ECE 453
Wireless Communication Systems

Network Parameters

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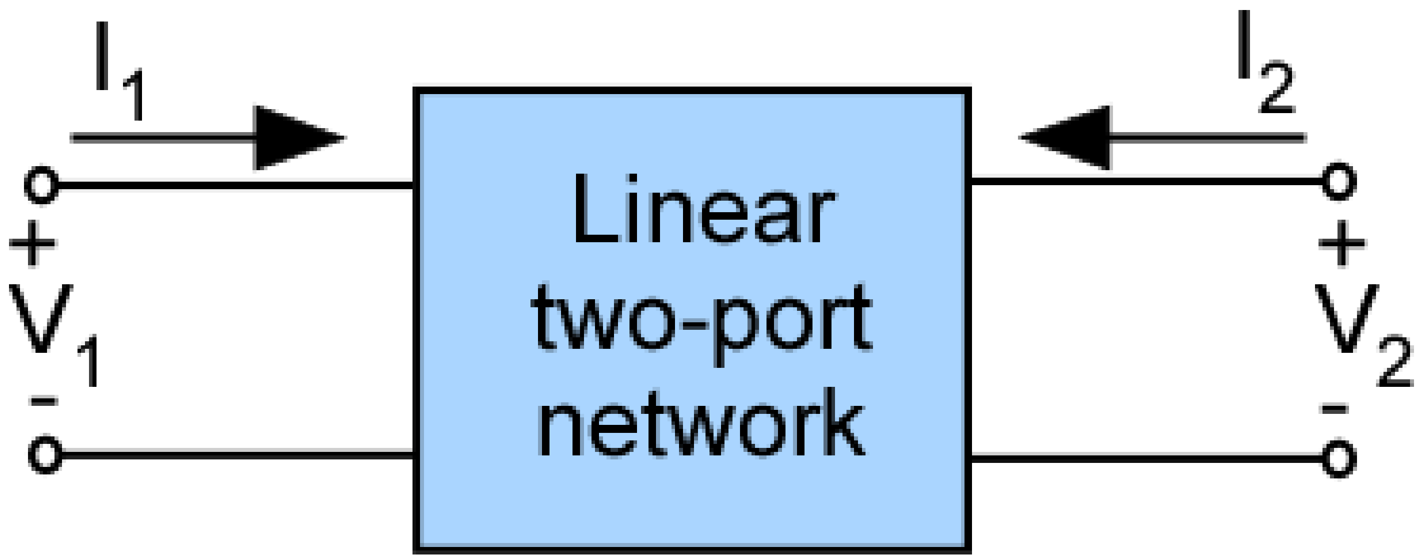
Properties of Real Networks

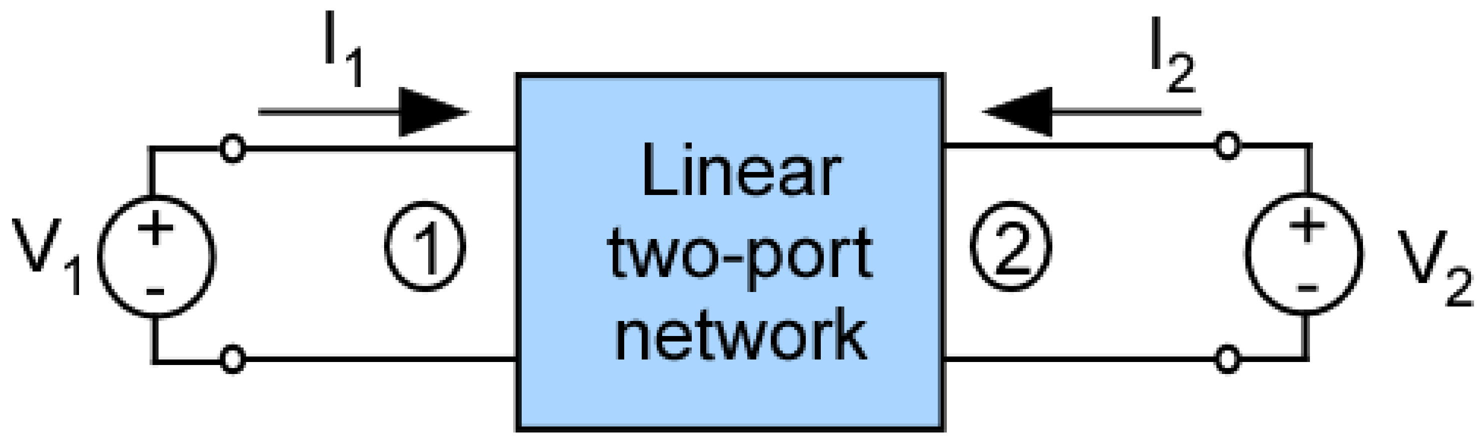
- Symmetry
- Reciprocity
- Reality
- Stability
- Causality
- Passivity

Transfer Function Representation



Use a two-terminal representation of system for input and output

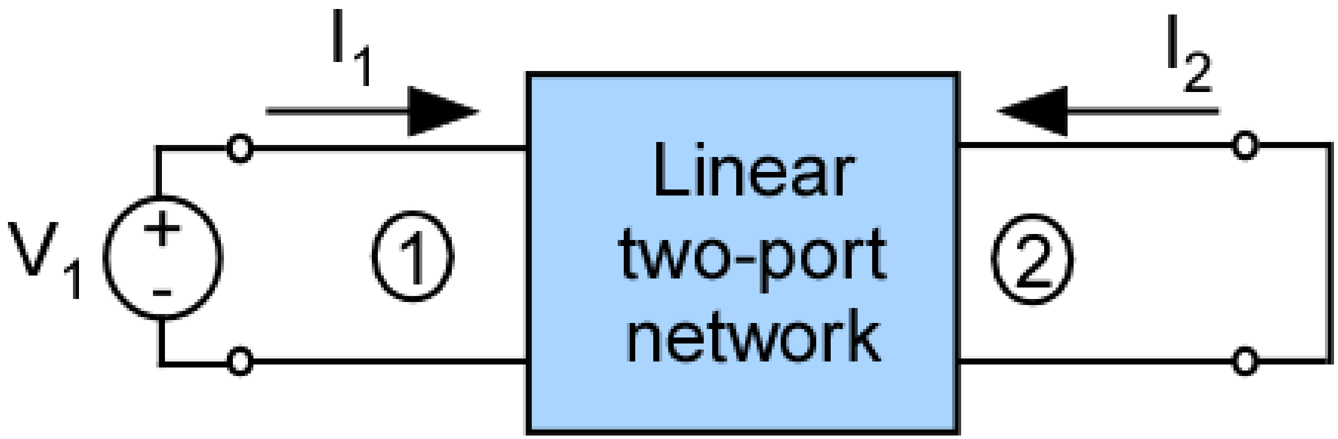
Y-parameter Representation



$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

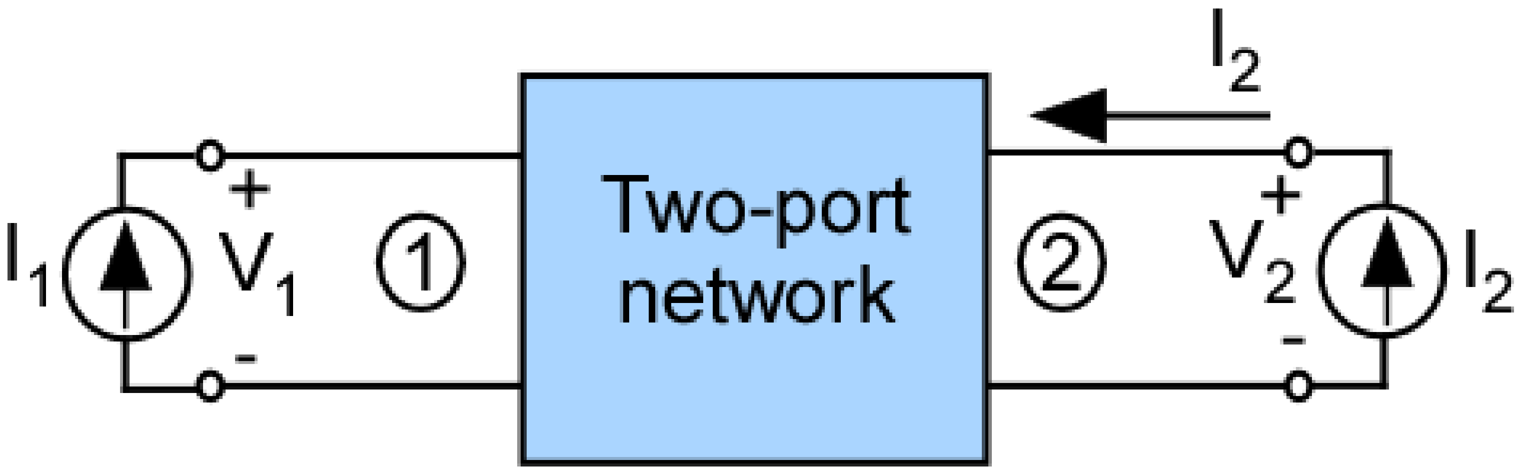
Y Parameter Calculations



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

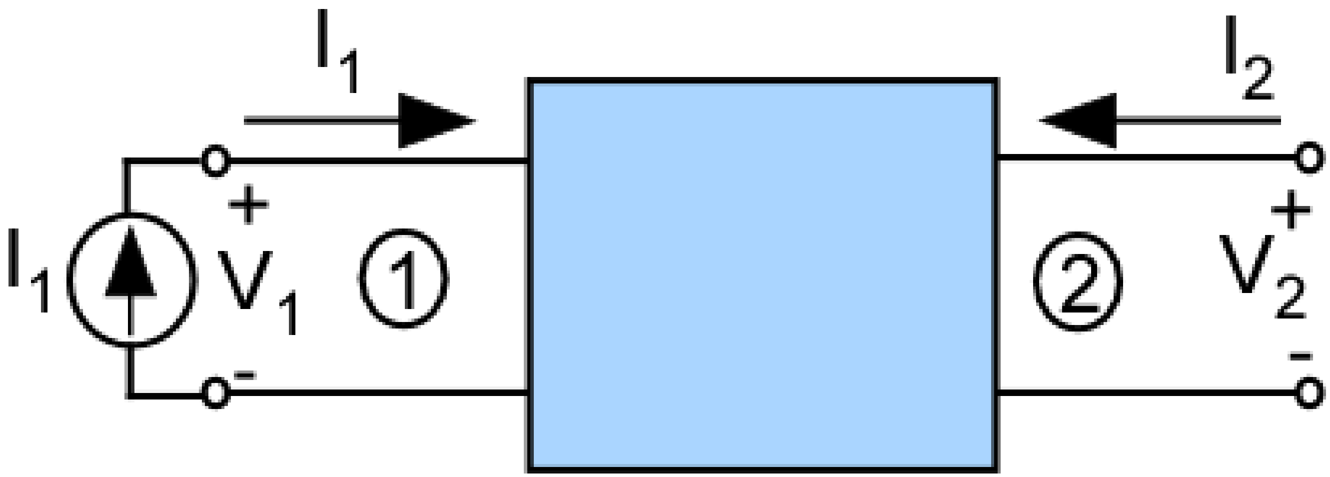
To make $V_2 = 0$, place a short at port 2

