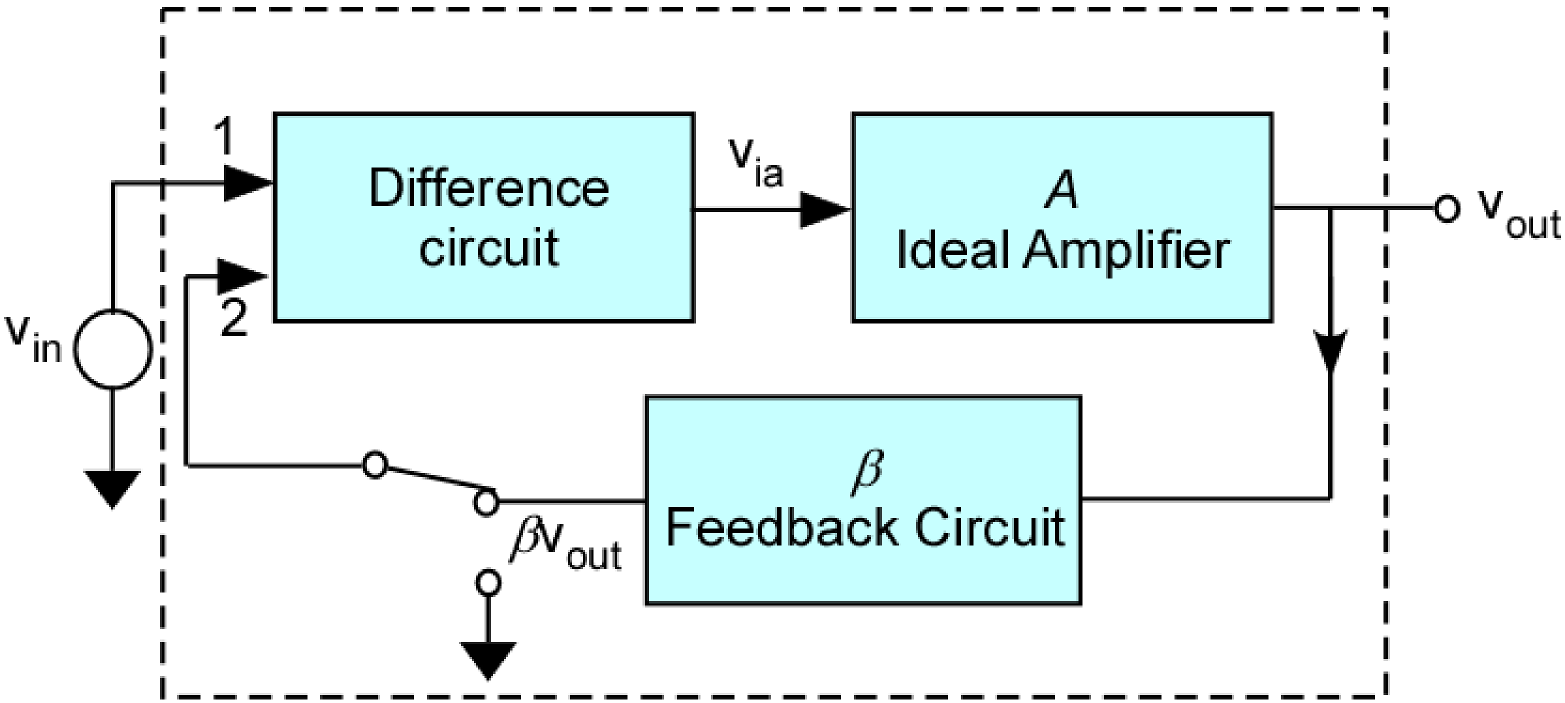
ECE 453
Wireless Communication Systems

Oscillators

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jesa@Illinois.edu

Feedback – Basic Concept



Feedback and Frequency Dependence

1. The closed-loop transfer function is a function of frequency

2. The manner in which the loop gain varies with frequency determines the stability or instability of the feedback amplifier

3. The frequency at which the phase of the transfer function is equal to 180^o will be unstable if the magnitude is greater than unity

Feedback and Stability

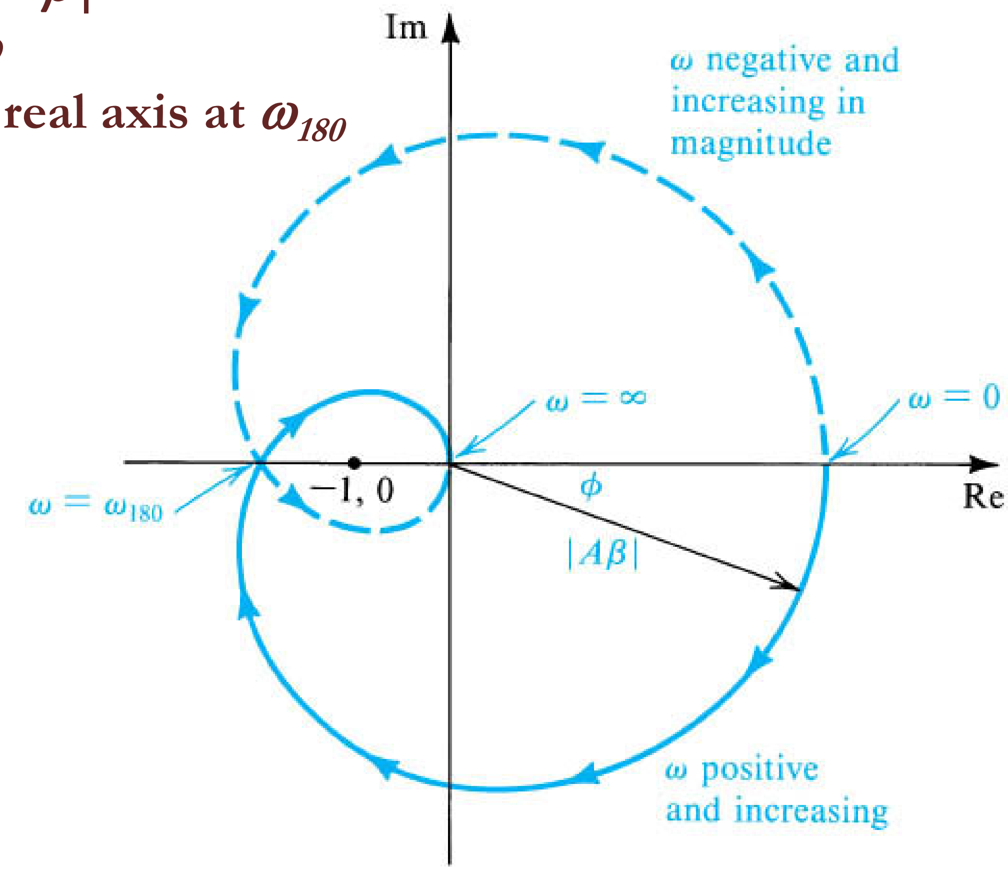
$$A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)}$$

$$A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)}$$

When loop gain $A(j\omega)\beta(j\omega)$ has 180° phase, we have positive feedback

