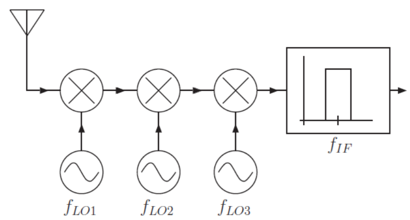
ECE 453: Wireless Communication Systems **Fall 2026**
Problem Set #3 **Due Date: 5pm, Fri, Sept 25, 2026**

Solutions

1. Consider a triple-conversion receiver with three ideal mixers in cascade. There are no filters between the mixers. The local-oscillator frequencies are f_{LO1} = 1250 MHz, f_{LO2} = 1370 MHz, and f_{LO3} = 185 MHz. The only filter is the final IF filter, which is ideal and centered at f_{IF} = 12.5 MHz.



(a) Find all positive input frequencies that can produce an output exactly at 12.5 MHz after the third mixer.

(b) If a fourth unfiltered conversion stage were added, how many input-frequency solutions would normally exist, assuming no accidental degeneracies?

2. The transfer function for a regenerative amplifier is:

$$T(\omega) = \frac{GF(\omega)}{1 - AGF(\omega)}$$

where A and G are positive real constants and $AG < 1$. Suppose that the filter is implemented with a single series or parallel RLC resonant circuit, in which case the frequency response function of the filter, $F(\omega)$, will have the form:

$$F(\omega) = \frac{1}{1 + jQ\left(\dfrac{\omega}{\omega_o} - \dfrac{\omega_o}{\omega}\right)},$$

where the factor Q is a dimensionless constant that controls the bandwidth of the filter.

(a) Find an expression for the bandwidth of $F(\omega)$ at attenuation level decibels — the "-α dB bandwidth" — where α is the attenuation expressed in dB. Denote this bandwidth by $\Delta f_{-\alpha dB}$ and express your result in terms of the power ratio $r = 10^{\alpha/10}$, the parameter Q, and the center frequency $f_o = \omega_o/2\pi$. Note that $|F(2\pi f)|$ is a bandpass function with peak at $f = f_o$. The -α dB bandwidth is defined to be the separation between the two frequencies f_1 and f_2 that satisfy $f_1 > 0, f_2 > 0$ and $\left|F(2\pi f_1)\right| = \left|F(2\pi f_2)\right| = \left|F(2\pi f_o)\right| / \sqrt{r}$.

(b) Using your answer to part a, write down the expression for the −3 dB bandwidth of $F(\omega)$. We will use this result often during the rest of the semester.

(c) Find an expression for the -α dB bandwidth of $T(\omega)$. Make sure that your result reduces to the result found in part a when $A \to 0$. Discuss how the magnitude of the loop gain (AG) affects the peak gain of $T(\omega)$ and the bandwidth of $T(\omega)$.

(d) Comment on how the product of the peak gain and bandwidth of $T(\omega)$ (the *gain-bandwidth product*) depends on the feedback gain parameter, A.

(e) Find an expression for the *shape factor* (SF) of the the regenerative amplifier's frequency response, $T(\omega)$, where

$$SF = \frac{\Delta f_{-30dB}}{\Delta f_{-3dB}}.$$

We already know that the gain of the system is increased and the bandwidth is decreased if we let the loop gain (AG) approach 1 from below. How does the shape factor of $T(\omega)$ change as the loop gain approaches 1? (Note - a good channel selection filter will have a shape factor smaller than 2. A really good channel selection filter might have shape factor of 1.2 or even smaller.)

(f) Find the numerical values of the shape factor of $T(\omega)$ when $A = 0$ and when A is chosen to make $AG = 0.99$.