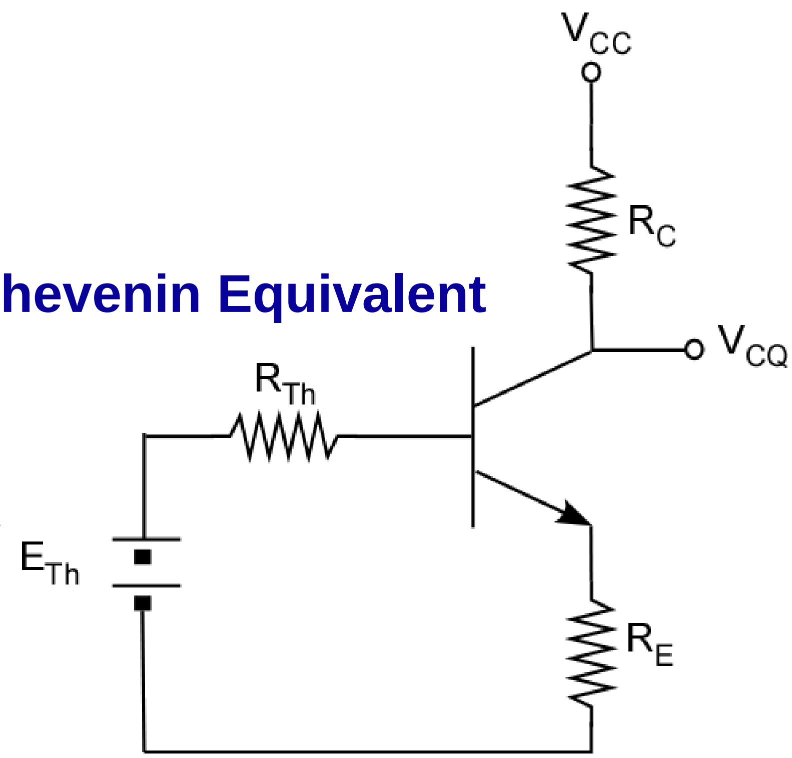
- **What is a small-signal incremental model?**
  - Equivalent circuit that only accounts for signal level fluctuations about the DC bias operating points
  - Fluctuations are assumed to be small enough so as not to drive the devices out of the proper range of operation
  - Assumed to be linear
  - Derives from superposition principle

BJT Bias

2. Emitter Bias

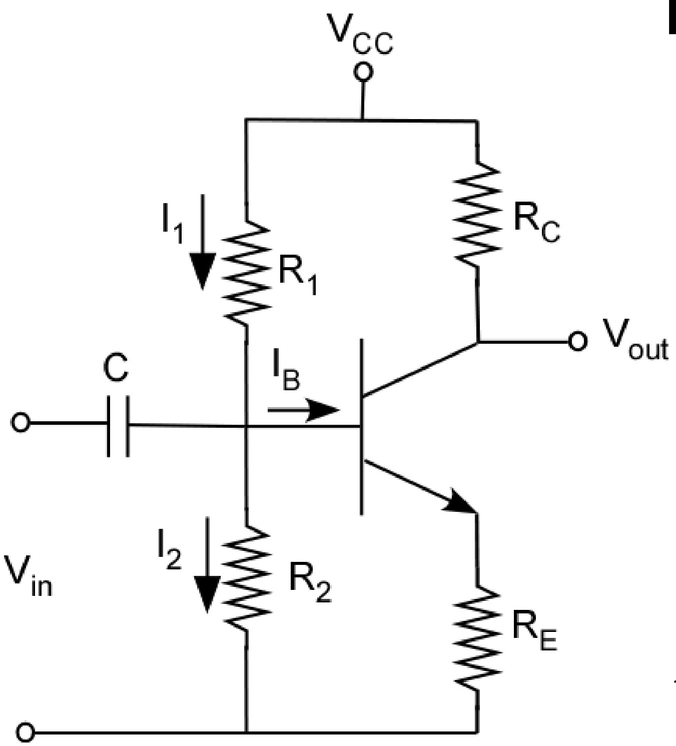


Thevenin Equivalent



Provides good stability with respect to changes in β with temperature

BJT Emitter Bias



$$E_{th} = R_{th} I_B + V_{BE} + R_E I_E$$

$$I_E = I_B + I_C = (\beta + 1) I_B$$

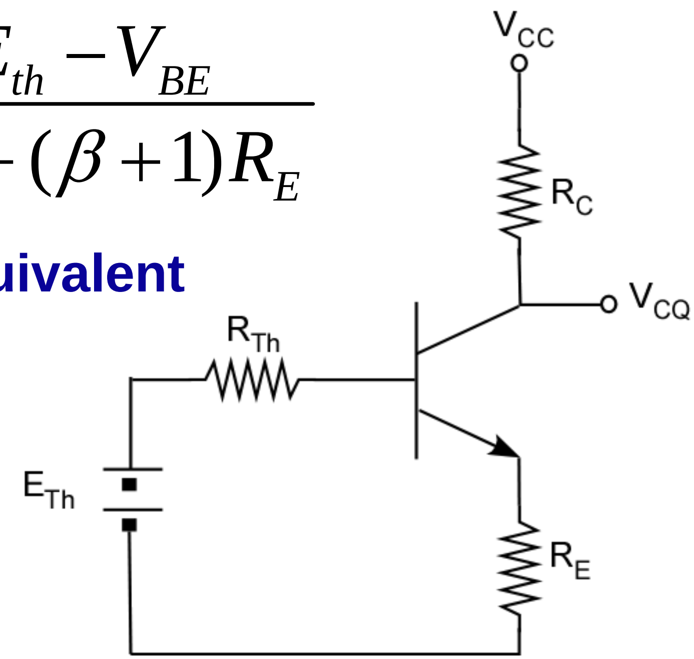
$$E_{th} - V_{BE} = R_{th} I_B + R_E (\beta + 1) I_B$$