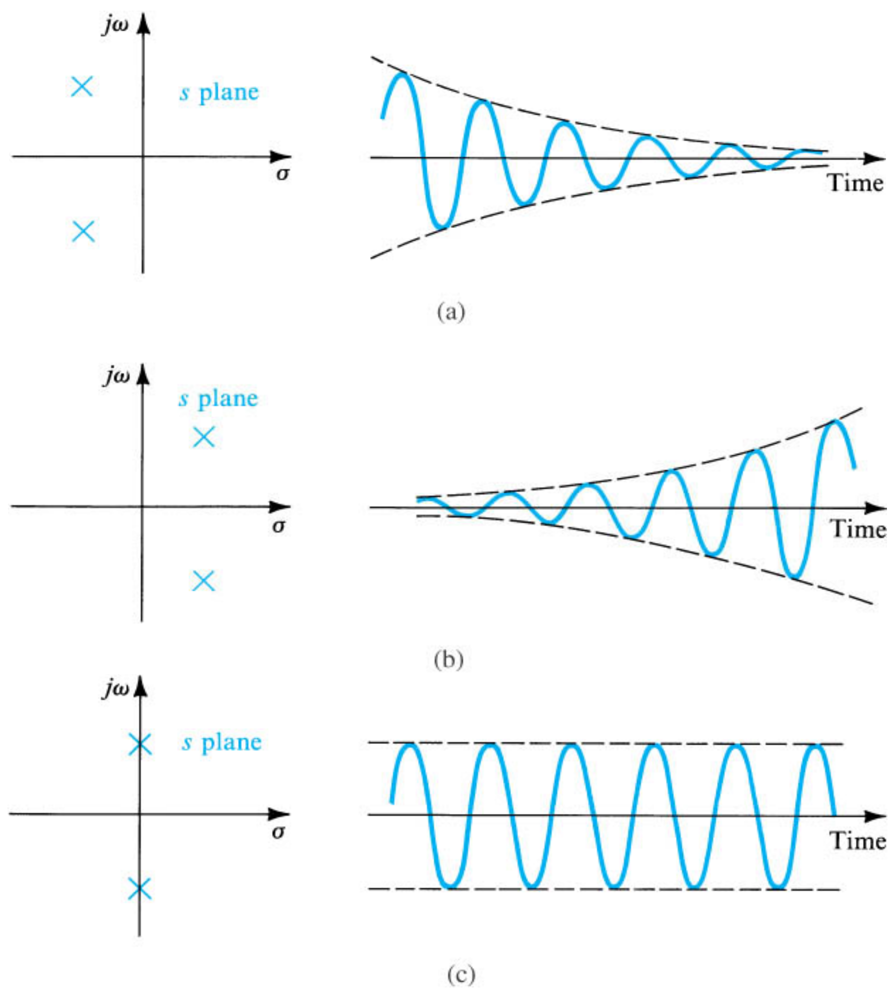
Nyquist Plot

- Radial distance is $|A\beta|$
- Angle is phase of ϕ
- Intersects negative real axis at ω_{180}

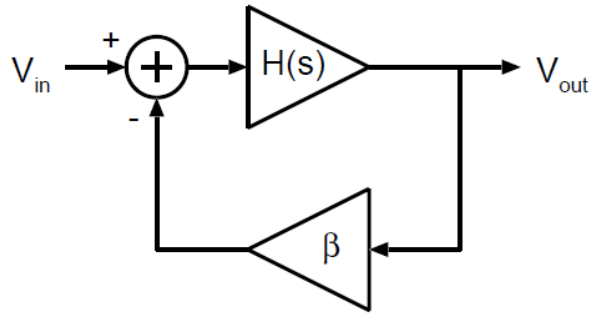


Stability and Pole Location

$$v(t) = e^{\sigma_o t}\left[e^{+j\omega t} + e^{-j\omega t}\right] = 2e^{\sigma_o t}\cos\left(\omega_n t\right)$$



Oscillator



- Closed-Loop Transfer function:

$$-\frac{V_{out}}{V_{in}}(s) = \frac{H(s)}{1+\beta H(s)}, where\ s = j\omega$$

- Barkhausen's criteria for oscillation:

  ➔ $|\beta H(j\omega_0)| = 1$

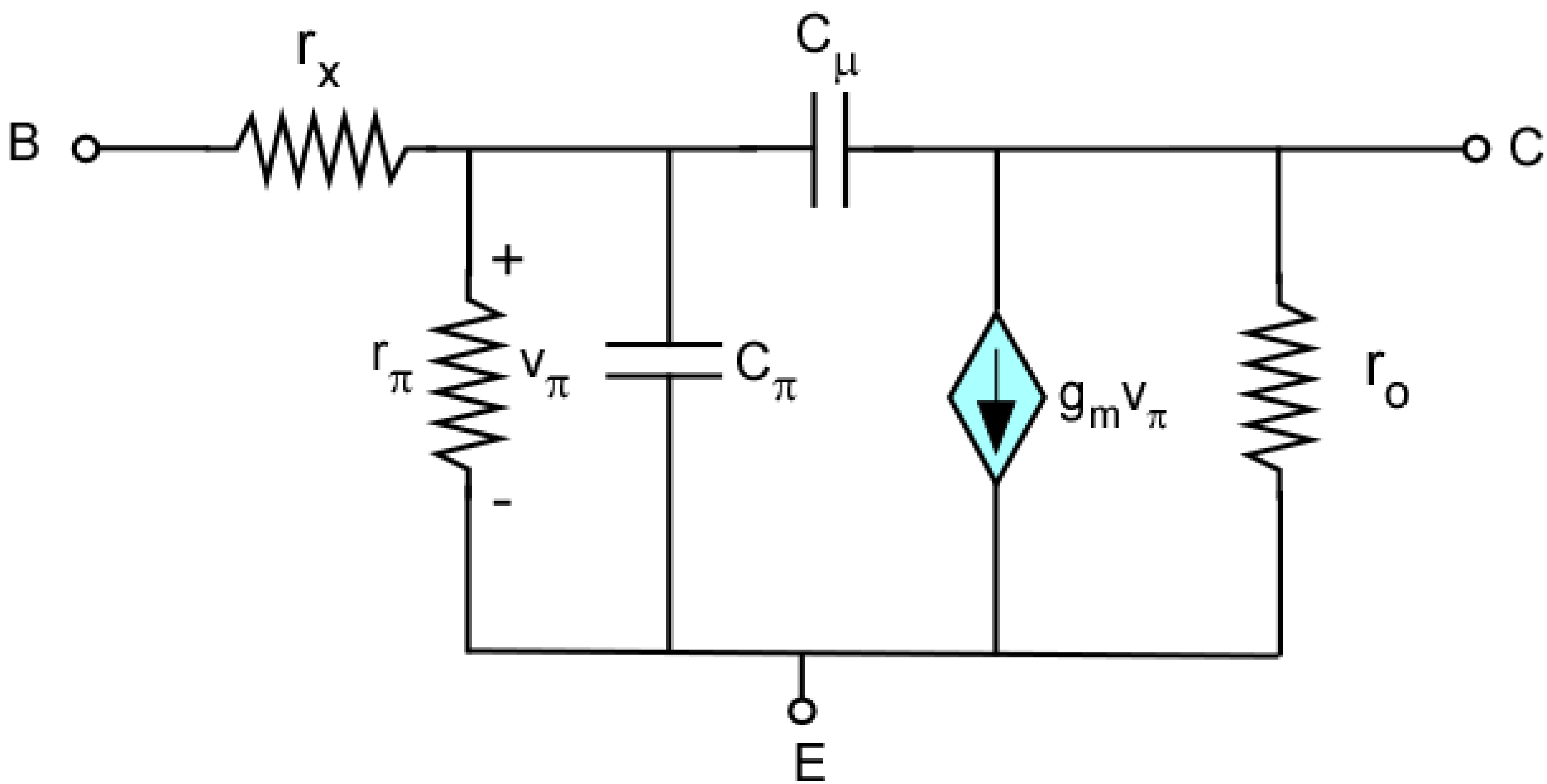
  ➔ $\arg\left(\beta H(j\omega_0)\right) = -180^0$.

- ω_0 = oscillation-frequency.