Z Parameters



$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

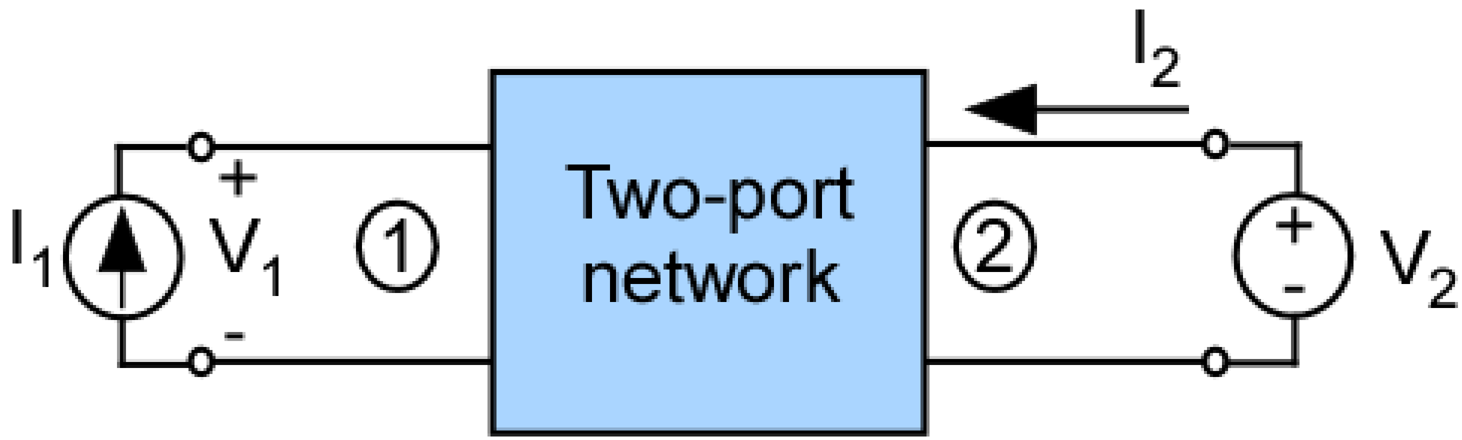
Z-parameter Calculations



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

To make $I_2 = 0$, place an open at port 2

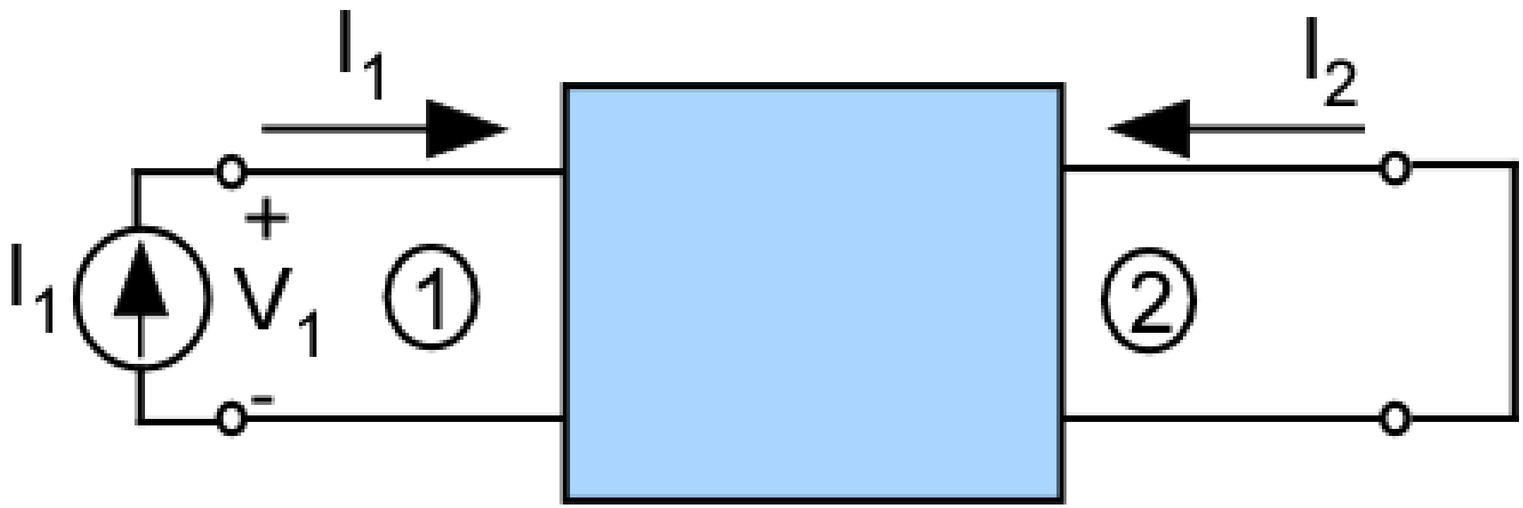
H Parameters



$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

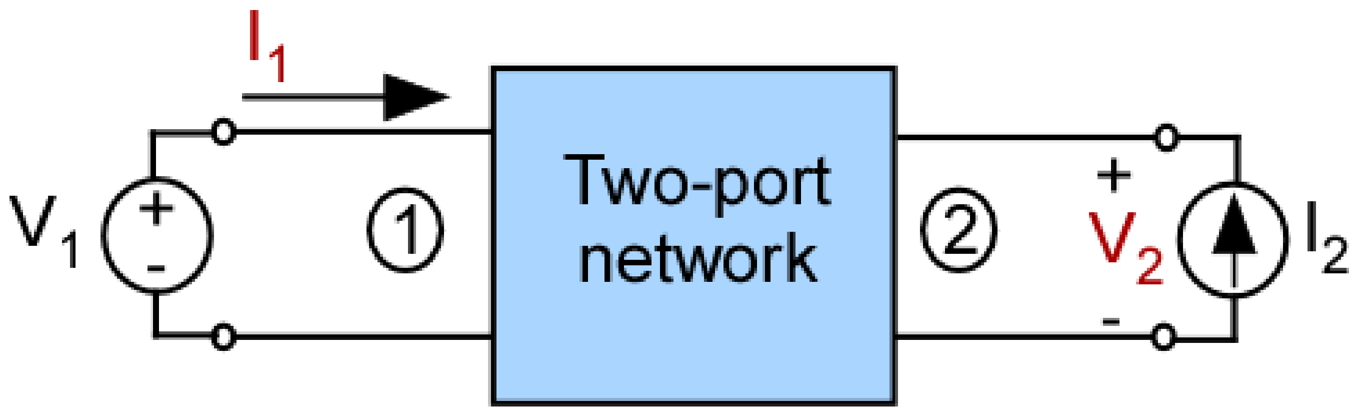
H Parameter Calculations



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

To make $V_2 = 0$, place a short at port 2

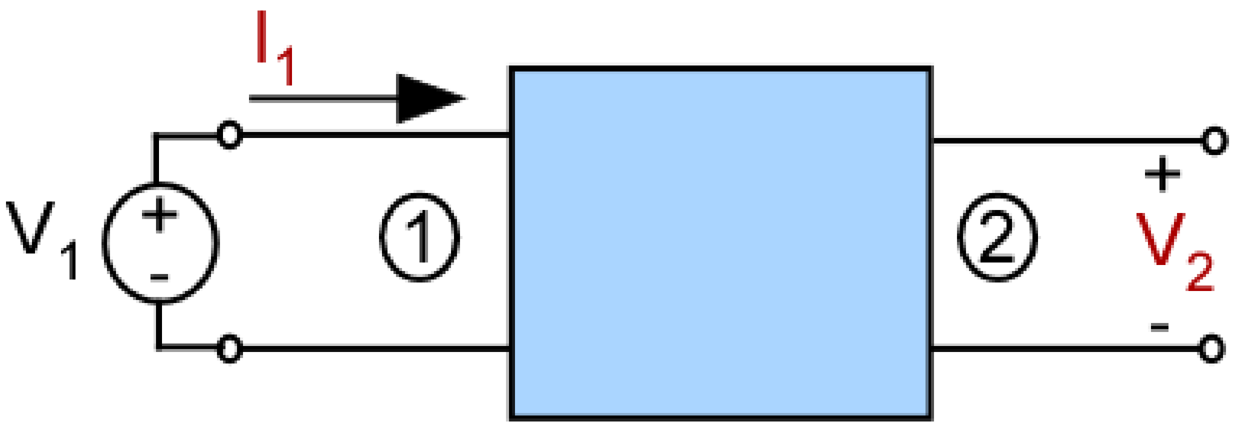
G Parameters



$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

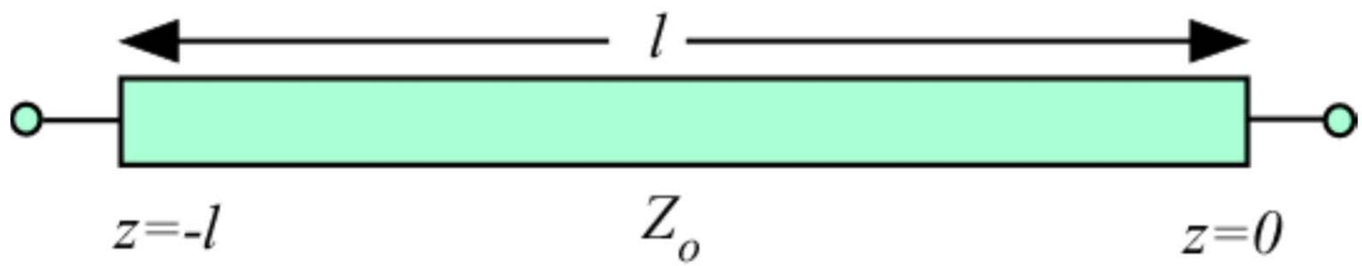
G-Parameter Calculations



$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \qquad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$

To make $I_2 = 0$, place an open at port 2

Y-Parameters of TL



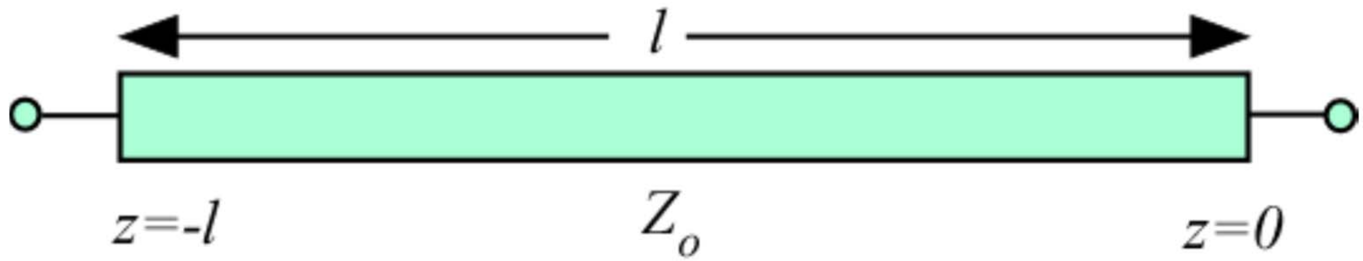
Find the Y-parameters of a lossless transmission line with propagation constant β and characteristic impedance Z_o (admittance Y_o)

$$V(z)=V_+e^{-j\beta z}+V_-e^{+j\beta z}$$

$$I(z)=Y_o\left(V_+e^{-j\beta z}-V_-e^{+j\beta z}\right)$$

Let port 1 be at $z=-l$ and port 2 at $z=0$

Y-Parameters of TL



at port 1

$$V_1 = V_+ e^{+j\beta l} + V_- e^{-j\beta l}$$

$$I_1 = Y_o \left(V_+ e^{+j\beta l} - V_- e^{-j\beta l} \right)$$

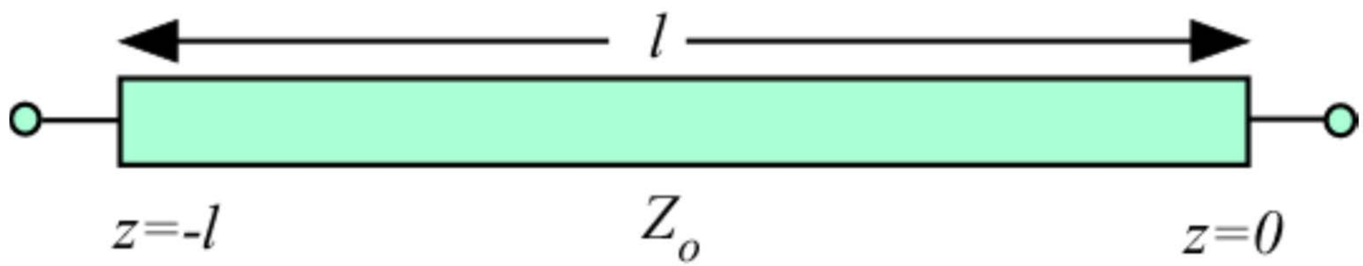
at port 2 (*z* = *0*)

$$V_2 = V_+ + V_-$$

$$I_2 = -Y_o \left(V_+ - V_- \right)$$

$$V_+ = \frac{V_2 - Z_o I_2}{2} \quad \text{and} \quad V_- = \frac{V_2 + Z_o I_2}{2}$$

Y-Parameters of TL



So that

$$V_1 = \left(\frac{V_2 - Z_o I_2}{2}\right)e^{+j\beta l} + \left(\frac{V_2 + Z_o I_2}{2}\right)e^{-j\beta l}$$

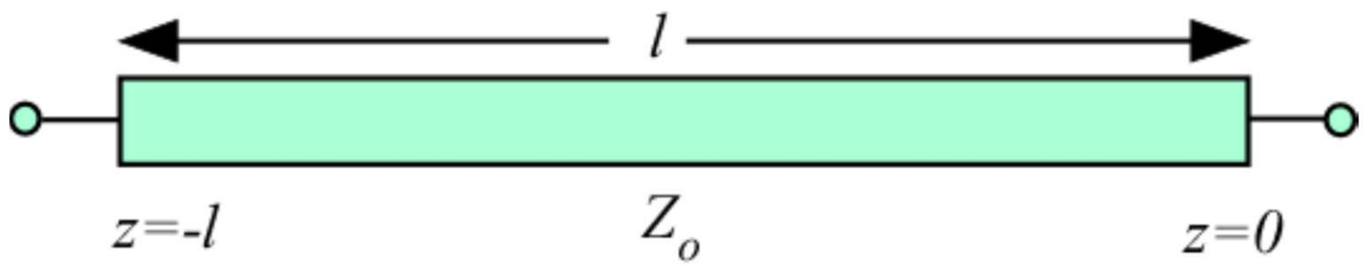
$$I_1 = Y_o\left(\frac{V_2 - Z_o I_2}{2}\right)e^{+j\beta l} - Y_o\left(\frac{V_2 + Z_o I_2}{2}\right)e^{-j\beta l}$$

and

$$V_1 = V_2 \cos\beta l - Z_o I_2 j \sin\beta l$$

$$I_1 = +Y_o V_2 j \sin\beta l - I_2 \cos\beta l$$

Y-Parameters of TL



Using definitions for Y_{11}

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{-I_2 \cos\beta l}{-jZ_o I_2 \sin\beta l} = \frac{-jY_o \cos\beta l}{\sin\beta l}$$