$$I_B = I_{BQ} = \frac{E_{th} - V_{BE}}{R_{th} + (\beta + 1) R_E}$$

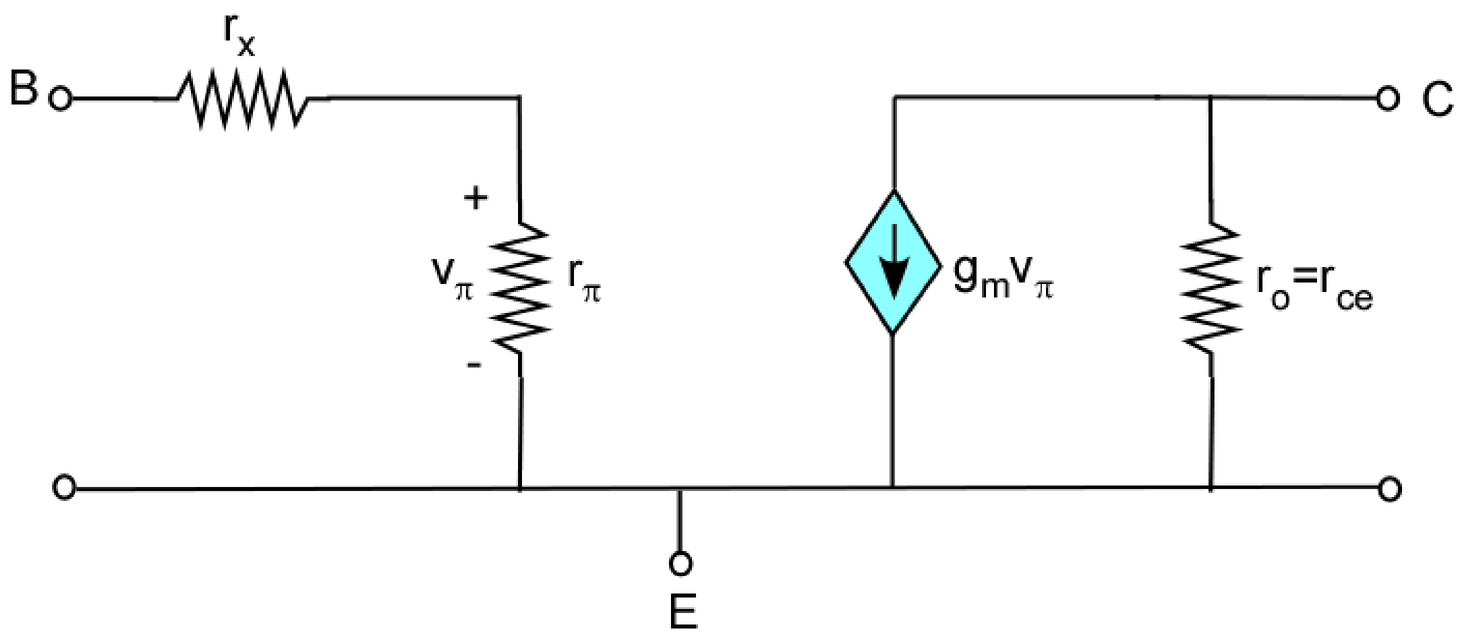
$$E_{th} = \frac{R_2}{R_1 + R_2} V_{CC}$$

$$R_{th} = R_1 \,||\, R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

Thevenin Equivalent



Hybrid-π Incremental Model for BJTs



r_π: input resistance looking into the base
r_x: parasitic series resistance looking into base – ohmic base resistance
g_m: BJT transconductance
r_o=r_{ce}: output collector resistance related to the Early effect

Hybrid-π Parameters

$$g_m = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{I_C = constant} = \frac{I_C}{V_T}$$

$$r_\pi \text{ is defined as} : r_\pi = \frac{v_\pi}{i_b}$$

$$\text{Since } i_b = \frac{g_m v_\pi}{\beta} \quad \text{then} \quad r_\pi = \frac{\beta}{g_m}$$