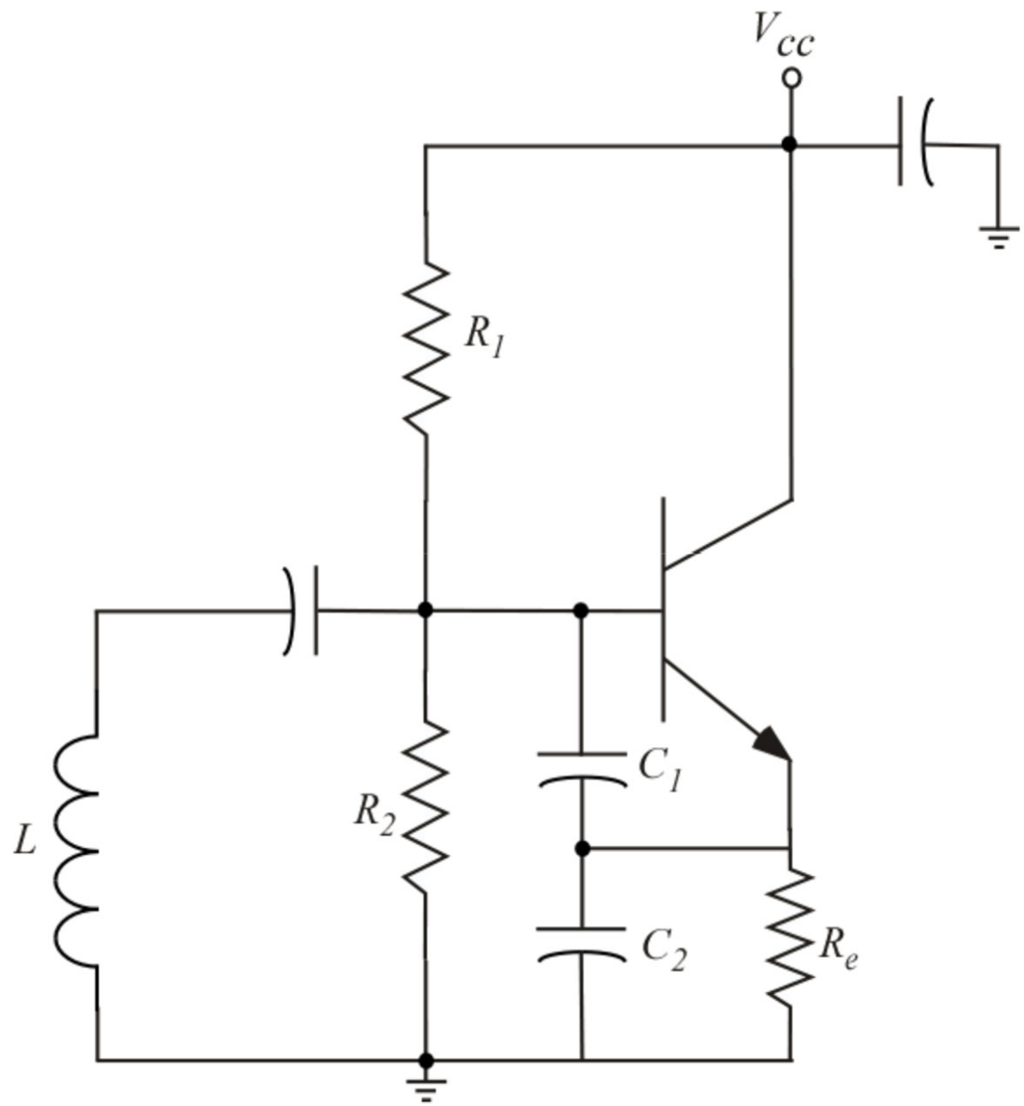
Oscillator Topologies

- **Common-base topology is usually preferred because**
  - Feedback within transistor is minimized (low Miller)
  - External feedback is dominant
  - Current gain has little phase shift and is constant with f
- **Oscillator built around 2 major requirements:**
  - Oscillation frequency f_o
  - Power P_o that must be delivered to a load
- **Other criteria**
  - Efficiency
  - Spectral purity
  - Frequency stability

BJT – High-Frequency Model



Common-Collector Colpitts

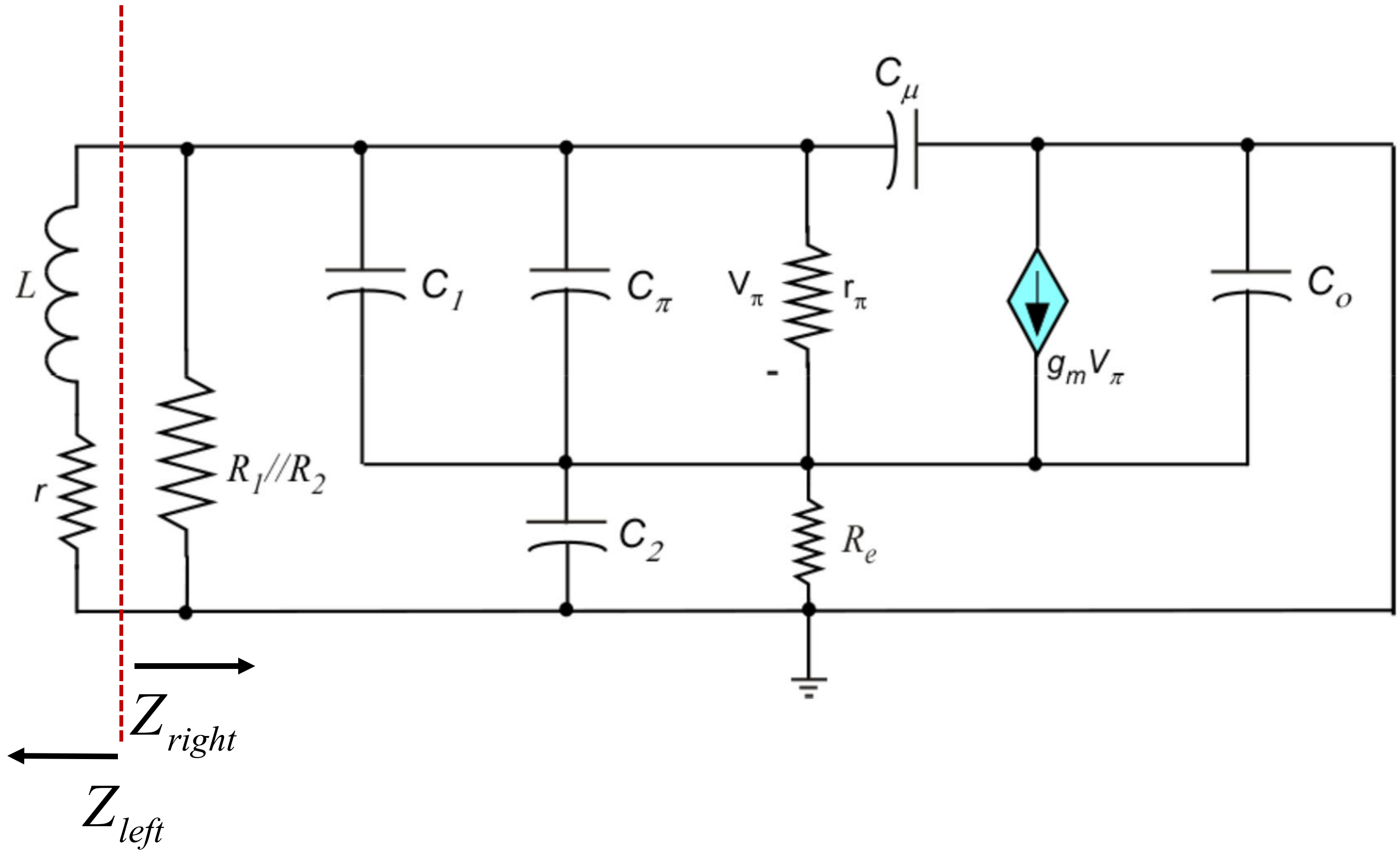


- **Minimize circuit dependence on transistor internal parameters**

- **Choose**

$$C_1 \gg C_\pi$$
$$C_2 \gg C_o$$

Incremental Model



For Oscillation, we want $Z_{left} = -Z_{right}$

Oscillation Criteria

Condition gives

$$\frac{1}{\omega^2 C_1^2 r_\pi} + \frac{1}{\omega^2 C_2^2 R_e} + r + g_m\left(\frac{1}{\omega^4 C_1^2 C_2^2 r_\pi R_e} - \frac{1}{\omega^2 C_1 C_2 R_e}\right) = 0$$

Term in ω^{-4} can be neglected. This gives

$$\frac{g_m}{\omega^2 C_1^2 \beta} + \frac{1}{\omega^2 C_2^2 R_e} + r - \frac{g_m}{\omega^2 C_1 C_2} = 0$$

Gives transconductance to set net resistance to zero

$$g_{ms} = \frac{\omega^2 C_1 C_2 r + \dfrac{C_1}{C_2 R_e}}{1 - \dfrac{C_2}{C_1 \beta}}$$