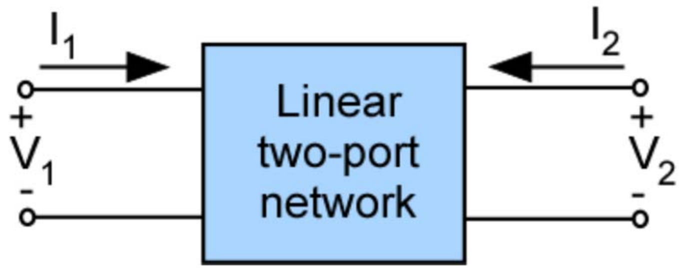
and

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-I_2}{-jZ_o I_2 \sin\beta l} = \frac{+jY_o}{\sin\beta l}$$

$Y_{22} = Y_{11}$ by symmetry

$Y_{12} = Y_{21}$ by reciprocity

TWO-PORT NETWORK REPRESENTATION



Z Parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Y Parameters

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

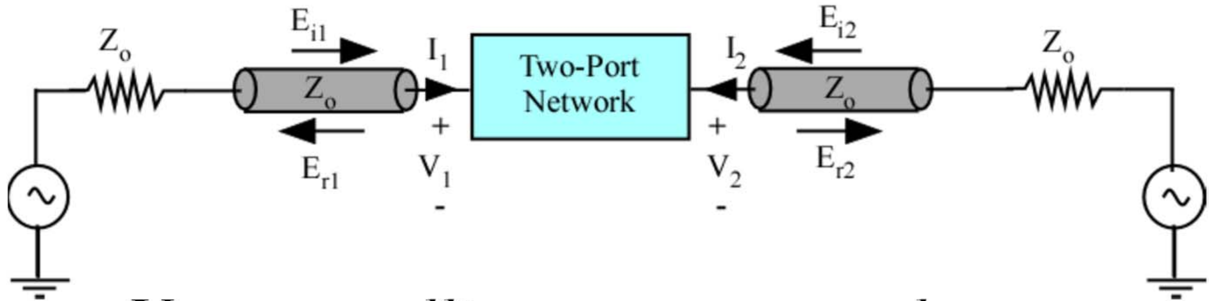
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

- At microwave frequencies, it is more difficult to measure total voltages and currents.

- Short and open circuits are difficult to achieve at high frequencies.

- Most active devices are not short- or open-circuit stable.

Wave Approach



Use a travelling wave approach

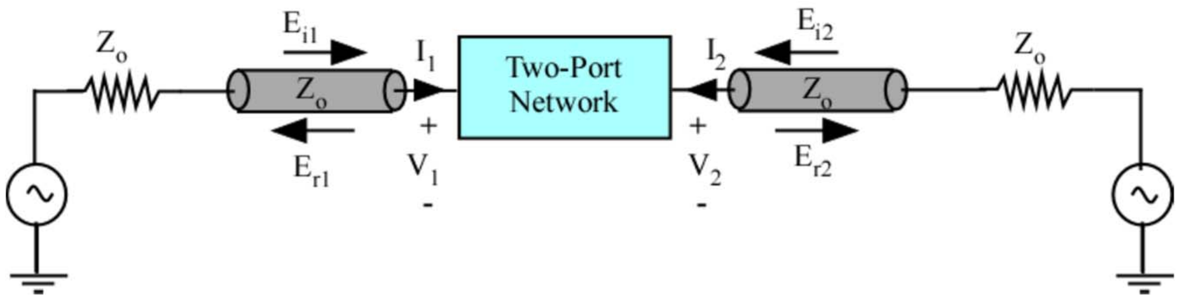
$$V_1 = E_{i1} + E_{r1} \qquad V_2 = E_{i2} + E_{r2}$$

$$I_1 = \frac{E_{i1} - E_{r1}}{Z_o} \qquad I_2 = \frac{E_{i2} - E_{r2}}{Z_o}$$

- Total voltage and current are made up of sums of forward and backward traveling waves.

- Traveling waves can be determined from standing-wave ratio.

Wave Approach



$$a_1 = \frac{E_{i1}}{\sqrt{Z_o}} \qquad a_2 = \frac{E_{i2}}{\sqrt{Z_o}}$$

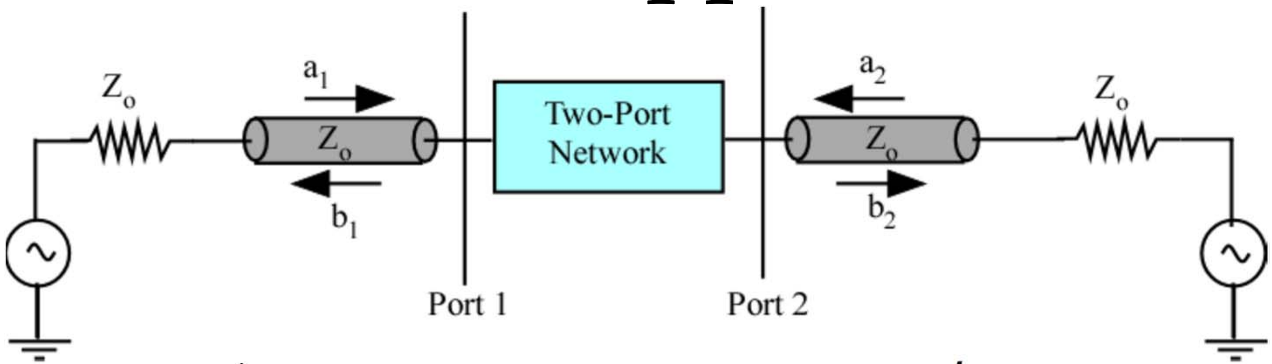
$$b_1 = \frac{E_{r1}}{\sqrt{Z_o}} \qquad b_2 = \frac{E_{r2}}{\sqrt{Z_o}}$$

Z_o is the reference impedance of the system

$$\mathbf{b_1 = S_{11}\, a_1 + S_{12}\, a_2}$$

$$\mathbf{b_2 = S_{21}\, a_1 + S_{22}\, a_2}$$

Wave Approach



$$S_{11} = \frac{b_1}{a_1}\Big|_{a2=0} \qquad S_{21} = \frac{b_2}{a_1}\Big|_{a2=0}$$

$$S_{12} = \frac{b_1}{a_2}\Big|_{a1=0} \qquad S_{22} = \frac{b_2}{a_2}\Big|_{a1=0}$$

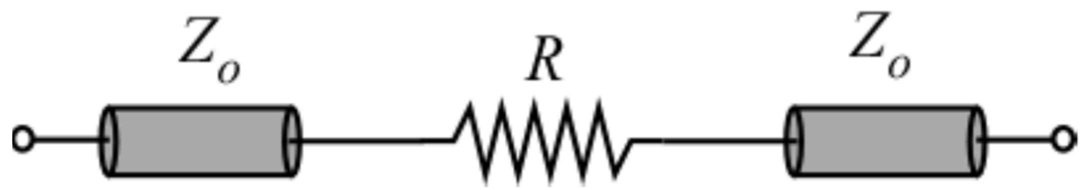
To make $a_i = 0$

1) Provide no excitation at port i

2) Match port i to the characteristic impedance of the reference lines.

CAUTION : a_i and b_i are the traveling waves in the <u>reference lines</u>.

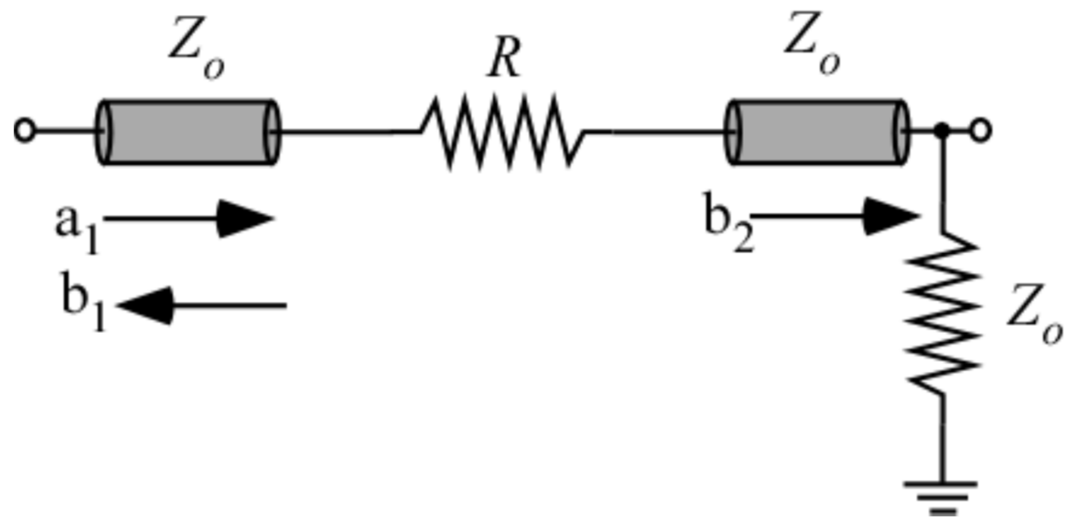
S-Parameters of Resistor



Determine S-Parameter of 2-port resistance

- Insert R between two reference TL
- Provide excitation at port 1 for S_{11} and S_{21}
- Provide excitation at port 2 for S_{12} and S_{22}
- Can use symmetry and reciprocity

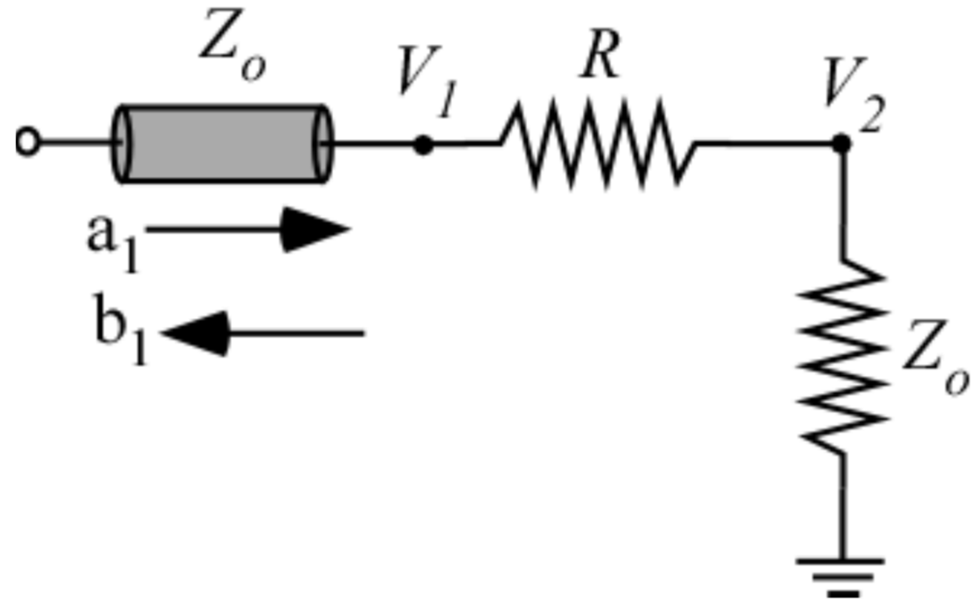
S-Parameters of Resistor



$$S_{11} = \frac{b_1}{a_1} = \Gamma = \frac{(R+Z_o)-Z_o}{(R+Z_o)+Z_o} = \frac{R}{R+2Z_o}$$

$$S_{11} = \frac{R}{R+2Z_o}$$ **and by symmetry,** $$S_{22} = \frac{R}{R+2Z_o}$$

Calculating S_{21} of Resistor

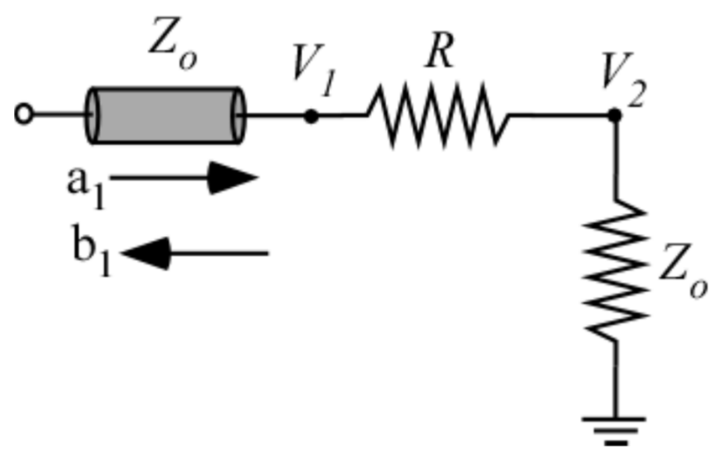


Since a_2=0, the total voltage in port 2 is: $V_2 = b_2\sqrt{Z_o}$

$$V_2 = \frac{V_1 Z_o}{R_1 + Z_o} = \frac{\sqrt{Z_o}\left(a_1 + b_1\right) Z_o}{R_1 + Z_o} = \frac{\sqrt{Z_o}\left(a_1 + S_{11} a_1\right) Z_o}{R_1 + Z_o}$$