Can show that

$$r_\pi = (\beta + 1) r_e$$

$$g_m = \frac{\alpha}{r_e}$$

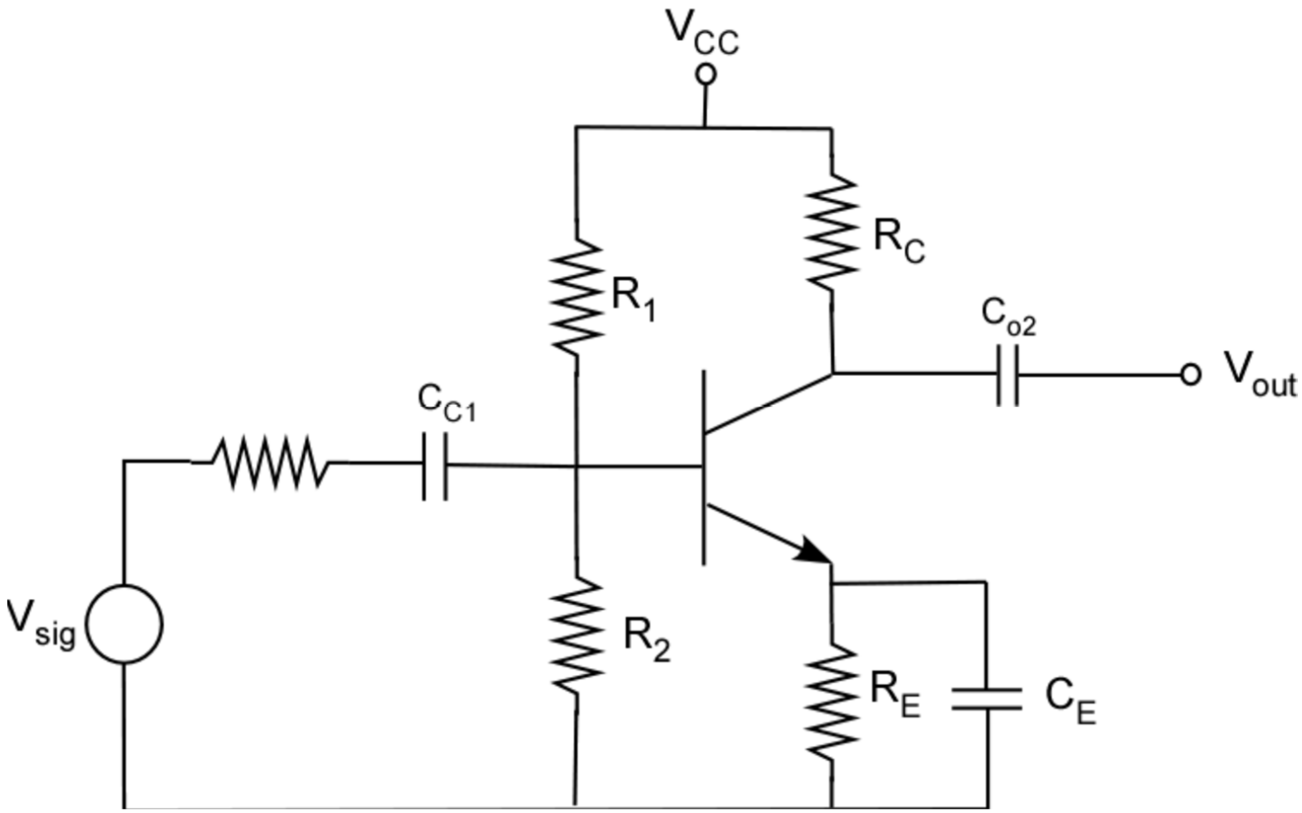
$$\beta = g_m r_\pi$$

$r_{ce} = r_o$ is associated with the Early effect

$$r_{ce} = r_o = \frac{|V_A|}{I_C} = \frac{|V_A|}{\beta I_B}$$

$$g_m + \frac{1}{r_\pi} = \frac{1}{r_e}$$

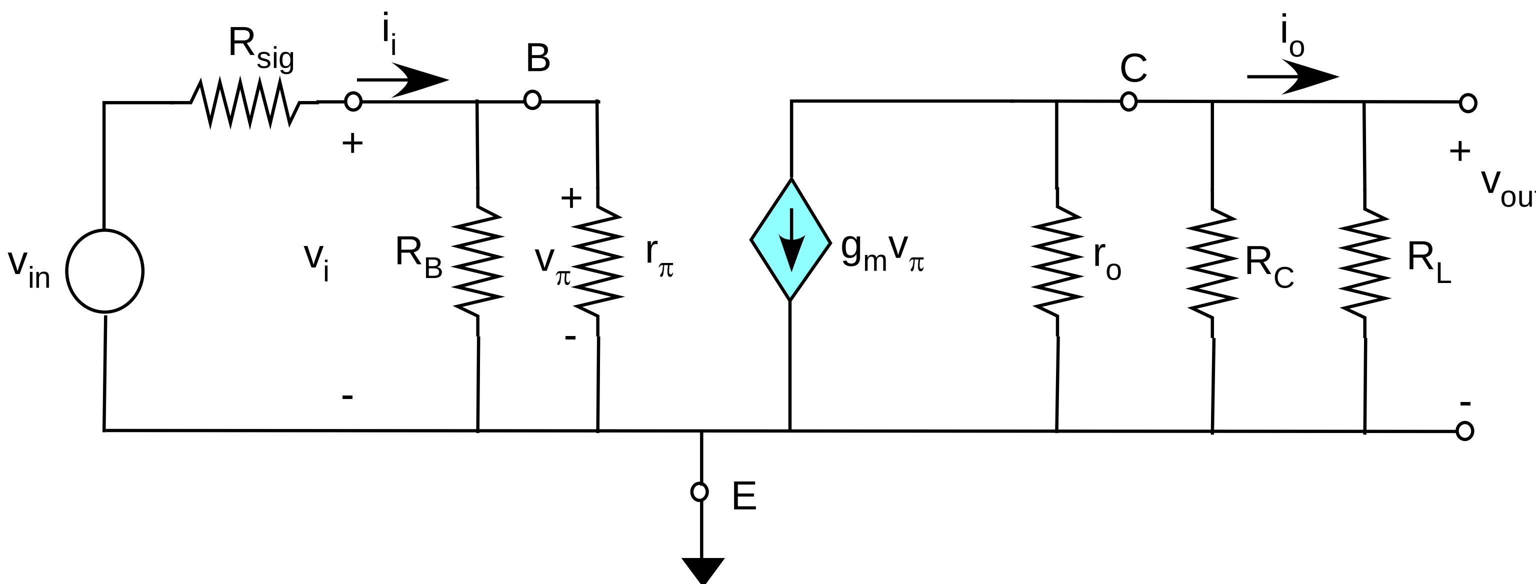
Common Emitter (CE) Amplifier



<u>Bias</u>: Choose *R_1 & R_2* to set *V_B*➔*V_E* is then set. Choose *R_E* to set *I_E~I_C*. Quiescent point of *V_{out}* will be determined by *R_C*. Emitter is an AC short.