Oscillation Conditions

g_{ms} **is the trans-conductance needed to support steady-state oscillations**

If $g_m \ll \omega C_1 \beta$ and assuming that β is large,

$$g_{ms} \simeq \omega^2 C_1 C_2 r + \frac{C_1}{C_2}\frac{1}{R_e}$$

In terms of the inductor quality factor Q_L

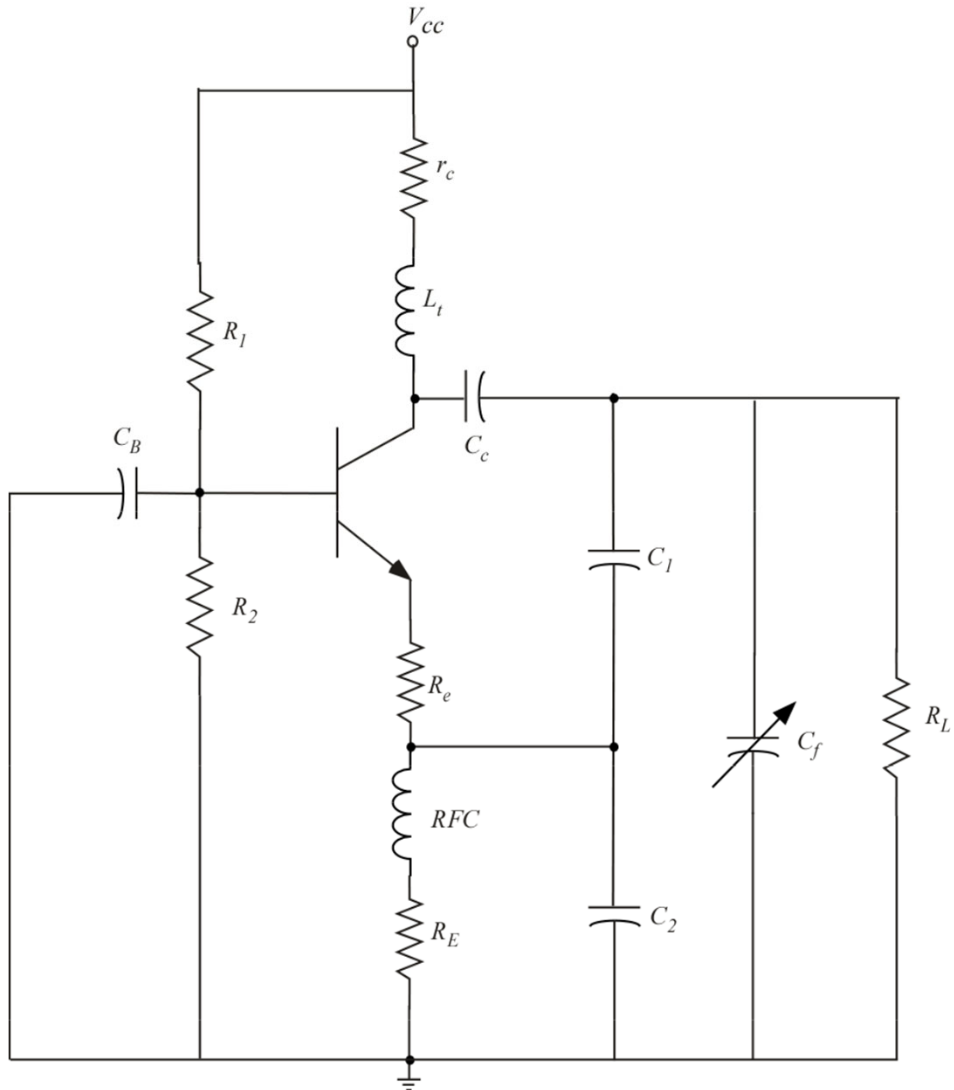
$$g_{ms} \simeq \frac{\omega_o \left(C_1 + C_2\right)}{Q_L} + \frac{C_1}{C_2}\frac{1}{R_e}$$

CC-Colpitts - Observation

- **To implement tunable oscillator**
  - Use variable capacitor in parallel or is series with inductor
- **To extract power from oscillator**
  - Place resistance in shunt with inductor
  - Account for loading effect in analysis and design

1. **Oscillation amplitude builds up and v_{be} increases**
2. **Transistor moves out of linear range and operation becomes nonlinear. Transconductance decreases.**
3. **When transconductance reaches g_{ms}, steady state operation is achieved.**

Common-Base Colpitts



C_f: tuning capacitor
C_1 & C_2: feedback ratio
RFC: RF choke to prevent power dissipation in R_E
R_P is equivalent resistance of coil L_t.

DEFINITIONS

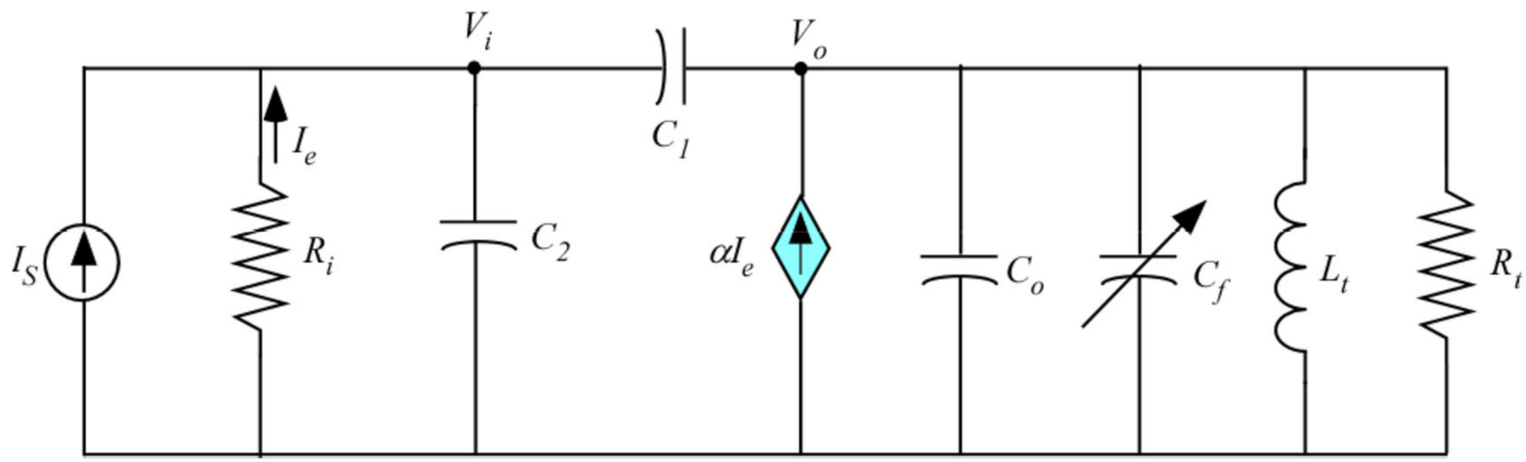
$$R_t = R_L \parallel R_p = \frac{R_L R_p}{R_L + R_p}$$

$$R_i = R_e + r_e$$

$$g_i = 1 / R_i$$

$$g_t = 1 / R_t$$

CB-Colpitts - Incremental Model



$$\begin{bmatrix} I_S \\ 0 \end{bmatrix} = \begin{bmatrix} g_i + s\left(C_1 + C_2\right) & -sC_1 \\ -\alpha g_i - sC_1 & g_t + s\left(C_1 + C_o + C_f\right) + \dfrac{1}{sL_t} \end{bmatrix} \begin{bmatrix} V_i \\ V_o \end{bmatrix}$$

Setting the determinant to zero provides the criterion for the onset of oscillation.

CB-Colpitts - Oscillation Conditions

Setting the determinant to zero gives.

$$\Delta(s) = g_i + s\left(L_t g_t g_i + C_a\right) + s^2\left(L_t g_i C_b + L_t g_t C_a - L_t C_1 \alpha g_i\right)$$
$$+ s^3\left(L_t C_a C_b - L_t C_1^2\right) = 0$$

Defining $C_a = C_1 + C_2$ **and** $C_b = C_1 + C_o + C_f$

we get a complex conjugate pair of roots. This leads to 2 equations for real and imaginary parts that must be equal to zero separately

$$\mathrm{Re}\left[\Delta(j\omega)\right] = g_i - \omega^2 L_t\left(C_b g_i + C_a g_t - C_1 \alpha g_i\right) = 0$$

$$\mathrm{Im}\left[\Delta(j\omega)\right] = L_t g_t g_i + C_a - \omega^2 L_t\left(C_a C_b - C_1^2\right) = 0$$

CB-Colpitts - Oscillation Conditions

From the equation for the imaginary part, we get

$$\omega_o^2 = \frac{g_t g_i + s\left(C_a / L_t\right)}{C_a C_b - C_t^2}$$

or

$$\omega_o^2 = \frac{1}{\underbrace{L_t\left[C_o + C_f + \left(C_1 C_2 / \left(C_1 + C_2\right)\right)\right]}_{LC\ tank\ circuit}} + \frac{1}{\underbrace{R_t R_i\left[\left(C_f + C_o\right)\left(C_1 + C_2\right) + C_1 C_2\right]}_{transistor\ and\ load}}$$

First term should predominate

CB-Colpitts - Oscillation Conditions

In that case

$$L_t \ll R_t R_i \left(C_1 + C_2\right)$$

which leads to $$\omega_o^2 = \frac{1}{L_t\left[C_o + C_f + C_1 C_2 / \left(C_1 + C_2\right)\right]}$$

Circuit oscillates at frequency determined by L_t and equivalent capacitance.