S-Parameters of Resistor

$$V_2 = \frac{Z_o\sqrt{Z_o}\left(1+S_{11}\right)a_1}{R_1+Z_o} = \frac{2Z_o a_1\sqrt{Z_o}}{R_1+2Z_o}$$



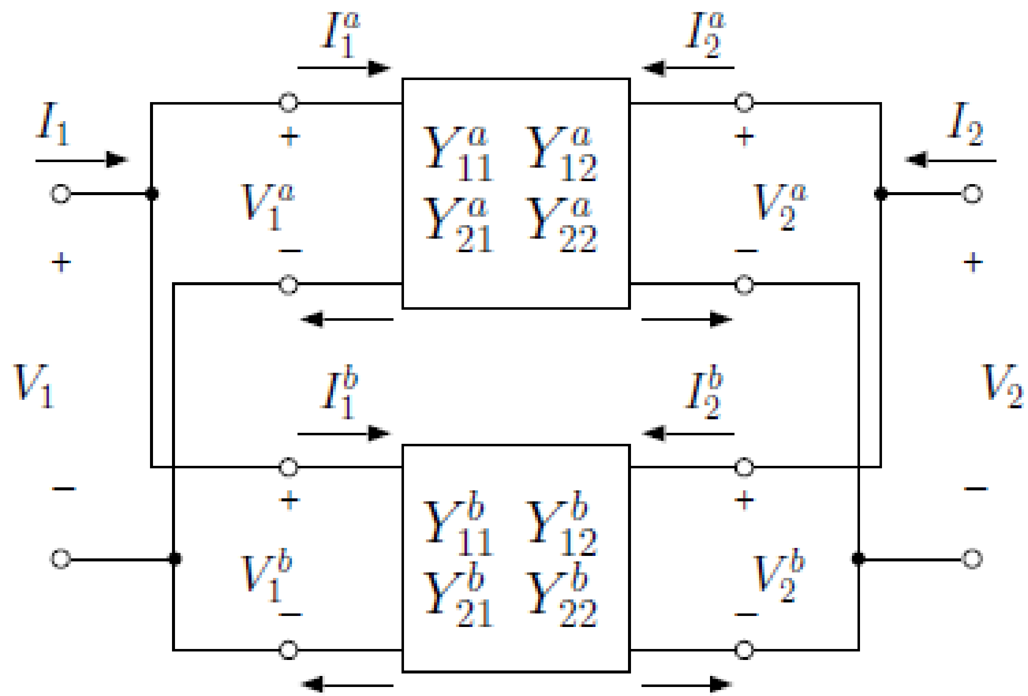
$$S_{21} = \frac{b_2}{a_1} = \frac{V_2}{\sqrt{Z_o}}\frac{1}{a_1} = \frac{2Z_o}{R+2Z_o}$$

$$S_{21} = \frac{2Z_o}{R+2Z_o}$$ and by reciprocity, $$S_{12} = \frac{2Z_o}{R+2Z_o}$$

S parameters of resistor R

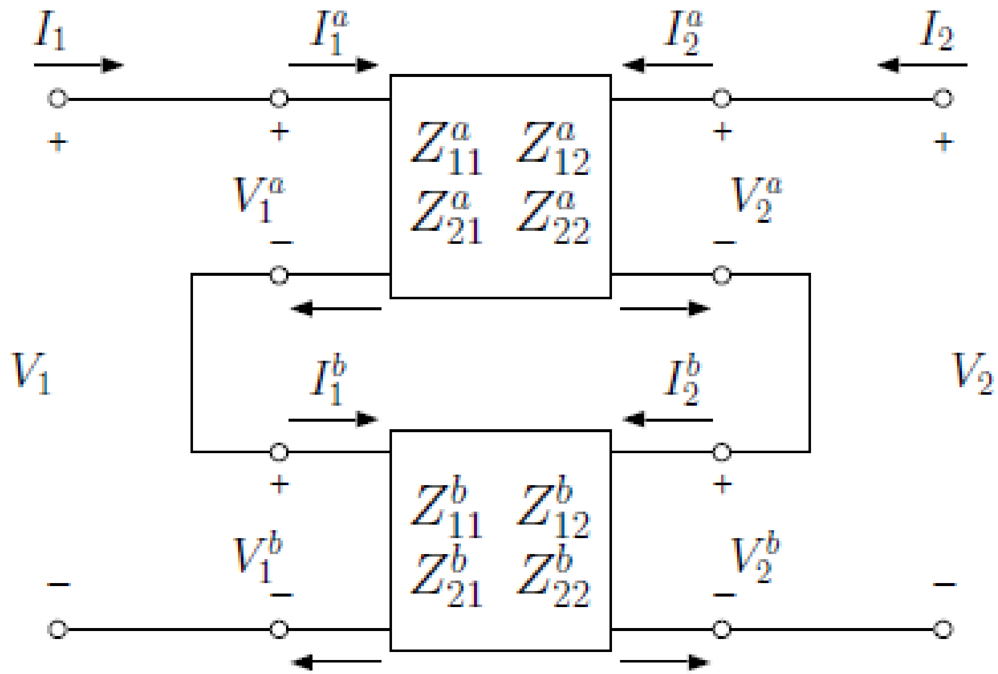
$$S = \begin{bmatrix} \frac{R}{R+2Z_o} & \frac{2Z_o}{R+2Z_o} \\ \frac{2Z_o}{R+2Z_o} & \frac{R}{R+2Z_o} \end{bmatrix}$$

Two-Ports in Parallel



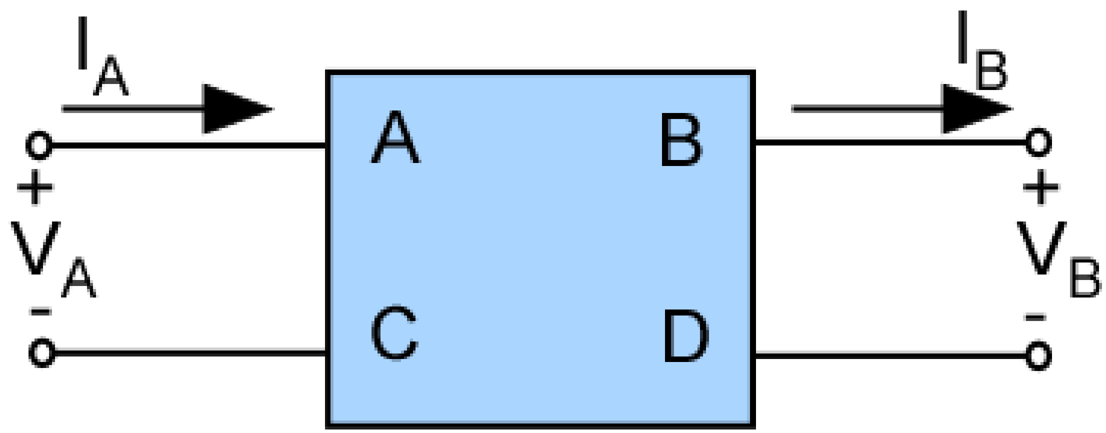
$$\mathbf{Y} = \mathbf{Y}^a + \mathbf{Y}^b$$

Two-Ports in Series



$$\mathbf{Z} = \mathbf{Z}^a + \mathbf{Z}^b$$

ABCD -Parameters



$$V_A = AV_B + BI_B$$

$$I_A = CV_B + DI_B$$

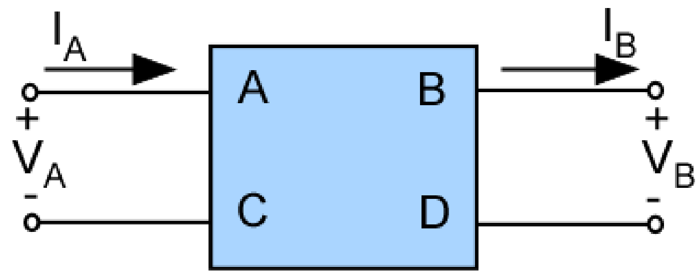
ABCD -Parameters

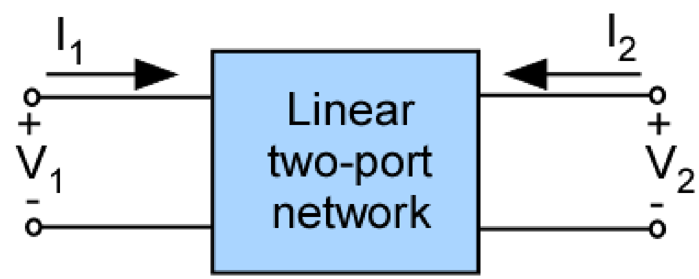
$$V_A = V_1$$

$$V_B = V_2$$

$$I_A = I_1$$

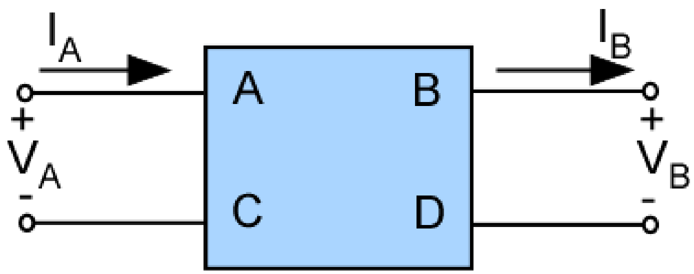
$$I_B = -I_2$$

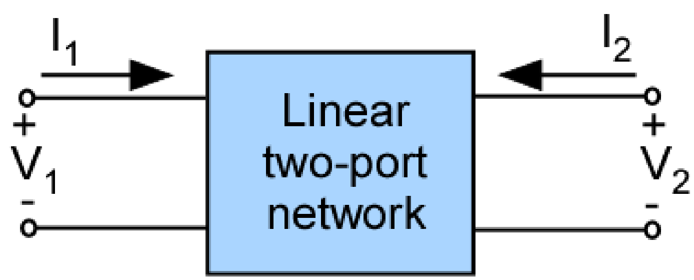




Relationship with Z parameters is obtained by first expressing ABCD parameters in terms of Z parameters

ABCD -Parameters





From

$$V_A = Z_{11} I_A - Z_{12} I_B$$

$$V_B = Z_{21} I_A - Z_{22} I_B$$

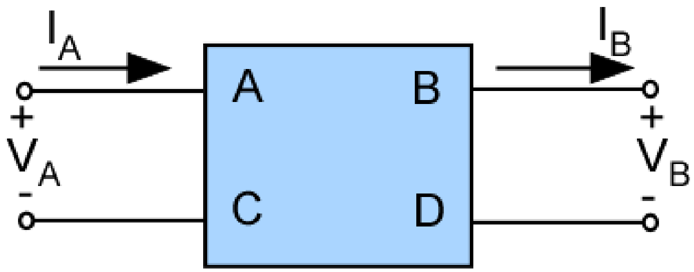
We get

$$A = \frac{Z_{11}}{Z_{21}} \qquad B = \frac{\Delta}{Z_{21}}$$

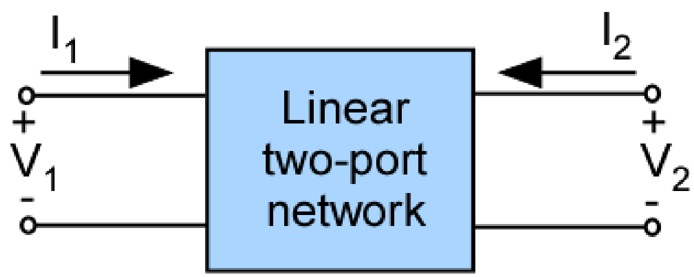
$$C = \frac{1}{Z_{21}} \qquad D = \frac{Z_{22}}{Z_{21}}$$