Incremental Model for CE Amplifier

Hybrid-π model (ignoring r_x)



$$R_B = R_1 \,||\, R_2$$

$$R_{in} = \frac{v_i}{i_i} = R_B \,||\, r_\pi$$

$$Sometimes\ R_B \gg r_\pi\ and\ R_{in} \simeq r_\pi$$

CE Amplifier

$$v_i = \frac{v_{sig} R_{in}}{R_{in} + R_{sig}} = \frac{v_{sig} R_B \parallel r_\pi}{\left(R_B \parallel r_\pi\right) + R_{sig}}$$

$$and\ if\ R_B \gg r_\pi,\ v_i \simeq \frac{v_{sig} r_\pi}{r_\pi + R_{sig}}$$

$$v_o = -g_m v_{sig} \frac{R_B \parallel r_\pi \left(r_o \parallel R_C \parallel R_L\right)}{\left(R_B \parallel r_\pi\right) + R_{sig}}$$

$$v_o = -g_m v_\pi \left(r_o \parallel R_C \parallel R_L\right)$$

$$v_i = v_\pi$$

$$A_v = \frac{v_o}{v_i} = -g_m \left(r_o \parallel R_C \parallel R_L\right)$$

gain from base to collector

CE Amplifier

Open-circuit voltage gain:

$$A_{vo} = -g_m \left(r_o \parallel R_C \right)$$

In most cases $r_o \gg R_C \Rightarrow A_{vo} = -g_m R_C$

$$G_v = -\frac{\left(R_B \parallel r_\pi \right)}{\left(R_B \parallel r_\pi \right) + R_{sig}} g_m \left(r_o \parallel R_C \parallel R_L \right)$$

and for the case where $R_B \gg r_\pi$

$$G_v = -\frac{\beta \left(r_o \parallel R_C \parallel R_L \right)}{r_\pi + R_{sig}}$$

CE Amplifier

Output Impedance

$$R_{out} = R_C \,||\, r_o$$

$$If\;\; r_o \gg R_C,\;\; R_{out} \simeq R_C$$

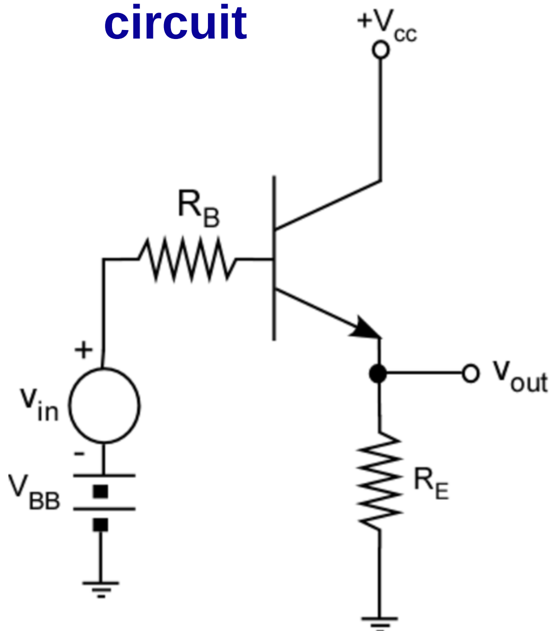
$$from\ which\ A_v = A_{vo}\left(\frac{R_L}{R_L + R_o}\right)$$

It can be seen that if $R_{sig} >> r_\pi$, the gain will be highly dependent on β. This is not good because of β variations

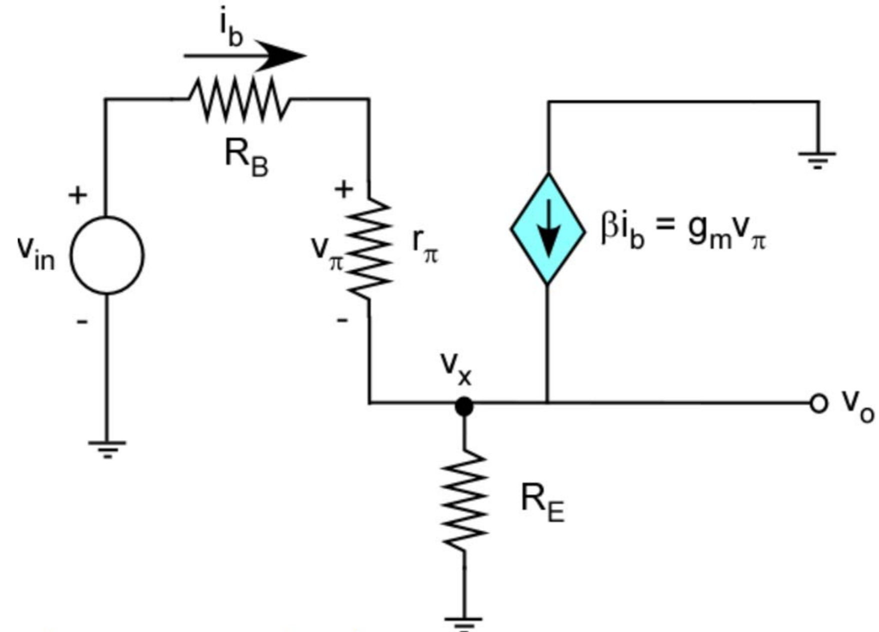
$$If\;\; R_{sig} \ll r_\pi,\quad G_v \simeq -g_m\left(R_C \,||\, R_L \,||\, r_o\right)$$

Emitter Follower (Common Collector)

circuit



Incremental model



$$v_o = \left(g_m v_\pi + \frac{v_\pi}{r_\pi} \right) R_E = v_\pi \left(g_m + \frac{1}{r_\pi} \right) R_E$$

$$v_{in} = v_\pi + R_B i_b + v_o = v_\pi + v_\pi R_E \left(g_m + \frac{1}{r_\pi} \right) + \frac{v_\pi}{r_\pi} R_B$$

Emitter Follower

$$v_{in} = v_\pi \left[1 + \frac{R_B}{r_\pi} + R_E \left(g_m + \frac{1}{r_\pi} \right) \right]$$