CB-Colpitts - Oscillation Conditions

The solution of the real part using $\omega = \omega_o$ gives

$$\alpha_{\min} = 1 + \frac{C_f + C_o}{C_1} + \frac{R_i}{R_t}\left(1 + \frac{C_2}{C_1}\right) - \frac{1}{\omega_o^2 L_t C_1}$$

$$\alpha_{\min} \approx 1 - \frac{C_2}{C_1 + C_2} + \frac{R_i}{R_t}\left(1 + \frac{C_2}{C_1}\right) = \frac{1}{1 + \left(C_2 / C_1\right)} + \frac{R_i}{R_t}\left(1 + \frac{C_2}{C_1}\right)$$

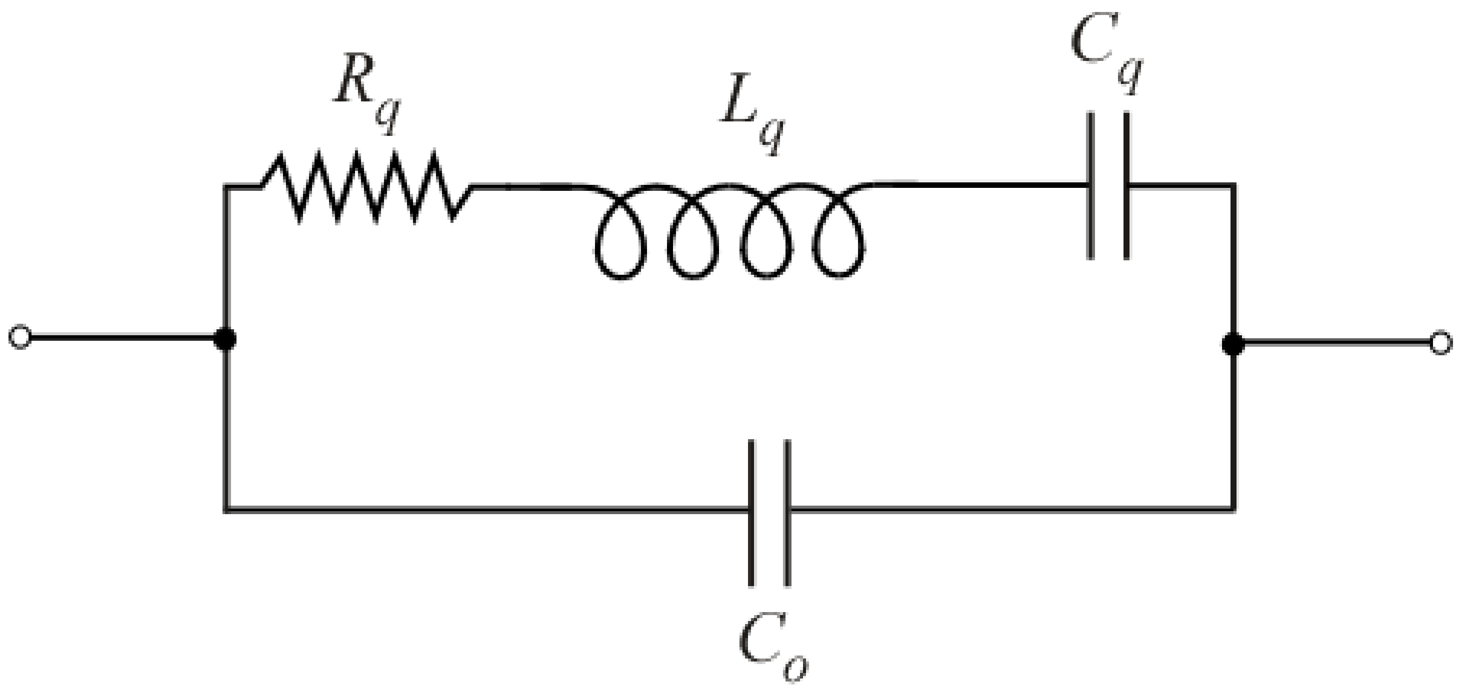
For oscillations to start, α of the transistor should be greater than α_{min}.

Crystal Oscillator

- **Quartz exhibits piezoelectric effect**
  - Reciprocal relationship between mechanical deformation and appearance of electrical potential
  - Deforming crystal produces voltage
  - Applying voltage produces deformation
- **When applied voltage is sinusoidal**
  - Mechanical oscillations that exhibit resonances at multiple frequencies
  - Extremely high Q that can reach 10^5 and 10^6
  - Do not work above 250 MHz

Crystal Resonator - Circuit Model

R_q, L_q and C_q describe mechanical resonance behavior
C_0 is capacitance due to external contacting



- L_q, C_q, R_q describe mechanical resonance
- C_0 *is* due to external contacting

Crystal Resonance Frequencies

Admittance from terminals is

$$Y = j\omega C_0 + \frac{1}{R_q + j\left[\omega L_q - 1/\left(\omega C_q\right)\right]} = G + jB$$

Resonance condition is expressed by

$$\omega_0 C_o - \frac{\omega_0 L_q - 1/\left(\omega_0 C_q\right)}{R_q^2 + \left[\omega_0 L_q - 1/\left(\omega_0 C_q\right)\right]^2} = 0$$

Crystal Resonance Frequencies

Series resonance frequency

$$\omega_0 = \omega_S \approx \omega_{S0}\left[1 + \frac{R_q^2}{2}\left(\frac{C_0}{L_q}\right)\right]$$

Parallel resonance frequency

$$\omega_0 = \omega_P \approx \omega_{P0}\left[1 - \frac{R_q^2}{2}\left(\frac{C_0}{L_q}\right)\right]$$

where

$$\omega_{S0} = 1/\sqrt{L_q C_q} \qquad \omega_{P0} = \sqrt{\left(C_q + C_0\right)/\left(L_q C_q C_0\right)}$$