$$\Delta = Z_{11} Z_{22} - Z_{12} Z_{21}$$

ABCD -Parameters



$$Z_{11} = \frac{A}{C} \qquad Z_{11} = \frac{(AD - BC)}{C}$$



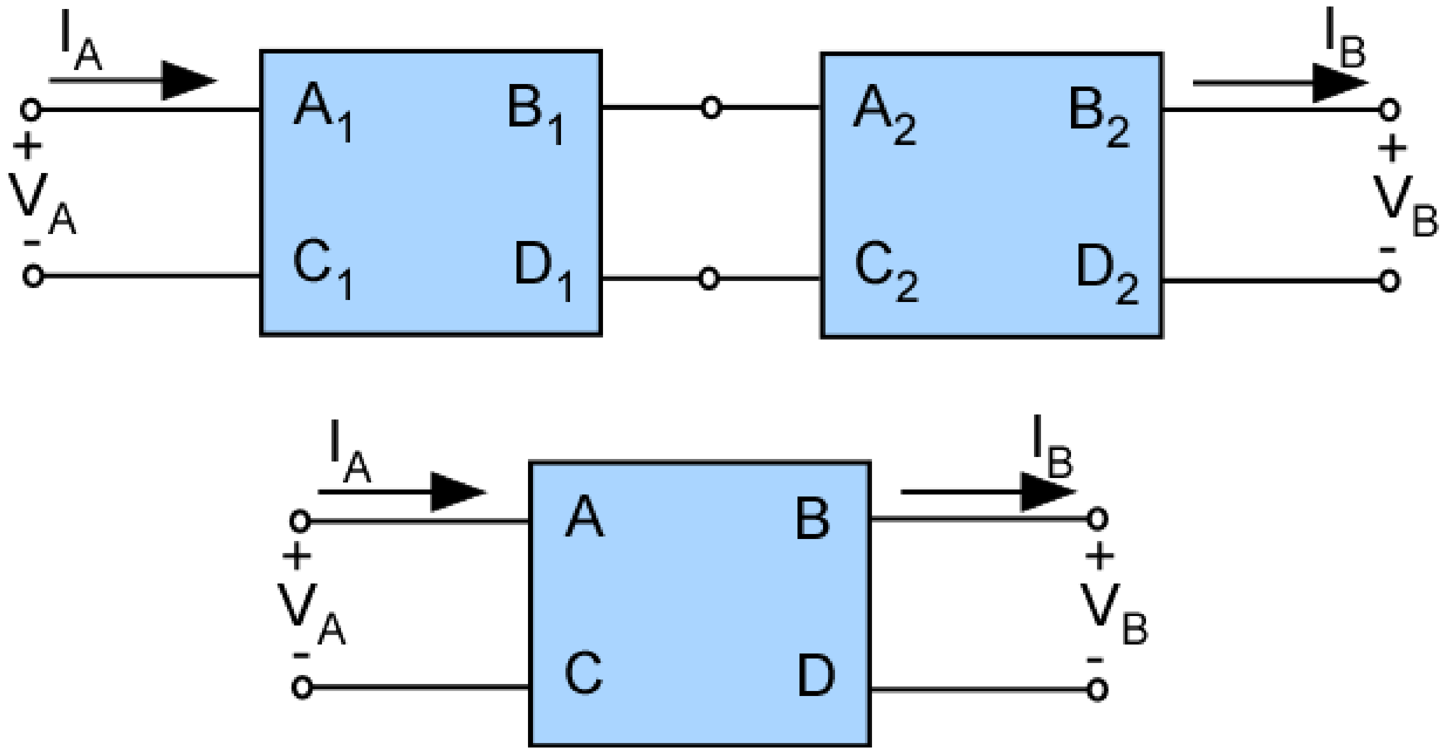
$$Z_{21} = \frac{1}{C} \qquad Z_{22} = \frac{1}{C}$$

For a reciprocal network, $Z_{21} = Z_{12}$, therefore

$$AD - BC = 1$$

Reciprocity condition for ABCD parameters

ABCD -Parameters



When cascading two-ports, it is best to use ABCD parameters. Put voltage and currents in cascadable form with the input variables in terms of the output variables

$$\mathbf{ABCD} = \left(\mathbf{ABCD}\right)_1 \cdot \left(\mathbf{ABCD}\right)_2$$

Scattering Transfer Parameters

In T-Parameters, traveling waves at the input are related to those at the output

$$b_1 = S_{11}a_1 + S_{12}a_2 \qquad\qquad b_1 = T_{11}a_2 + T_{12}b_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \qquad\qquad a_1 = T_{21}a_2 + T_{22}b_2$$

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} T_{12}T_{22}^{-1} & T_{11} - T_{12}T_{21}T_{22}^{-1} \\ T_{22}^{-1} & -T_{21}T_{22}^{-1} \end{pmatrix}$$

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} S_{12} - S_{11}S_{22}S_{21}^{-1} & S_{11}S_{21}^{-1} \\ -S_{22}S_{21}^{-1} & S_{21}^{-1} \end{pmatrix}$$

T parameters can be cascaded $\mathbf{T} = \mathbf{T_A} \cdot \mathbf{T_B}$

Parameter Conversion

7.7.1 Converting to Y-parameters

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{D_Z} & -\frac{Z_{12}}{D_Z} \\ -\frac{Z_{21}}{D_Z} & \frac{Z_{11}}{D_Z} \end{bmatrix} = \begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{D_h}{h_{11}} \end{bmatrix} = \begin{bmatrix} \frac{D}{B} & -\frac{D_{ABCD}}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$$

7.7.2 Converting to Z-parameters

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{D_Y} & -\frac{Y_{12}}{D_Y} \\ -\frac{Y_{21}}{D_Y} & \frac{Y_{11}}{D_Y} \end{bmatrix} = \begin{bmatrix} \frac{D_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix} = \begin{bmatrix} \frac{A}{C} & \frac{D_{ABCD}}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$$

7.7.3 Converting to h-parameters

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{D_Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix} = \begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{D_Y}{Y_{11}} \end{bmatrix} = \begin{bmatrix} \frac{B}{D} & \frac{D_{ABCD}}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$$

7.7.4 Converting to ABCD-parameters

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{D_Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix} = \begin{bmatrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{D_Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{bmatrix} = \begin{bmatrix} -\frac{D_h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{bmatrix}$$

N-Port S Parameters

$$\begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdot & \cdot \\ S_{21} & S_{22} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ a_n \end{bmatrix}$$

$$\mathbf{b} = \mathbf{Sa}$$

If $b_i = 0$, then no reflected wave on port *i* ➔ port is matched

$$a_i = \frac{V_i^+}{\sqrt{Z_{oi}}}$$

$$b_i = \frac{V_i^-}{\sqrt{Z_{oi}}}$$

V_i^+ **: incident voltage wave in port *i***

V_i^- **: reflected voltage wave in port *i***

Z_{oi} **: impedance in port *i***

N-Port S Parameters

$$\mathbf{v} = \sqrt{Z_o}\left(\mathbf{a}+\mathbf{b}\right) \quad (1)$$

$$\mathbf{i} = \frac{1}{\sqrt{Z_o}}\left(\mathbf{a}-\mathbf{b}\right) \quad (2)$$

$$\mathbf{v} = \mathbf{Z}\mathbf{i} \quad (3)$$

Substitute (1) and (2) into (3)

$$\sqrt{Z_o}\left(\mathbf{a}+\mathbf{b}\right) = \mathbf{Z}\frac{1}{\sqrt{Z_o}}\left(\mathbf{a}-\mathbf{b}\right)$$

Defining S such that b = Sa and substituting for b

$$Z_o\left(\mathbf{U}+\mathbf{S}\right)\mathbf{a} = Z_o\left(\mathbf{U}-\mathbf{S}\right)\mathbf{a}$$

U : unit matrix

S→Z

$$\mathbf{Z} = Z_o\left(\mathbf{U}+\mathbf{S}\right)\left(\mathbf{U}-\mathbf{S}\right)^{-1}$$

Z→S

$$\mathbf{S} = \left(\mathbf{Z}+Z_o\mathbf{U}\right)^{-1}\left(\mathbf{Z}-Z_o\mathbf{U}\right)$$

N-Port S Parameters

If the port reference impedances are different, we define k as

$$\mathbf{k} = \begin{bmatrix} \sqrt{Z_{o1}} & & & \\ & \sqrt{Z_{o2}} & & \\ & & \cdot & \\ & & & \sqrt{Z_{on}} \end{bmatrix}$$

$\mathbf{v} = \mathbf{k}(\mathbf{a}+\mathbf{b})$ and $\mathbf{i} = \mathbf{k}^{-1}(\mathbf{a}-\mathbf{b})$ and $\mathbf{k}(\mathbf{a}+\mathbf{b}) = \mathbf{Z}\mathbf{k}^{-1}(\mathbf{a}-\mathbf{b})$

Z→S

$$\mathbf{S} = \left(\mathbf{Z}\mathbf{k}^{-1}+\mathbf{k}\right)\left(\mathbf{Z}\mathbf{k}^{-1}-\mathbf{k}\right)$$

S→Z

$$\mathbf{Z} = \mathbf{k}\left(\mathbf{U}+\mathbf{S}\right)\left(\mathbf{U}-\mathbf{S}\right)^{-1}\mathbf{k}$$

Normalization

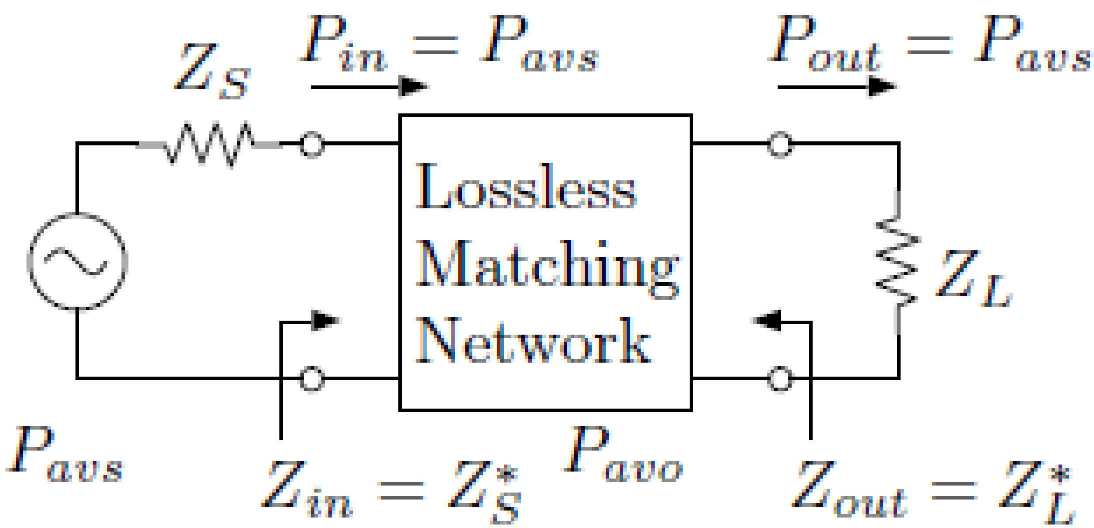
Assume original S parameters as S_1 with system k_1. Then the representation S_2 on system k_2 is given by

Transformation Equation

$$\mathbf{S}_2 = \left[\mathbf{k}_1(\mathbf{U}+\mathbf{S}_1)(\mathbf{U}-\mathbf{S}_1)^{-1}\mathbf{k}_1\mathbf{k}_2+\mathbf{k}_2\right]^{-1}\left[\mathbf{k}_1(\mathbf{U}+\mathbf{S}_1)(\mathbf{U}-\mathbf{S}_1)^{-1}\mathbf{k}_1\mathbf{k}_2-\mathbf{k}_2\right]$$

If Z is symmetric, S is also symmetric

Power Definitions



P_{in}: Power delivered to input of 2-port

P_{out}: Power delivered to the load

P_{avs}: Power available from the source

Power Gain Definitions

Operating Power Gain

$$G = \frac{\text{Power delivered to load}}{\text{Power delivered to input of 2-port}} = \frac{P_{out}}{P_{in}}$$

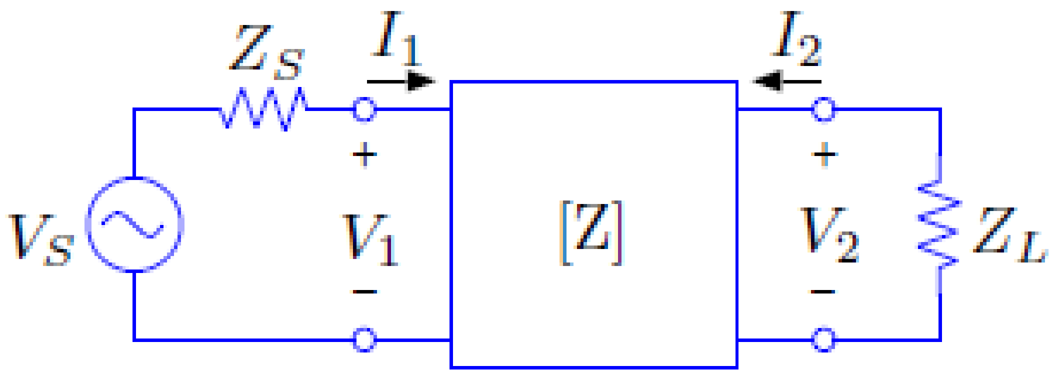
Transducer Power Gain

$$G_T = \frac{\text{Power delivered to load}}{\text{Power available from source}} = \frac{P_{out}}{P_{avs}}$$