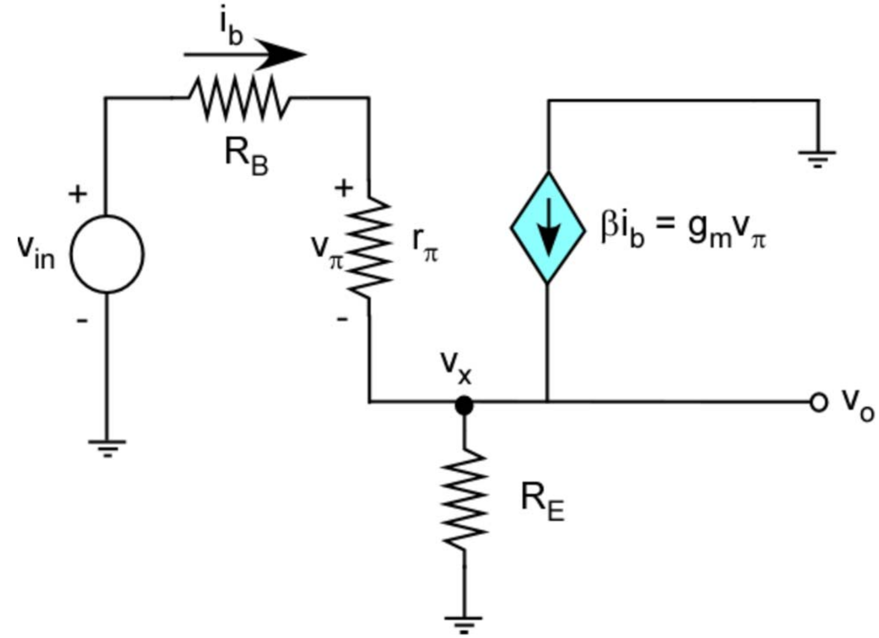
$$\frac{v_o}{v_{in}} = \frac{\left(g_m + \frac{1}{r_\pi} \right) R_E}{\left(g_m + \frac{1}{r_\pi} \right) R_E + 1 + \frac{R_B}{r_\pi}} = \frac{\left(g_m r_\pi + 1 \right) R_E}{\left(g_m r_\pi + 1 \right) R_E + r_\pi + R_B}$$

Using $g_m r_\pi = \beta$

$$\frac{v_o}{v_{in}} = \frac{\left(\beta + 1 \right) R_E}{\left(\beta + 1 \right) R_E + r_\pi + R_B} \simeq 1$$

Emitter follower has unity voltage gain

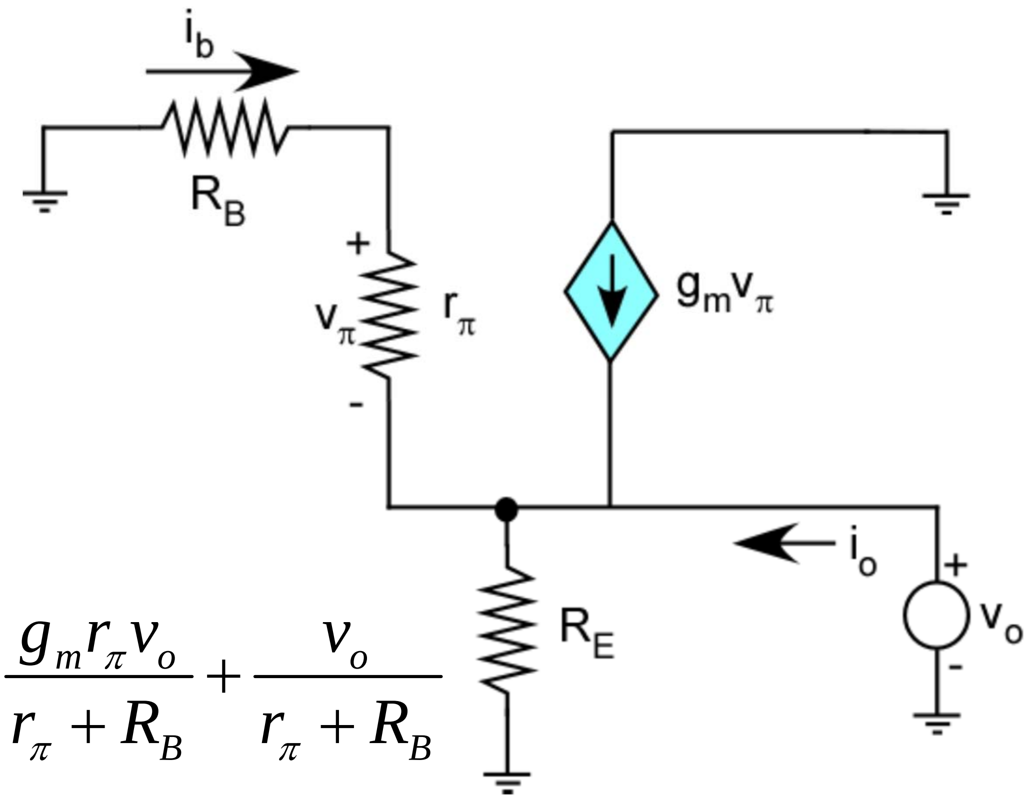
Emitter Follower – Input Impedance



$$r_{in} = \frac{v_{in}}{i_b} = \frac{v_\pi \left[1 + R_B / r_\pi + R_E \left(g_m + 1 / r_\pi \right) \right]}{v_\pi / r_\pi}$$

$$r_{in} = r_\pi + R_B + R_E \left(\beta + 1 \right)$$

Emitter Follower – Output Impedance



$$i_B = -\frac{v_o}{r_\pi + R_B}$$

$$i_o = \frac{v_o}{R_E} - g_m v_\pi + \frac{v_o}{r_\pi + R_B} = \frac{v_o}{R_E} + \frac{g_m r_\pi v_o}{r_\pi + R_B} + \frac{v_o}{r_\pi + R_B}$$

$$i_o = v_o\left[\frac{1}{R_E} + \frac{g_m}{r_\pi + R_B} + \frac{1}{r_\pi + R_B}\right] = v_o\left[r_\pi + R_B + R_E(\beta+1)\right]\frac{1}{R_E\left(r_\pi + R_B\right)}$$

Output Impedance (cont')