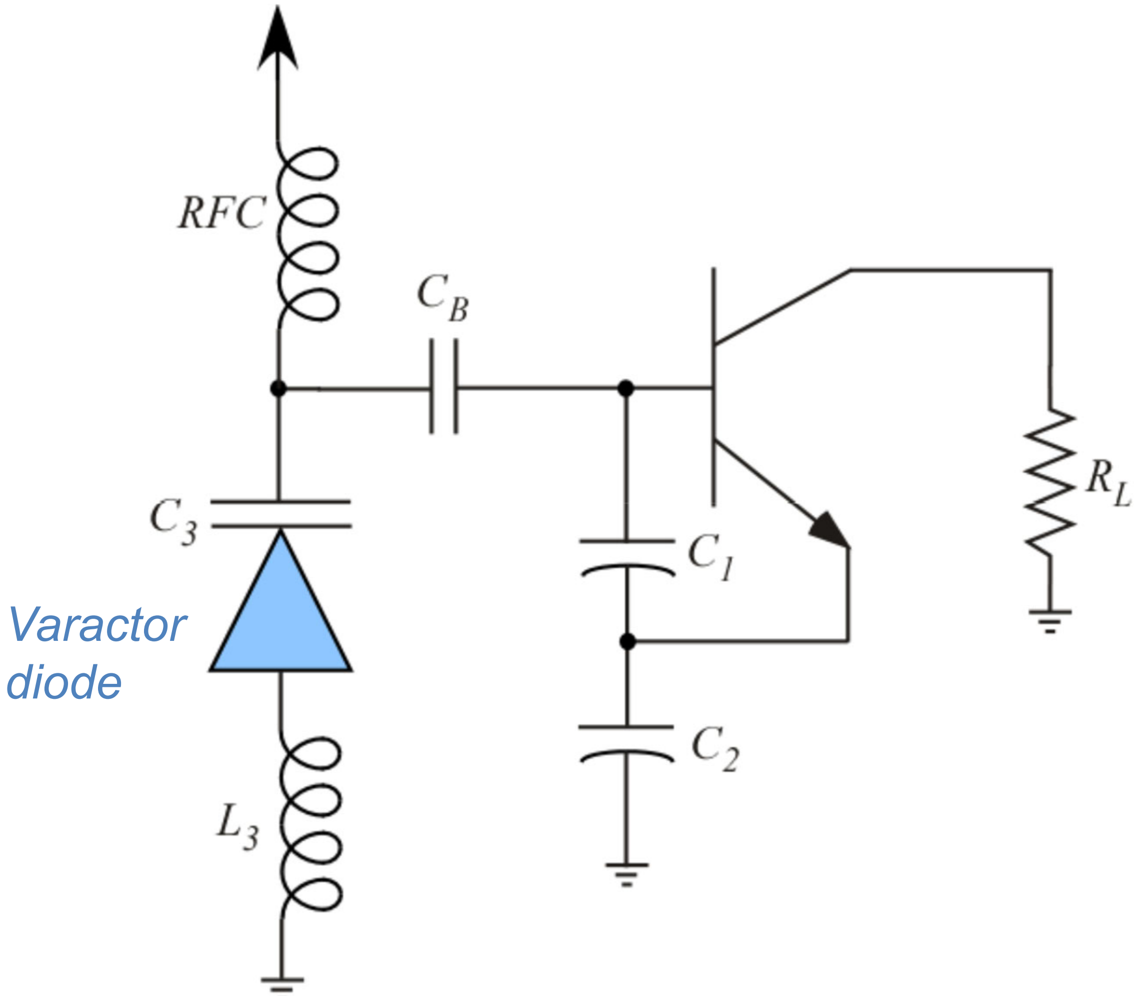
Crystal Oscillator - Example

Crystal is characterized by L_q = 0.1H, R_q = 25Ω, C_q = 0.3pF, and C_0 = 1 pF. Find series and parallel resonance frequencies and compare them against the susceptance formula.

$$f_S = f_{S0}\left[1+\frac{R_q^2}{2}\left(\frac{C_0}{L_q}\right)\right] = \frac{1}{2\pi\sqrt{L_q C_q}}\left[1+\frac{R_q^2}{2}\left(\frac{C_0}{L_q}\right)\right] = 0.919\ MHz$$

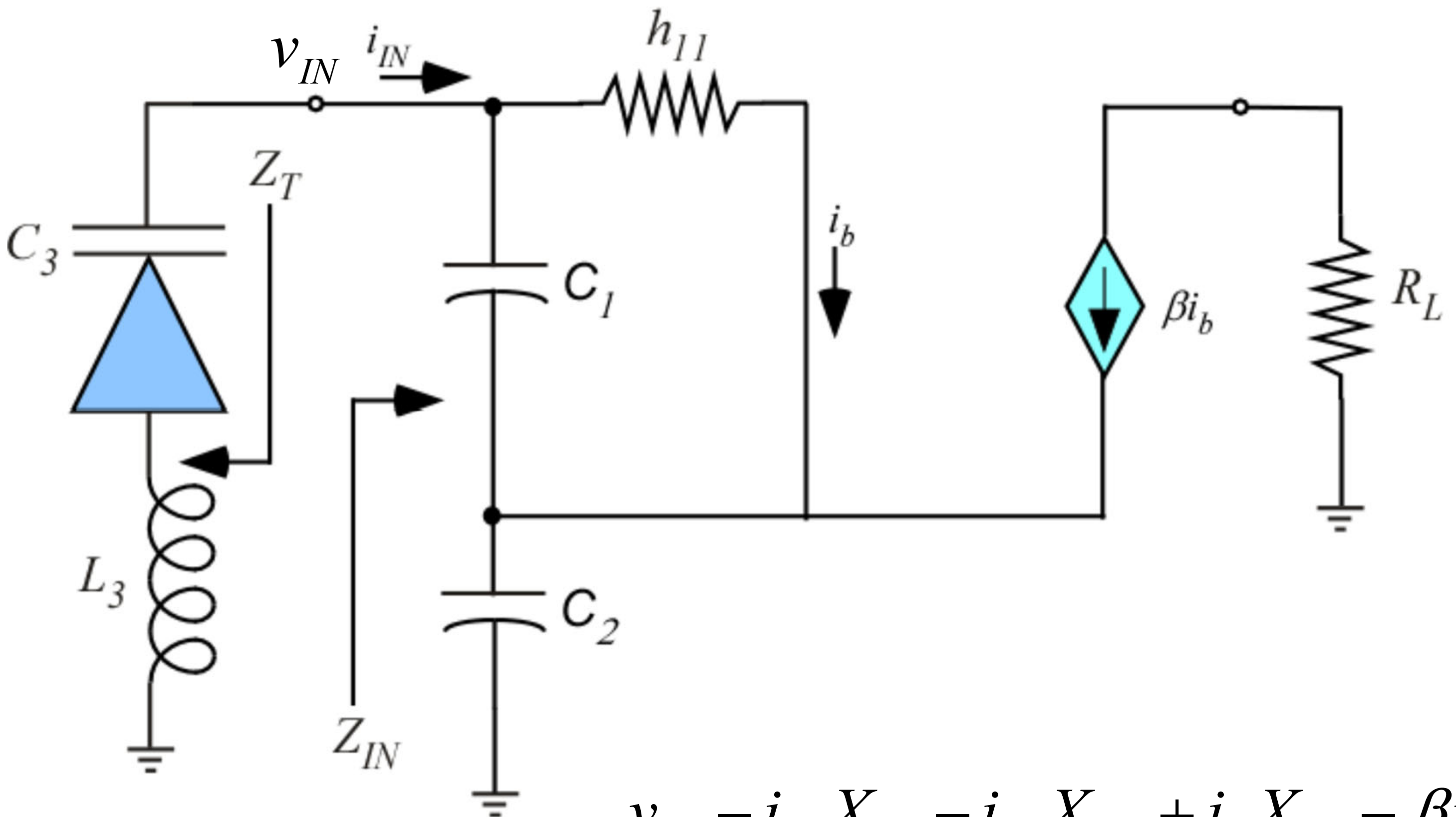
$$f_P = f_{P0}\left[1-\frac{R_q^2}{2}\left(\frac{C_0}{L_q}\right)\right] = \frac{1}{2\pi}\sqrt{\frac{C_q + C_0}{L_q C_q C_0}}\left[1-\frac{R_q^2}{2}\left(\frac{C_0}{L_q}\right)\right] = 1.048\ MHz$$

Voltage Controlled Oscillator



Varactor diode exhibits large change in capacitance in response to an applied bias voltage

Voltage Controlled Oscillator



$$v_{IN} - i_{IN}X_{C1} - i_{IN}X_{C2} + i_B X_{C1} - \beta i_B X_{C2} = 0$$

$$h_{11}i_B + i_B X_{C1} - i_{IN}X_{C1} = 0$$

Voltage Controlled Oscillator

Finding input impedance

$$Z_{IN} = \frac{1}{h_{11} + X_{C1}}\left[h_{11}\left(X_{C1} + X_{C2}\right) + X_{C1}X_{C2}\left(1+\beta\right)\right]$$

Re-write as (assuming $h_{11} >> X_{C1}$ and $\beta+1 \sim \beta$)

$$Z_{IN} = \frac{1}{j\omega}\left[\frac{1}{C_1} + \frac{1}{C_2}\right] - \frac{\beta}{h_{11}}\left(\frac{1}{\omega^2 C_1 C_2}\right)$$