Available Power Gain

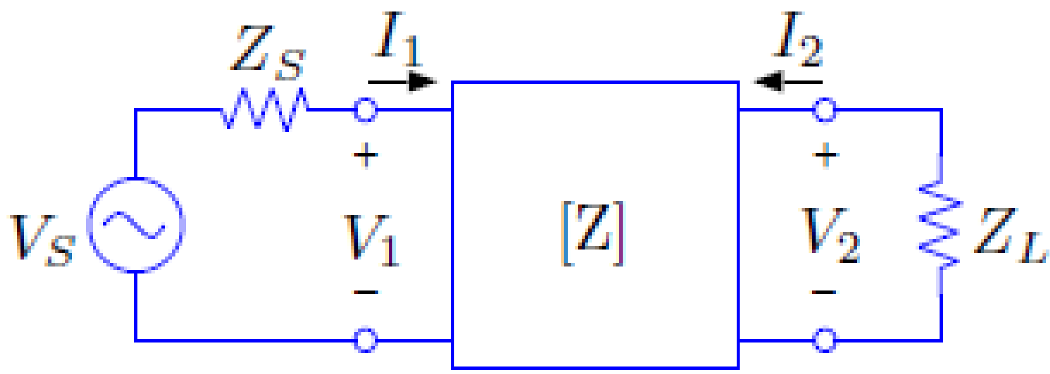
$$G_A = \frac{\text{Power available from output}}{\text{Power available from source}} = \frac{P_{avo}}{P_{avs}}$$

Power Available from a Source



$$P_{avs} = \frac{|V_S|^2}{8R_S}$$

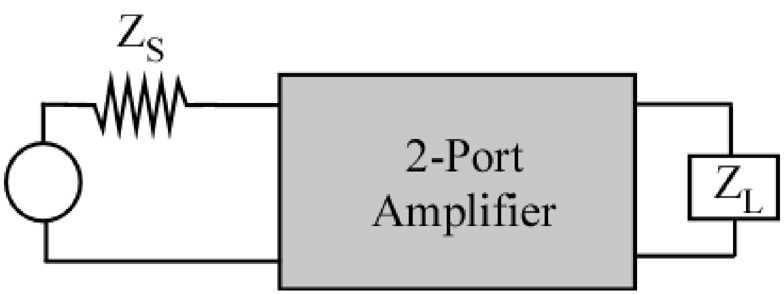
Transducer Gain with Z-Parameters



$$G_T = 4\frac{|Z_{21}|^2 R_L R_S}{\left|(Z_{11}+Z_S)(Z_{22}+Z_L)-Z_{12}Z_{21}\right|^2}$$

Linear Amplifiers

The transducer power gain is defined as the power delivered to the load divided by the power available from the source.



$$P_{avs} = \frac{|b_s|^2}{1-|\Gamma_S|^2}$$

Transducer Gain

Definition of transduced gain

$$G_T = \frac{P_{del}}{P_{avs}} = \frac{|b_2|^2\left(1-|\Gamma_L|^2\right)}{|b_s|^2 / \left(1-|\Gamma_S|^2\right)}$$

In terms of two-port scattering parameters

$$G_T = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{\left|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_S\Gamma_L\right|^2}$$

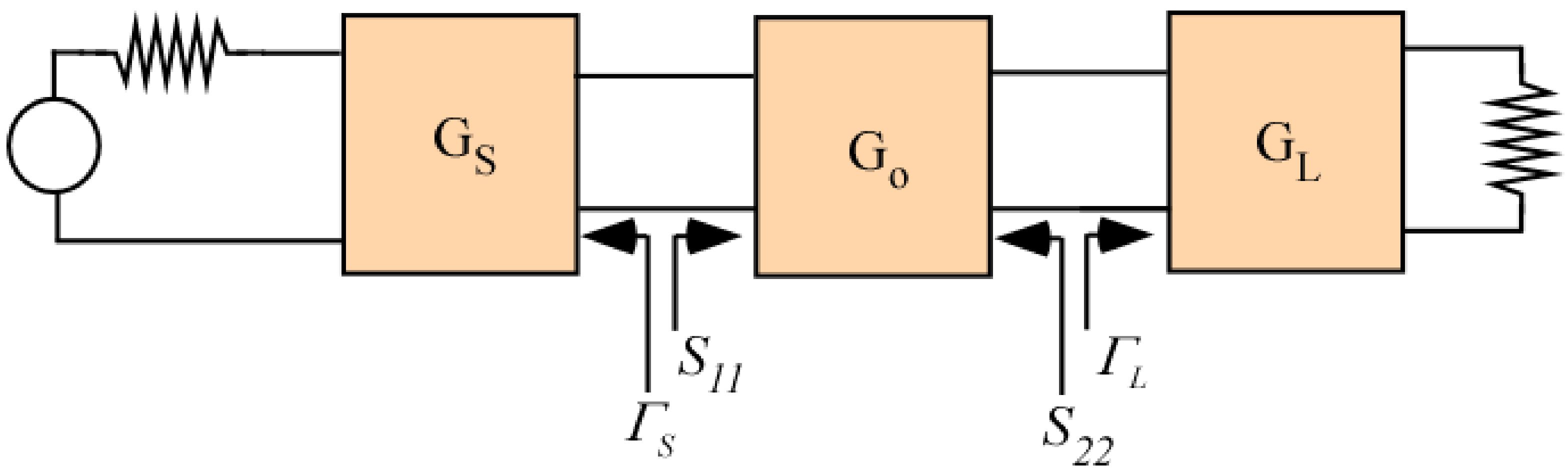
Linear Amplifiers

If we assume that the network is unilateral, then we can neglect S_{12} and get the unilateral transducer gain for $S_{12}=0$.

$$G_{TU} = \left|S_{21}\right|^2 \frac{\left(1-\left|\Gamma_S\right|^2\right)}{\left|1-S_{11}\Gamma_S\right|^2} \frac{\left(1-\left|\Gamma_L\right|^2\right)}{\left|1-S_{22}\Gamma_L\right|^2}$$

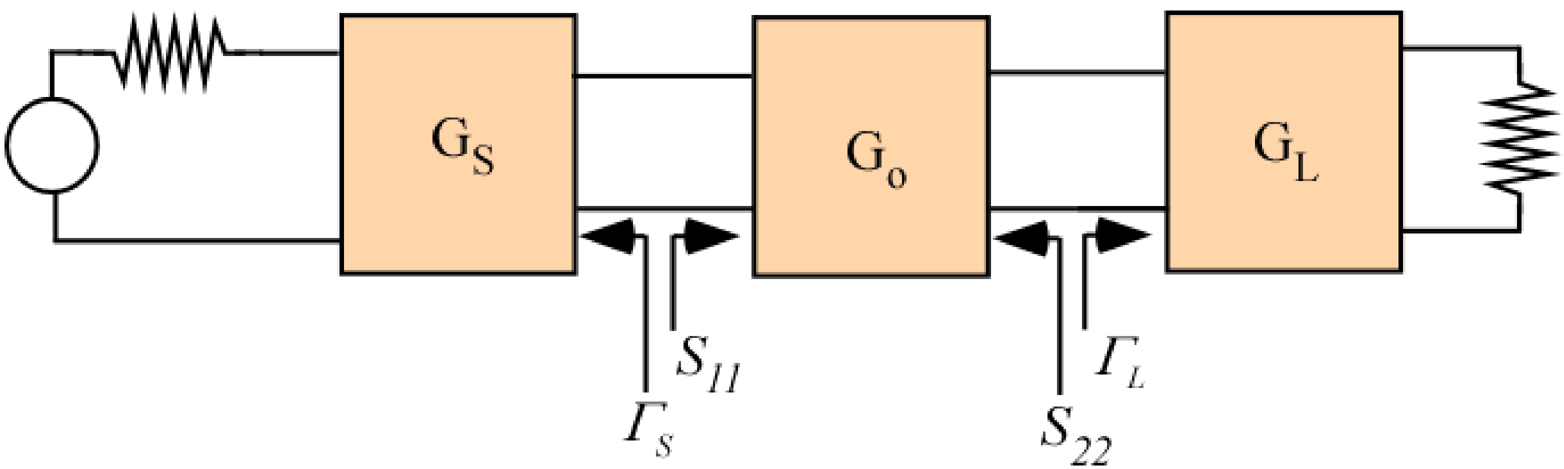
The first term ($|S_{21}|^2$) depends on the transistor. The other 2 terms depend on the source and the load.

Linear Amplifiers



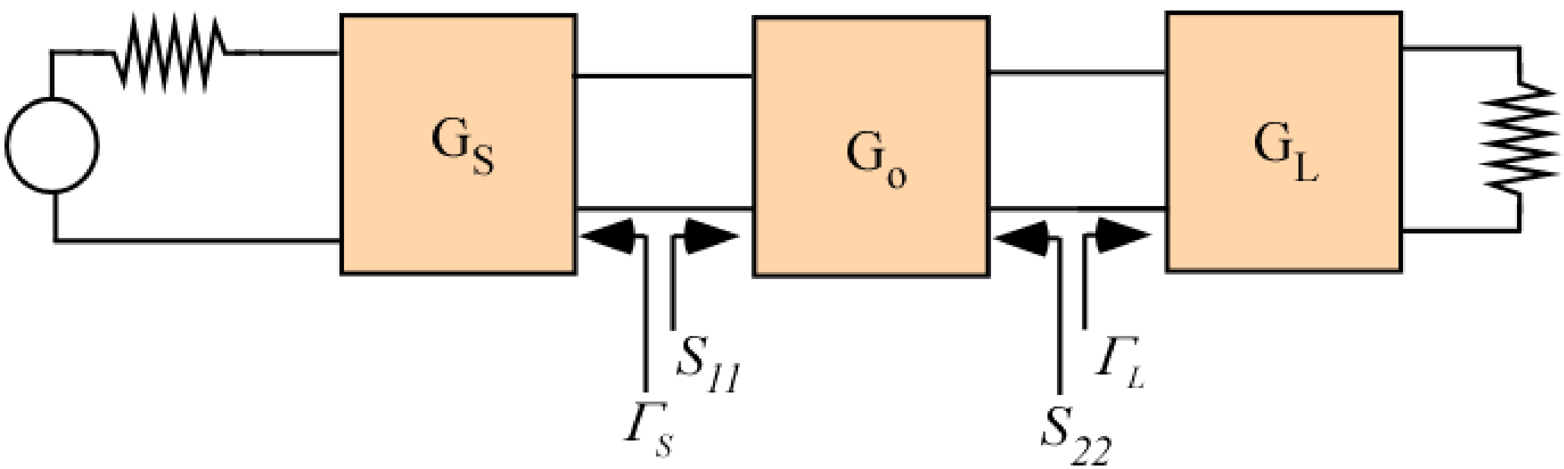
G_s affects the degree of mismatch between the source and the input reflection coefficient of the two-port.

Linear Amplifiers



G_L affects the degree of mismatch between the load and the output reflection coefficient of the 2-port.

Linear Amplifiers



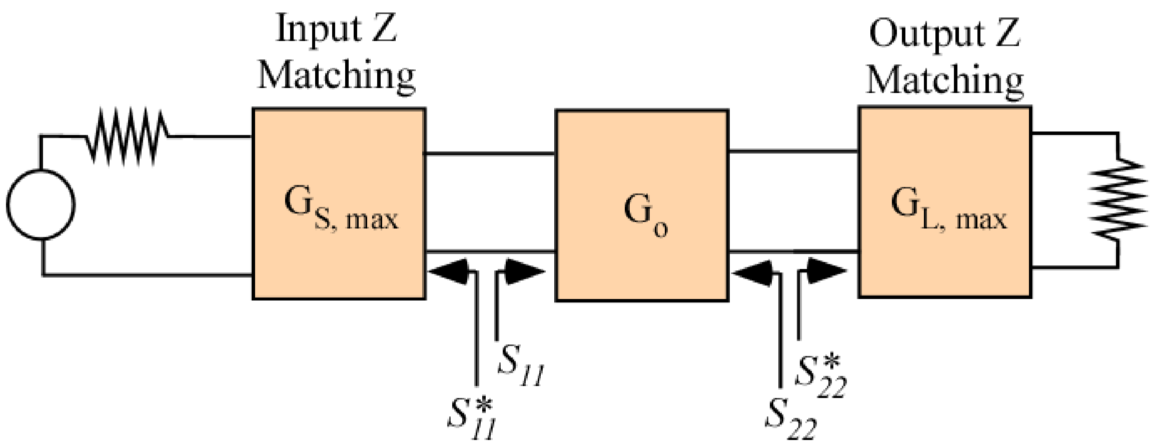
G_o depends on the device and bias conditions