Using $g_m r_\pi = \beta$

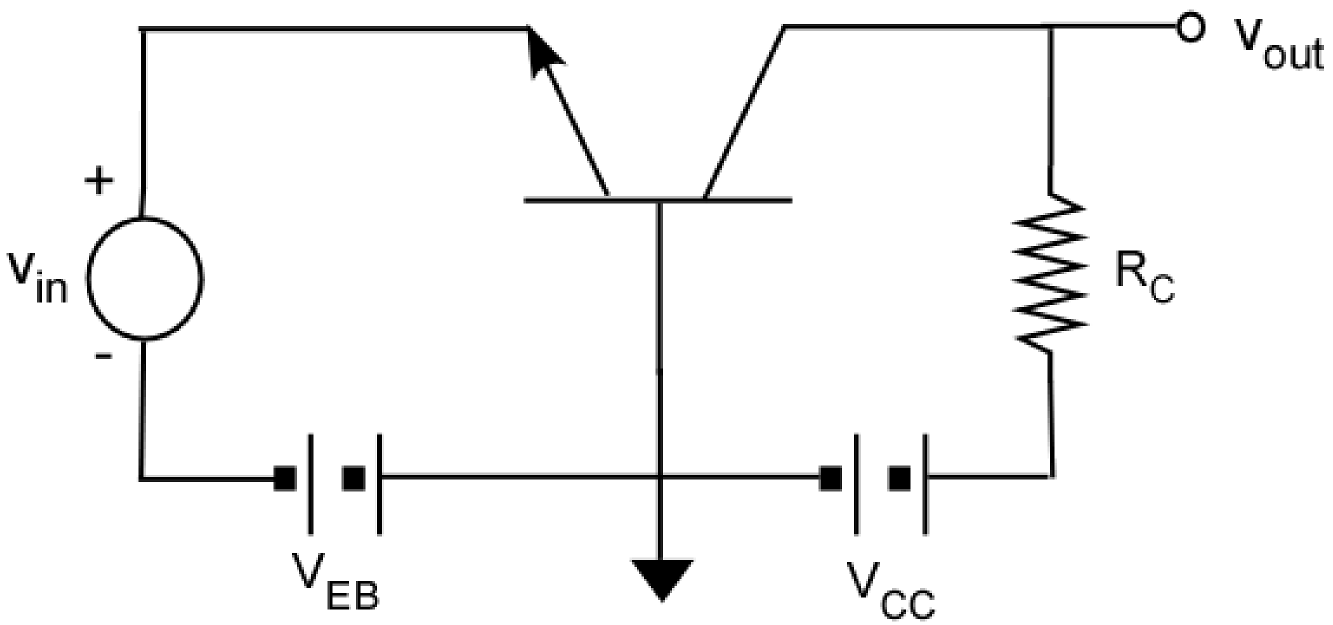
$$\frac{v_o}{i_o} = R_{out} = \frac{R_E\left(r_\pi + R_B\right)}{r_\pi + R_B + R_E(\beta+1)} = \frac{R_E\left(r_\pi + R_B\right)/(\beta+1)}{R_E + \left(r_\pi + R_B\right)/(\beta+1)}$$

$$R_{out} = R_E \,||\left(r_\pi + R_B\right)/(\beta+1)$$

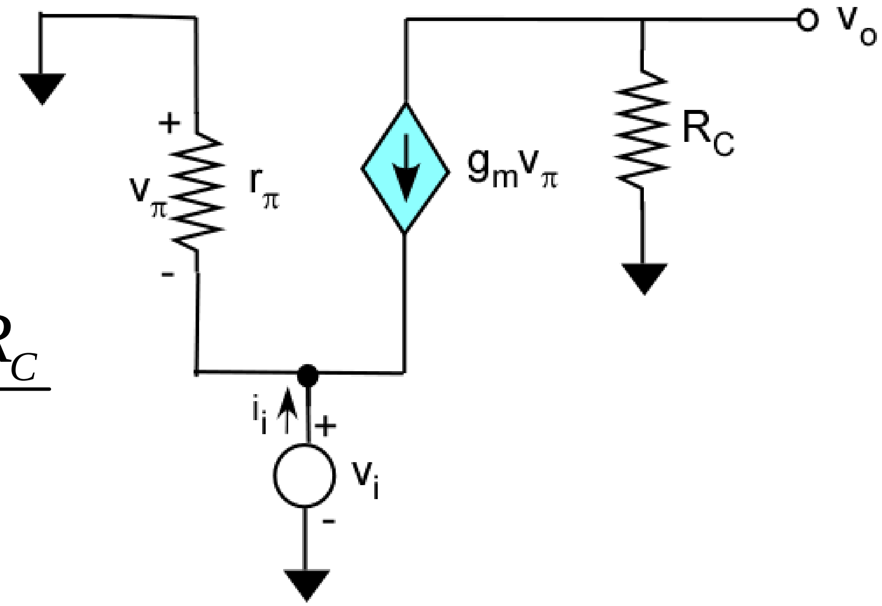
If we neglect R_B

$$A_{MB}^{'} = \frac{(\beta+1)R_E}{r_\pi + (\beta+1)R_E} \quad and \quad R_{out}^{'} = R_E \,||\, \frac{r_\pi}{\beta+1}$$

Common Base Configuration



Common Base Configuration



$$v_i = -v_\pi, \quad v_o = -g_m v_\pi R_C = g_m v_i R_C$$

$$Voltage \;\; gain = \frac{v_o}{v_i} = g_m R_C = \frac{\alpha R_C}{r_e}$$

$$Current \; gain = \frac{i_o}{i_i} = \frac{g_m v_\pi}{i_i} = \frac{-g_m v_\pi}{\left(g_m + \frac{1}{r_\pi}\right)(-v_\pi)} = \frac{\beta}{\beta+1} = \alpha$$

