## Quadrature FM Demodulator



- The demodulator operates with the RLC circuit tuned to resonance at the carrier frequency of $v_{i n}, \omega_{o}$.
- The voltage divider consisting of $C^{\prime}$ and the resonant RLC circuit converts small frequency deviations in $v_{i n}$ to phase shifts of $v^{\prime}$ relative to $v_{i n}$.
- For small frequency deviations, $\delta \omega, v^{\prime}$ is phase shifted by $\pi / 2-2 Q_{p} \frac{\delta \omega}{\omega_{o}}$.
- The multiplier and LPF act as a phase comparator, producing output proportional to $2 Q_{p} \frac{\delta \omega}{\omega_{o}}$ for small frequency deviations, $\delta \omega$.


## Quadrature FM Demodulator



Using upper-case symbols $V_{i n}, V^{\prime}$ for phasors:

$$
V^{\prime}=\frac{Z_{p}}{Z_{p}+\frac{1}{j \omega C^{\prime}}} V_{i n} ; \quad Z_{p}=j \omega L\left\|\frac{1}{j \omega C}\right\| R
$$

If $C^{\prime}$ is small, such that $\frac{1}{\omega C^{\prime}} \gg\left|Z_{p}\right|$, then $V^{\prime} \simeq j \omega C^{\prime} Z_{p} V_{i n}$.
Let $\theta^{\prime}, \theta_{p}$, and $\theta_{i n}$ represent the phase angles of $V^{\prime}, Z_{p}$, and $V_{i n}$, respectively. Then:

$$
\theta^{\prime}=\frac{\pi}{2}+\theta_{p}+\theta_{i n}
$$

Phase difference between $V_{i n}$ and $V^{\prime}$ is $\frac{\pi}{2}+\theta_{p}$ where $\theta_{p}$ is the phase angle of $Z_{p}$.

Impedance of a parallel RLC circuit is:

$$
\begin{gathered}
Z_{p}(\omega)=\frac{R}{1+j Q_{p}\left(\frac{\omega}{\omega_{o}}-\frac{\omega_{o}}{\omega}\right)}=\left|Z_{p}(\omega)\right| e^{j \theta_{p}(\omega)} \\
\theta_{p}(\omega)=-\tan ^{-1} Q_{p}\left(\frac{\omega}{\omega_{o}}-\frac{\omega_{o}}{\omega}\right)
\end{gathered}
$$

Let $\omega=\omega_{o}+\delta \omega$. If $\delta \omega \ll \omega_{o}$, then

$$
\theta_{p}(\omega) \simeq-2 Q_{p} \frac{\delta \omega}{\omega_{o}}
$$

For small frequency deviations away from resonance the phase of $Z_{p}$ is proportional to the frequency deviation, $\delta \omega$.


$$
\omega C^{\prime}\left|Z_{p}(\omega)\right| \cos \left(\omega_{o} t+\delta \omega t+\pi / 2-2 Q_{p} \frac{\delta \omega}{\omega_{o}}\right)
$$

The multiplier is usually configured such that the lower input is "saturated", in which case the output does not depend on the magnitude of the lower input.

The lowpass filter rejects the double-frequency term in the mixer output, leaving the cosine of the phase difference. The output is:

$$
v_{\text {out }} \sim \cos \left(\frac{\pi}{2}-2 Q_{p} \frac{\delta \omega}{\omega_{o}}\right)=\sin \left(2 Q_{p} \frac{\delta \omega}{\omega_{o}}\right)
$$

If $2 Q_{p} \frac{\delta \omega}{\omega_{o}} \ll 1$, then $v_{o u t} \sim 2 Q_{p} \frac{\delta \omega}{\omega_{o}}$.

## Summary:

- For small frequency deviations $(\delta \omega)$ away from $\omega_{o}, v_{\text {out }} \sim 2 Q_{p} \frac{\delta \omega}{\omega_{o}}$.
- For large frequency deviations, the output is a nonlinear function of deviation. See section 4.5 of the course notes for details.


Figure 4.23: Output voltage verses normalized frequency deviation for a quadrature demodulator employing a saturated input for $v^{\prime}$. The dotted line shows an ideal linear response with the same slope as the actual response at $x=0$.