Linear Amplifiers

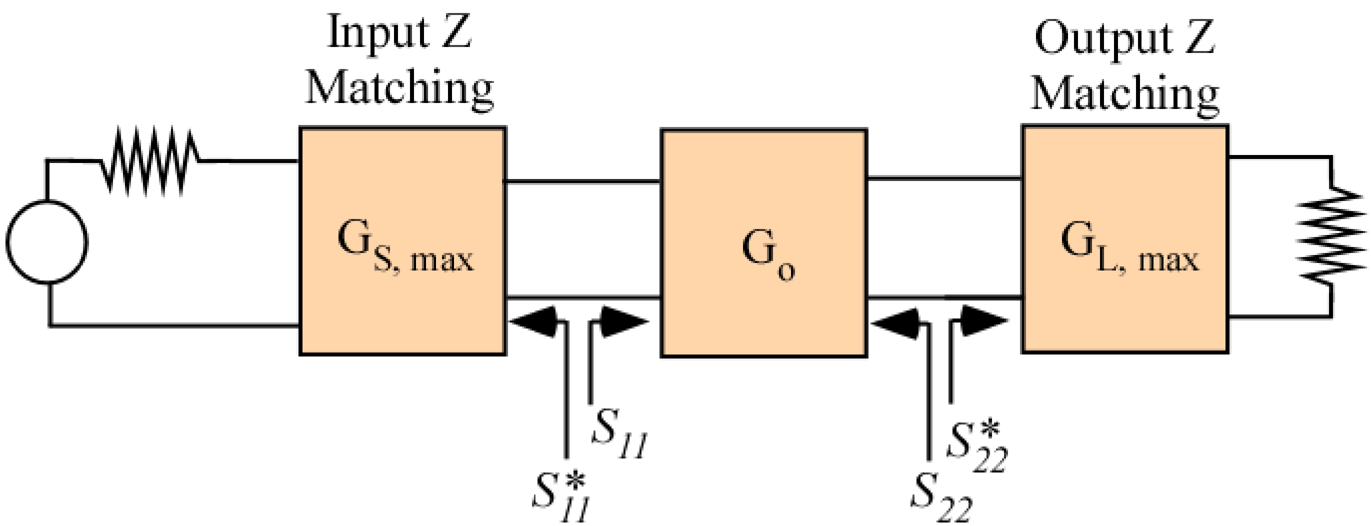
Maximum unilateral transducer gain can be accomplished by choosing impedance matching networks such that.

$$\Gamma_S = S_{11}^* \qquad \Gamma_S = S_{22}^*$$

$$G_{UMAX} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2}$$



Linear Amplifiers



$$G_{UMAX}(dB) = G_{S\max}(dB) + G_o(dB) + G_{L\max}(dB)$$

For $\Gamma_S = S_{11}^*$, **G_S is a maximum**

For $|\Gamma_S| = 1$, **G_S is 0**

Dissipated Power

$$P_d = \frac{1}{2}\mathbf{a}^T\left(\mathbf{U} - \mathbf{S}^T\mathbf{S}^*\right)\mathbf{a}^*$$

The dissipation matrix D is given by:

$$\mathbf{D} = \mathbf{U} - \mathbf{S}^T\mathbf{S}^*$$

Passivity insures that the system will always be stable provided that it is connected to another passive network

For passivity

- **(1) the determinant of D must be** ≥ 0
- **(2) the determinant of the principal minors must be** ≥ 0

Dissipated Power

When the dissipation matrix is 0, we have a lossless network➔

$$\mathbf{S}^T\mathbf{S}^* = \mathbf{U}$$

The S matrix is unitary.

For a lossless two-port:

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

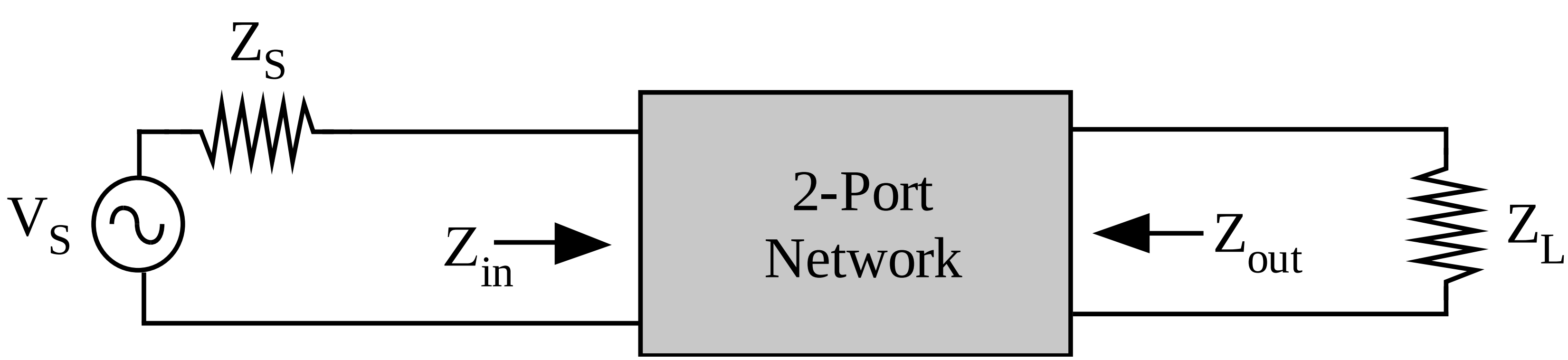
$$|S_{22}|^2 + |S_{12}|^2 = 1$$

If in addition the network is reciprocal, then

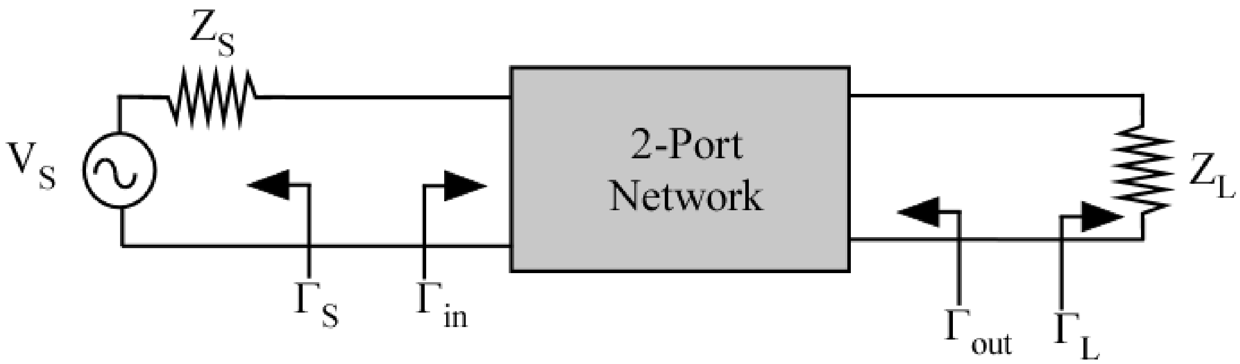
$$S_{12} = S_{21} \text{ and } |S_{11}| = |S_{22}| = \sqrt{1-|S_{12}|^2}$$

Stability Considerations

Before maximizing transducer gain, and perform conjugate match, it is necessary to study stability of two-port



Reflection Coefficients



Input reflection coefficient associated with Z_{in}

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

Output reflection coefficient associated with Z_{out}

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

Stability

A network is **conditionally stable** if the real part of Z_{in} and Z_{out} is greater than zero for ***some*** positive real source and load impedances at a specific frequency

A network is **unconditionally stable** if the real part of Z_{in} and Z_{out} is greater than zero for ***all*** positive real source and load impedances at a specific frequency

Stability Factor

Positive real source and load impedances imply that

$$|\Gamma_S| \; and \; |\Gamma_L| \leq 1$$

If we want to match input and output for maximum power transfer, we have

$$\Gamma_S = \Gamma_{in}^* \qquad \Gamma_L = \Gamma_{out}^*$$

The *K* or *Rollet Stability Factor* for stability requires that

$$K = \frac{1 + |S_{11}S_{22} - S_{12}S_{21}|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}||S_{21}|} > 1$$

***K* factor must not be considered alone**

Stability Circle

$$\left|\Gamma_{in}\right| = \left|S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L}\right| = 1$$

The solution for Γ_L will lie on a circle

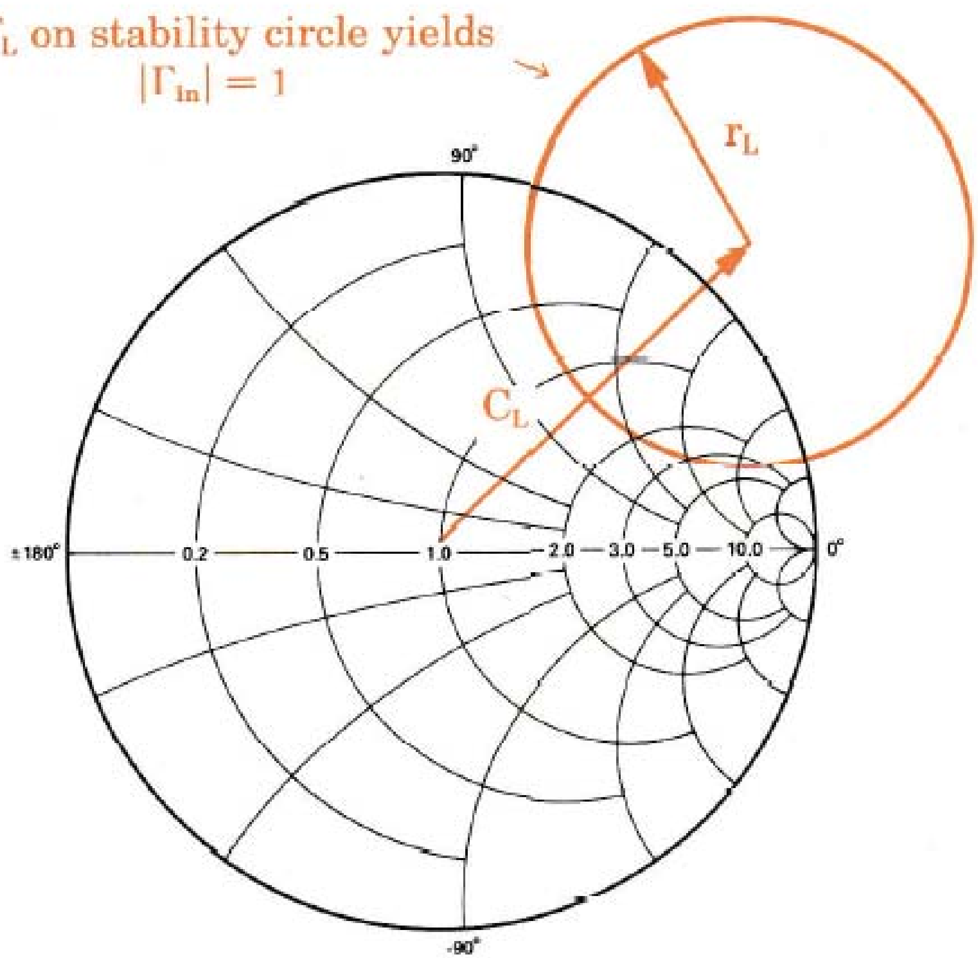
$$radius = r_L = \left|\frac{S_{21}S_{12}}{\left|S_{22}\right|^2 - \left|\Delta\right|^2}\right|$$

$$center = C_L = \frac{\left(S_{22} - \Delta S_{11}^*\right)^*}{\left|S_{22}\right|^2 - \left|\Delta\right|^2}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

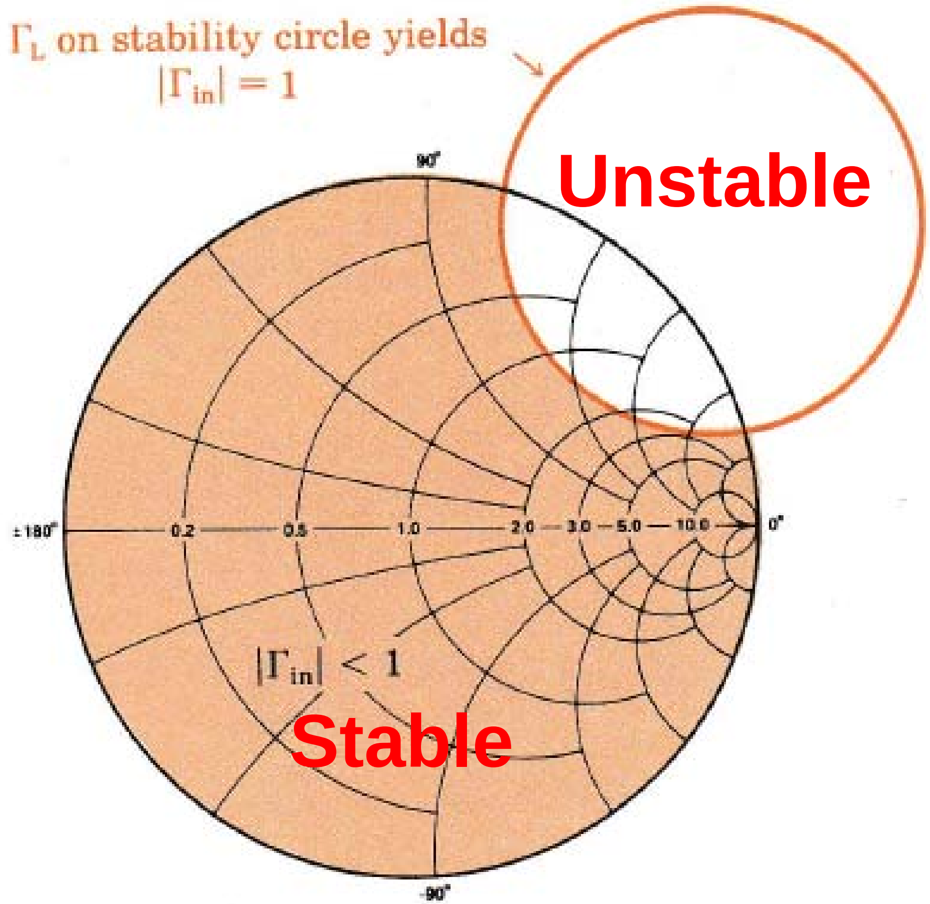
Stability Circle for Γ_L

Area inside or outside stability circle will represent a stable operating condition