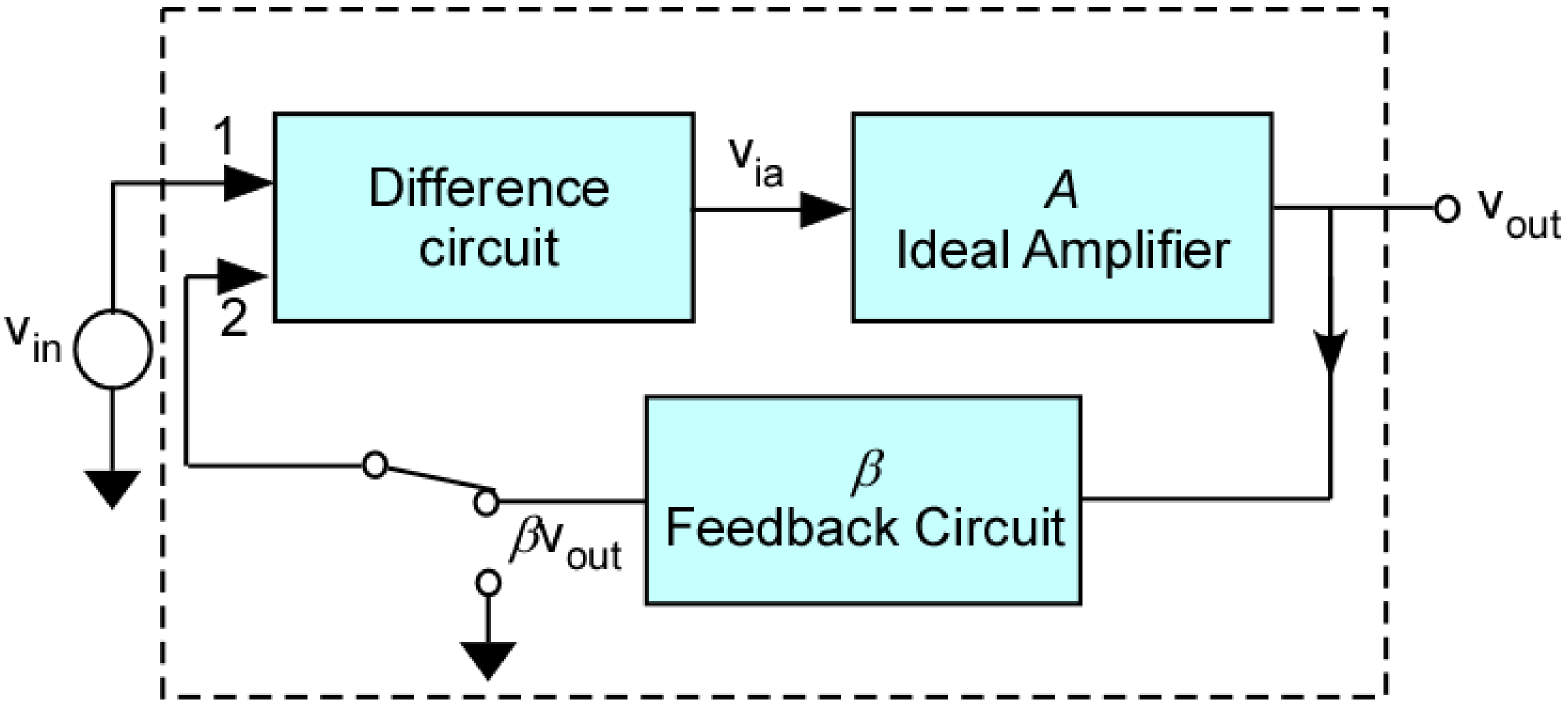
$$R_{out} = R_C$$

$$r_{in} = \frac{r_\pi}{\beta+1}$$

BJT Topologies - Summary

	CE	CB	EF
A_{vo}	$-g_m R_C$	$g_m R_C$	1
R_{in}	r_π	$\dfrac{r_\pi}{\beta+1}$	$r_\pi + R_E\left(\beta+1\right)$
R_{out}	R_C	R_C	$R_E \parallel r_\pi/(\beta+1)$

Feedback – Basic Concept



Feedback and Frequency Dependence

1. The closed-loop transfer function is a function of frequency

2. The manner in which the loop gain varies with frequency determines the stability or instability of the feedback amplifier

3. The frequency at which the phase of the transfer function is equal to 180º will be unstable if the magnitude is greater than unity

Feedback and Stability

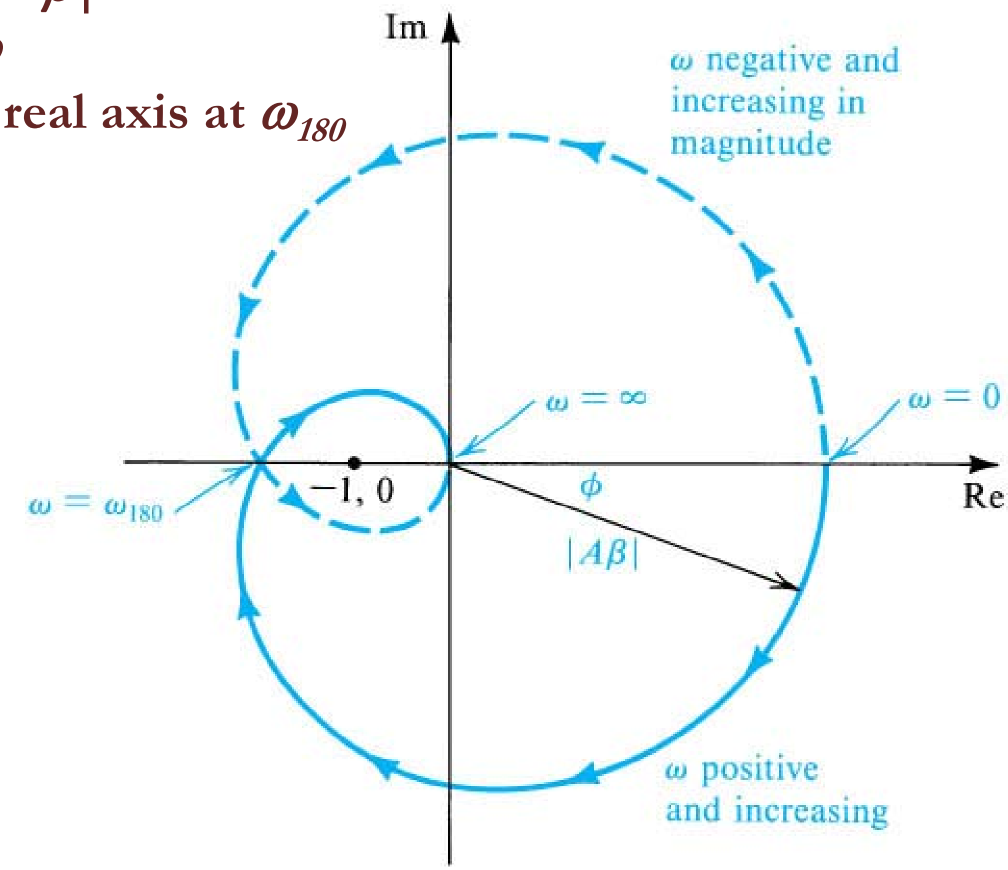
$$A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)}$$