Using $g_m = \beta/h_{11}$

$$R_{IN} = -\frac{g_m}{\omega^2 C_1 C_2}$$

$$X_{IN} = \frac{1}{j\omega C_{IN}}$$

Negative resistance

VCO – Resonance Frequency

$$C_{IN} = \frac{C_1 C_2}{C_1 + C_2}$$

Resonance frequency follows from condition:

$$X_{IN} = -X_{VARACTOR}$$ **where** $$X_{VARACTOR} = j\left(\omega_0 L_3 - \frac{1}{\omega_0 C_3}\right)$$

which gives

$$-j\left(\omega_0 L_3 - \frac{1}{\omega_0 C_3}\right) - \frac{1}{j\omega_0}\left[\frac{1}{C_1} + \frac{1}{C_2}\right] = 0$$

and

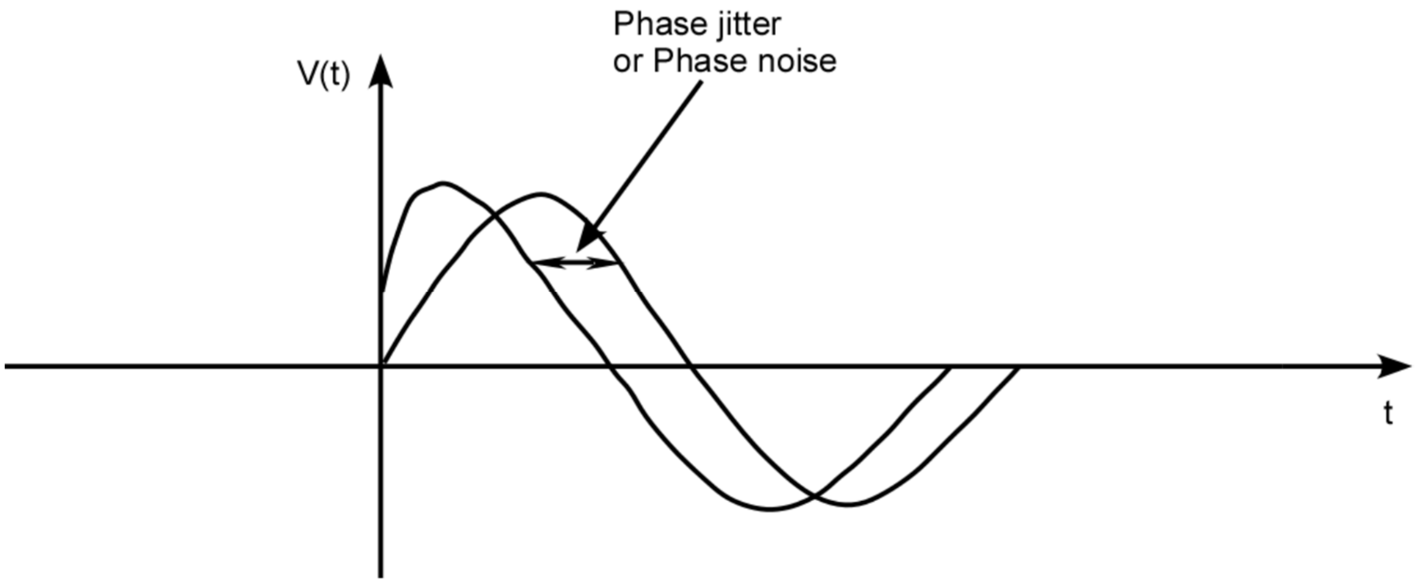
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{L_3}\left(\frac{1}{C_3} + \frac{1}{C_2} + \frac{1}{C_1}\right)}$$

Combined resistance of varactor diode must be equal to or less than $|R_{IN}|$ to create sustained oscillations

Oscillator Phase Noise

- **Most important performance characteristic**
  - Frequency-domain equivalent of jitter
  - Originates from thermal, shot and *1/f* noise
  - Random variation in phase angle of oscillator
  - Affects frequency stability
- **Phase noise quantification**
  - Compare phase noise power to carrier power
  - Determine phase noise spectral density
  - Can be characterized in time or frequency domain

Phase Jitter in Time Domain



If the phase varies, the waveform V(t) shifts back and forth along the time axis and this creates phase jitter