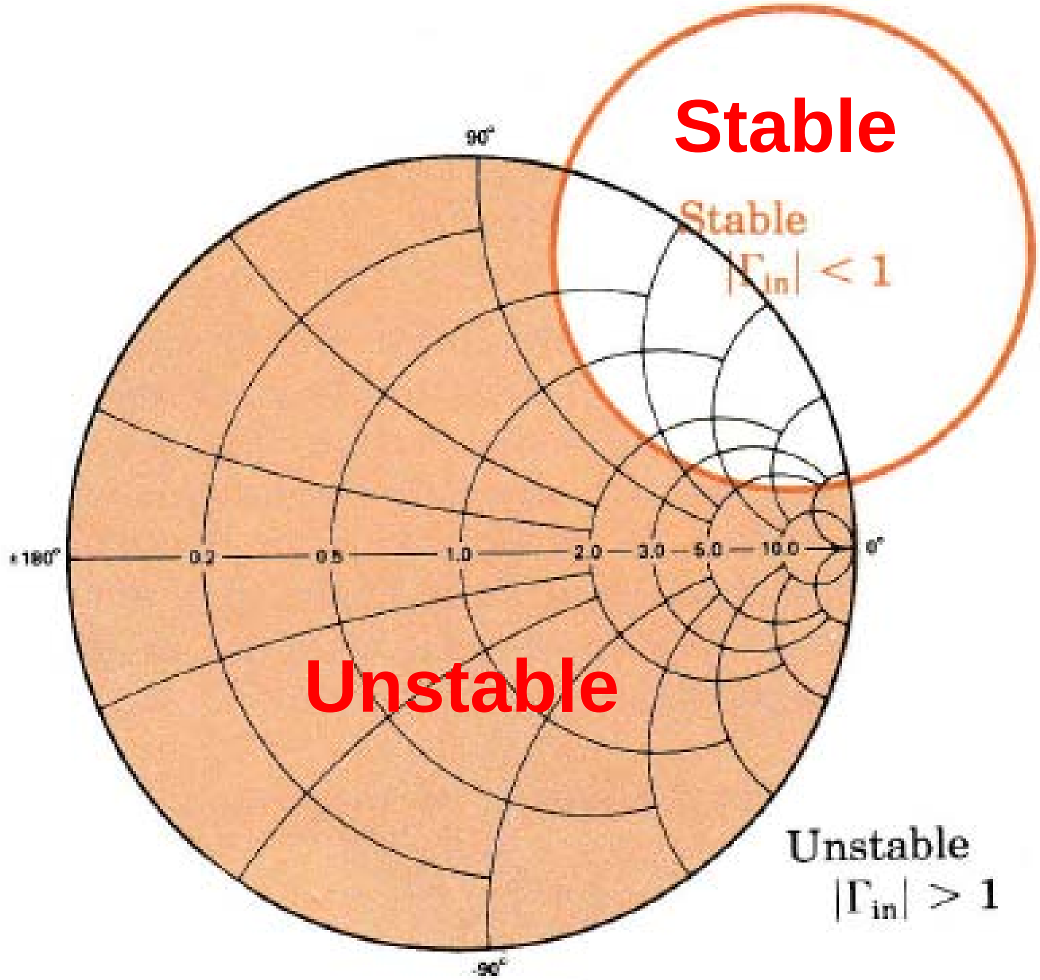
Stability Circle for Γ_L

To determine stable area, make $Z_L = Z_o$ or $\Gamma_L = 0$. If $|\Gamma_{in}| < 1$, then area corresponding to center of Smith chart is stable.

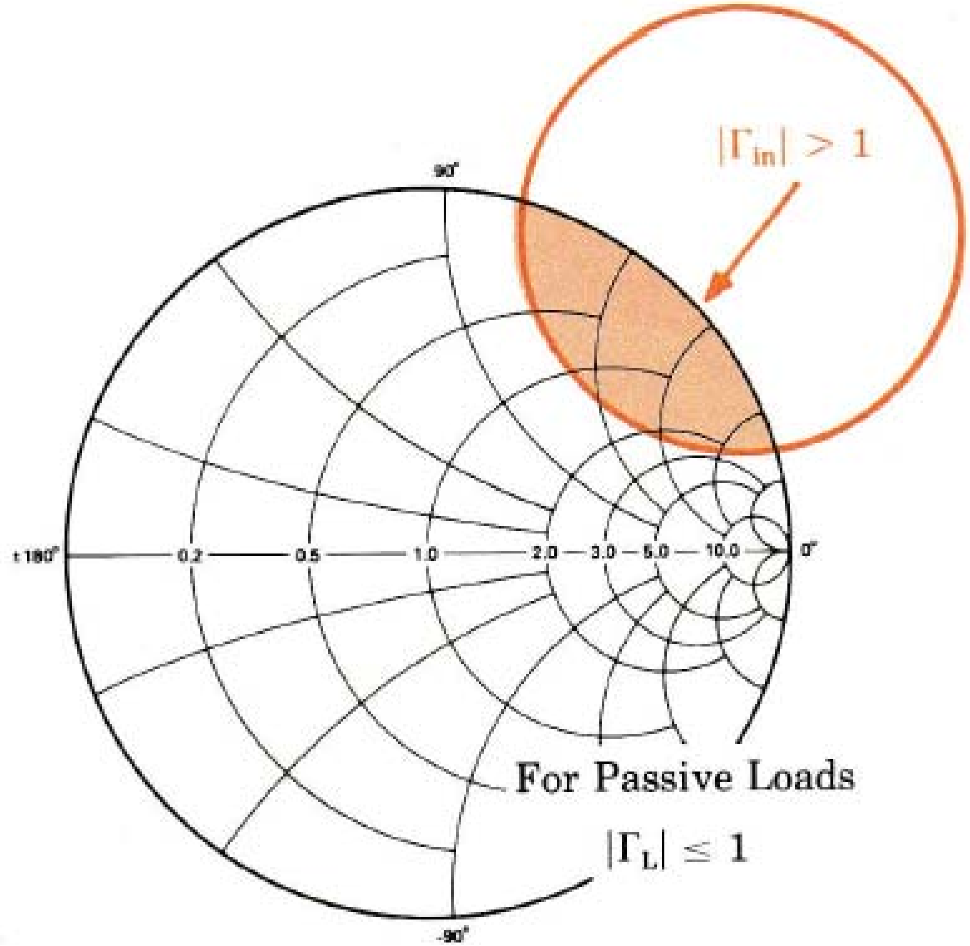


Stability Circle for Γ_L

To determine unstable area, make $Z_L = Z_o$ or $\Gamma_L = 0$. If $|\Gamma_{in}| > 1$, then area corresponding to center of Smith chart is unstable.

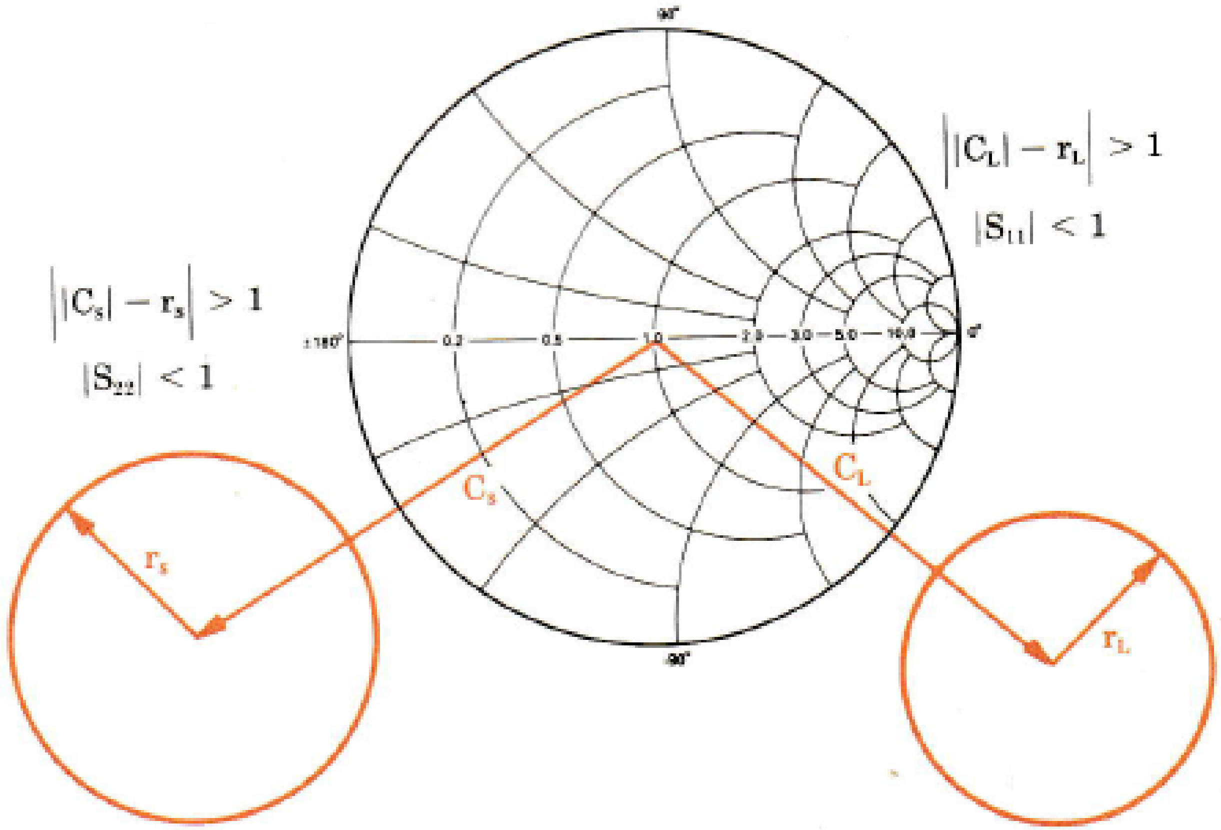


Stability Circle for Γ_L



Unconditional Stability

To insure unconditional stability for any passive load, stability circles must lie completely out of the Smith chart.



Unconditional Stability

1. Case where center of Smith chart is outside of stability circle

$$|S_{22}|^2 - |\Delta|^2 > 0$$

$$\frac{\left|S_{22}^* - \Delta^* S_{11}\right| - \left|S_{12}S_{21}\right|}{\left||S_{22}|^2 - |\Delta|^2\right|} > 1$$

2. Case where center of Smith chart is inside of stability circle

$$|S_{22}|^2 - |\Delta|^2 < 0$$

$$\frac{\left|S_{12}S_{21}\right| - \left|S_{22}^* - \Delta^* S_{11}\right|}{\left||S_{22}|^2 - |\Delta|^2\right|} > 1$$

Unconditional Stability

Both cases can be combined into a single inequality

$$\frac{\left|S_{22}^{*}-\Delta^{*}S_{11}\right|-\left|S_{12}S_{21}\right|}{\left|\left|S_{22}\right|^{2}-\left|\Delta\right|^{2}\right|}>1$$

which is valid for either case

Unconditional Stability

Criteria for unconditional stability

$$K > 1, \quad |S_{12}S_{21}| < 1 - |S_{11}|^2$$

$$K > 1, \quad |S_{12}S_{21}| < 1 - |S_{22}|^2$$

$$K > 1, \quad B_1 > 0$$

$$K > 1, \quad B_2 > 0$$

$$K > 1, \quad |D| < 1$$

$$\mu_{ES} = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* D| + |S_{12}S_{21}|} > 1$$

$$\mu_{ES}' = \frac{1 - |S_{22}|^2}{|S_{11} - S_{22}^* D| + |S_{12}S_{21}|} > 1$$

$$B_1 = 1 + |S_{11}|^2 - |D|^2 - |S_{22}|^2$$

$$B_2 = 1 + |S_{22}|^2 - |D|^2 - |S_{11}|^2$$

$$D = S_{11}S_{22} - S_{12}S_{21}$$

$$K = \frac{1 + |S_{11}S_{22} - S_{12}S_{21}|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}||S_{21}|} > 1$$

Stability Circle for Γ_L

Stability circles are functions of frequency.