$$A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)}$$

When loop gain $A(j\omega)\beta(j\omega)$ has 180º phase, we have positive feedback

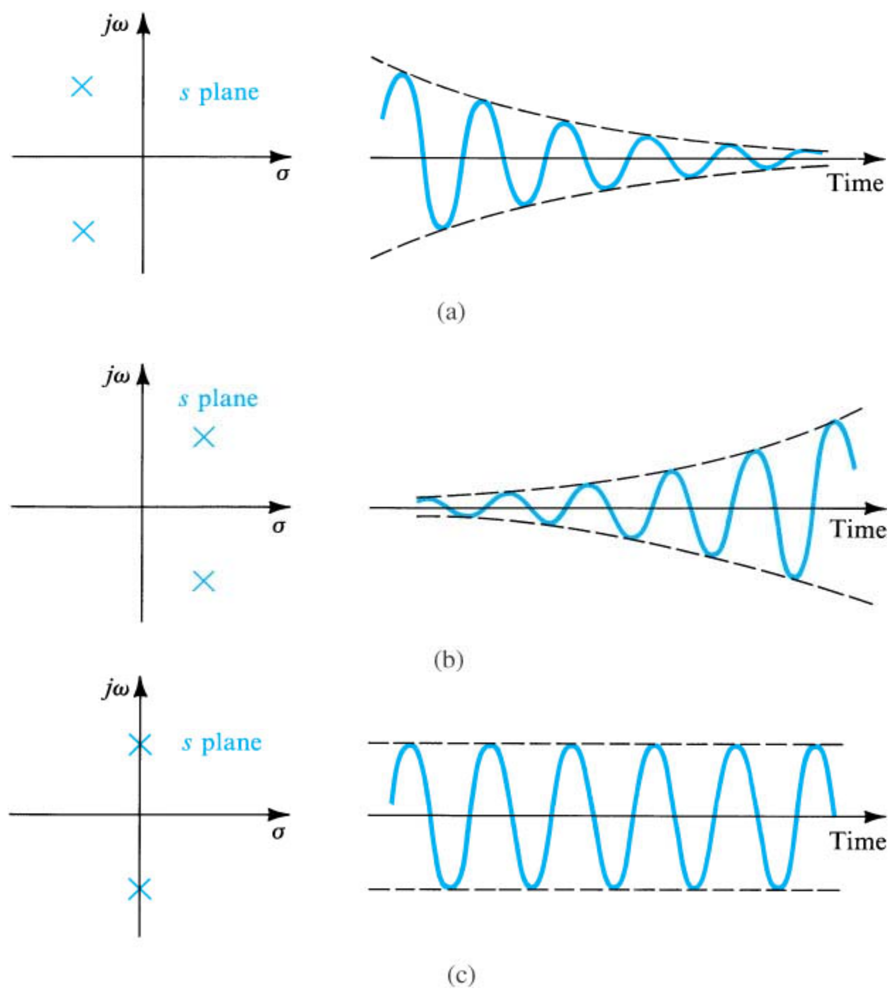
Nyquist Plot

- Radial distance is $|A\beta|$
- Angle is phase of ϕ
- Intersects negative real axis at ω_{180}

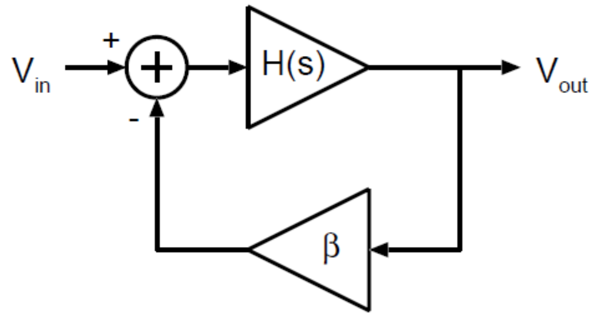


Stability and Pole Location

$$v(t) = e^{\sigma_o t}\left[e^{+j\omega t} + e^{-j\omega t}\right] = 2e^{\sigma_o t}\cos\left(\omega_n t\right)$$



Oscillator



- Closed-Loop Transfer function:

  – $\frac{V_{out}}{V_{in}}(s) = \frac{H(s)}{1+\beta H(s)}, where\ s = j\omega$

- Barkhausen's criteria for oscillation:

  – $|\beta H(j\omega_0)| = 1$

  – $\arg\left(\beta H(j\omega_0)\right) = -180^0.$

- ω_0 = oscillation-frequency.