Oscillator Phase Noise

Oscillator operates as

$$V(t) = V_o \cos\left[2\pi f_o t + \phi(t)\right]$$

***$\phi(t)$* is a random noise process. The instantaneous frequency is**

$$f_{inst} = f_o + \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

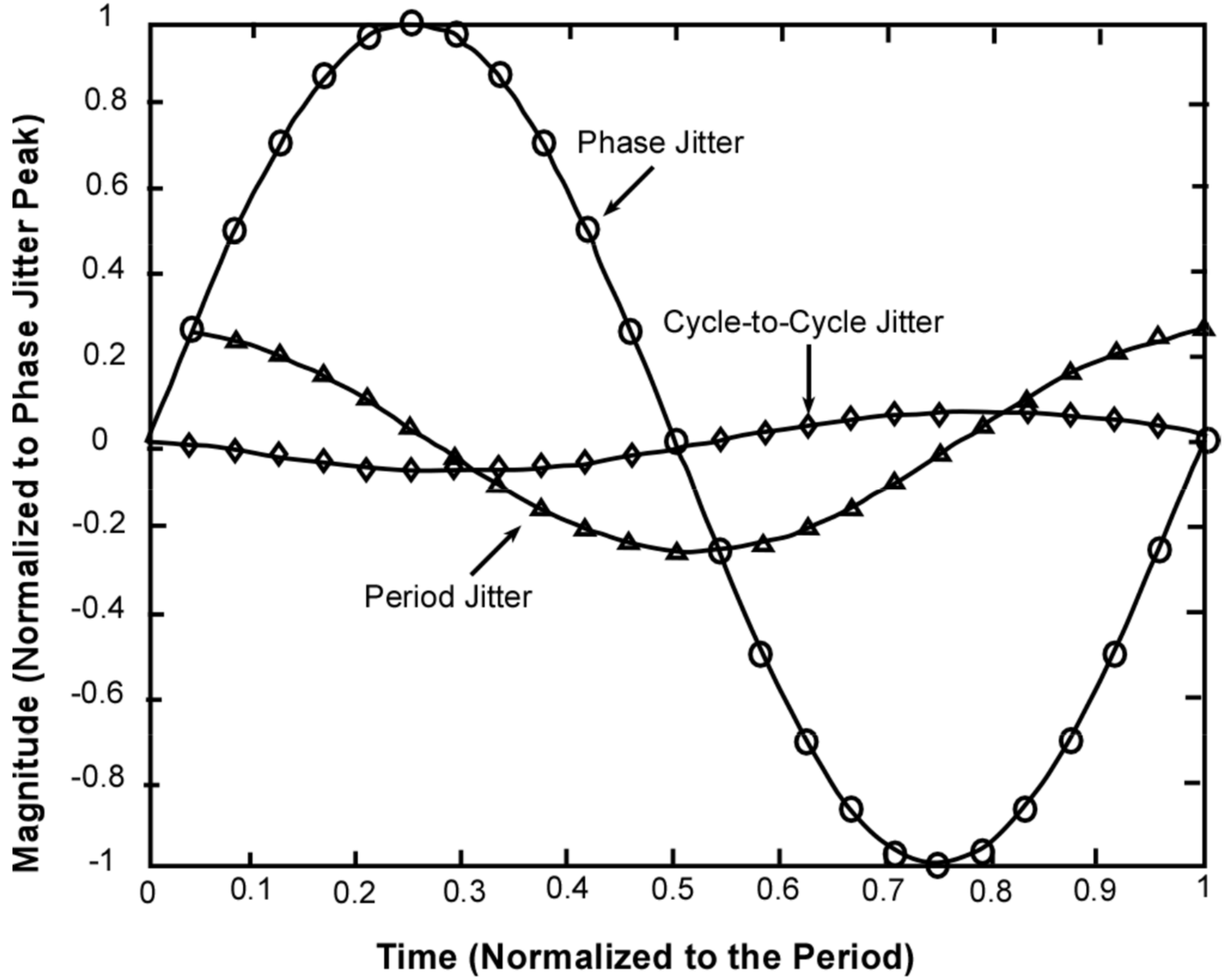
Instantaneous frequency variation is

$$\delta f(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

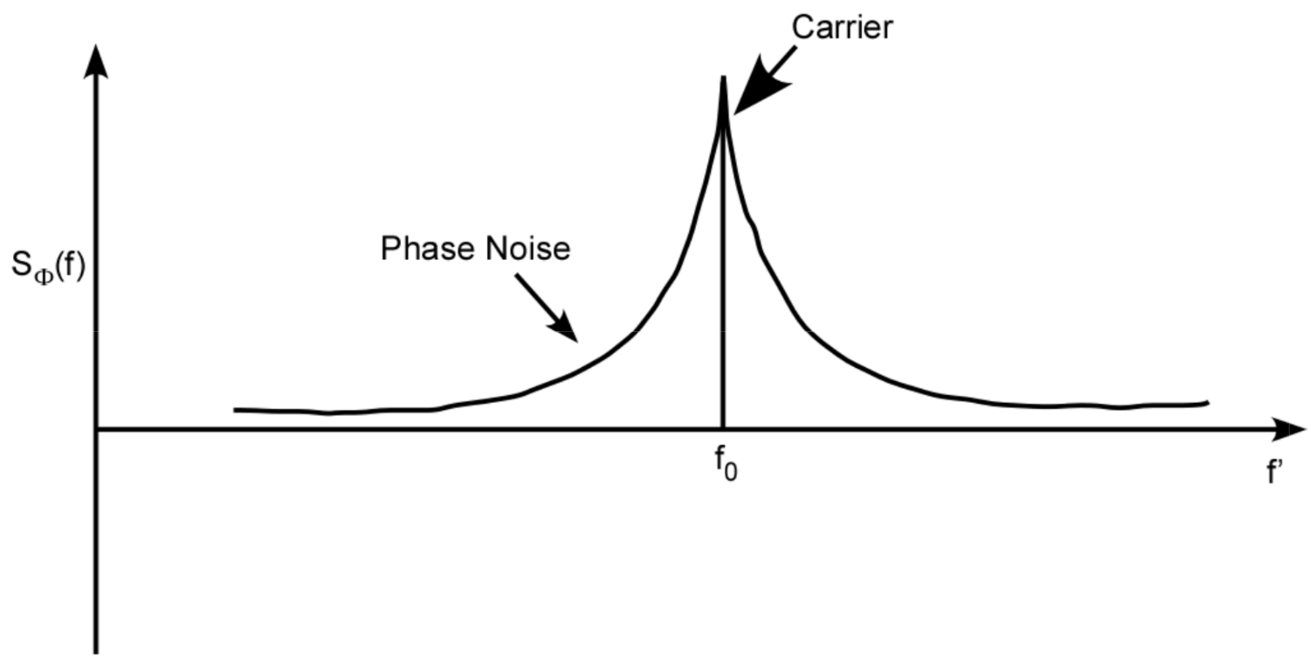
Fractional deviation in instantaneous frequency is

$$y(t) = \frac{\delta f(t)}{f_o} = \frac{1}{2\pi f_o}\frac{d\phi(t)}{dt}$$

Phase, Period and CTC Jitter



Phase Noise in Spectral Domain



Phase noise appears as sidebands centered around the carrier frequency

Phase Noise Specification

Phase noise magnitude is specified relative to the carrier's power on a per-hertz basis

$$L(f) = \frac{P_n(f)}{P_o \Delta f}$$

$P_n(f)$: phase noise power (in watts)

P_o : carrier's power (in watts)

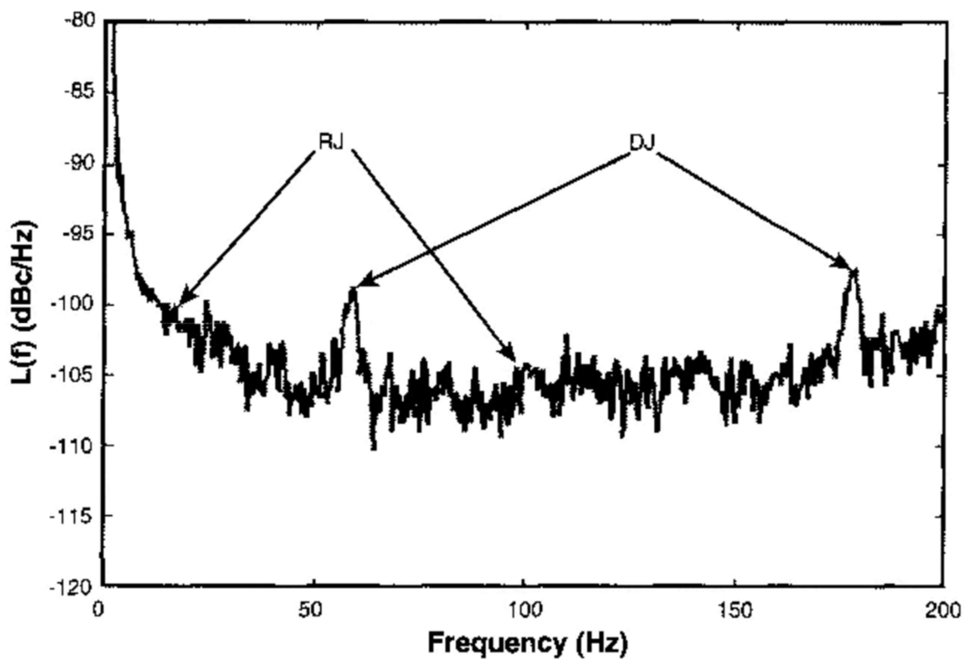
Δf : phase noise bandwidth (in hertz)

$$L(f) = \frac{1}{2} S_\Phi(f) \qquad \text{or} \qquad L(f) = 10\log_{10}\left(\frac{S_\Phi(f)}{2}\right)$$

$S_\Phi(f)$: PSD of phase noise

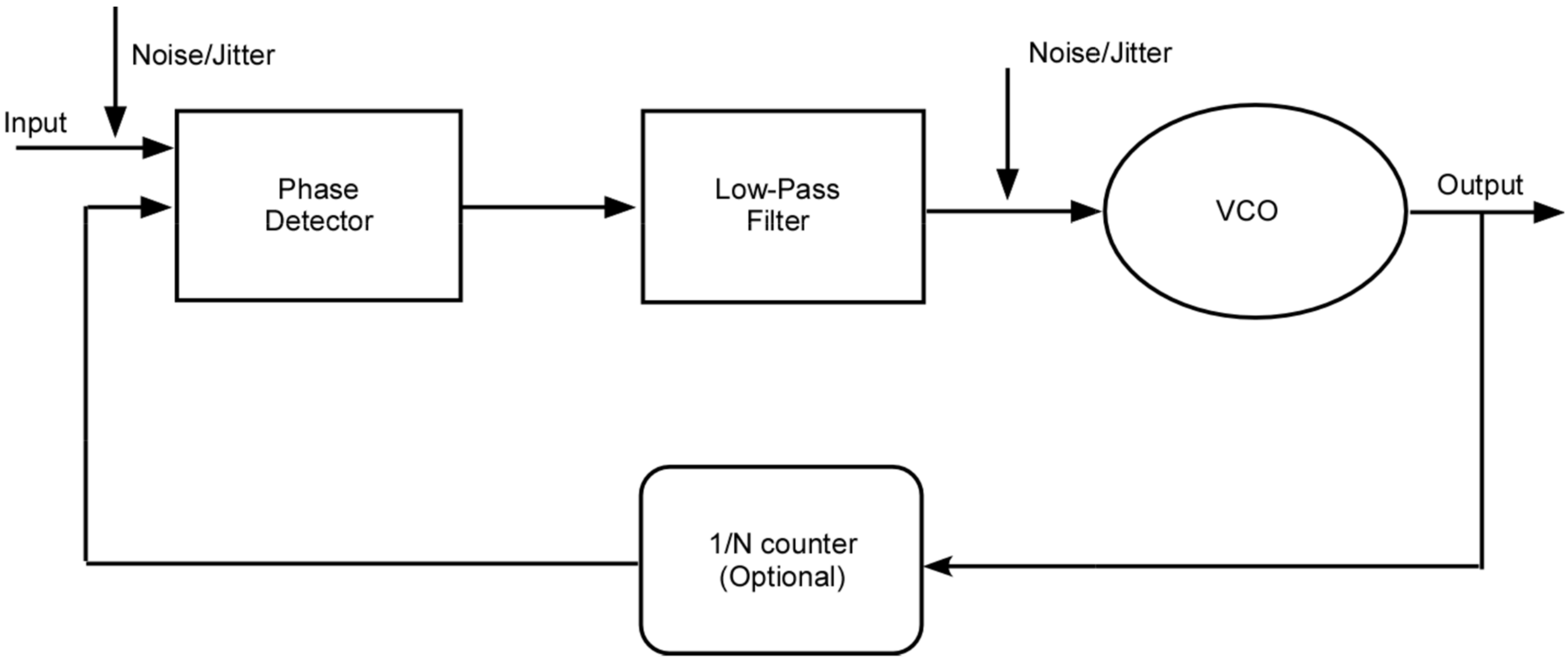
Phase Noise to Phase Jitter

Need: convert phase noise measured in the frequency domain to phase jitter for PLLs, clocks and oscillators



From the phase noise PSD, random jitter and deterministic jitter can be identified

Phase Noise Impact on PLL



Phase noise is the key metric for evaluating the performance of a PLL system