ECE 453
Wireless Communication Systems

Noise

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Noise

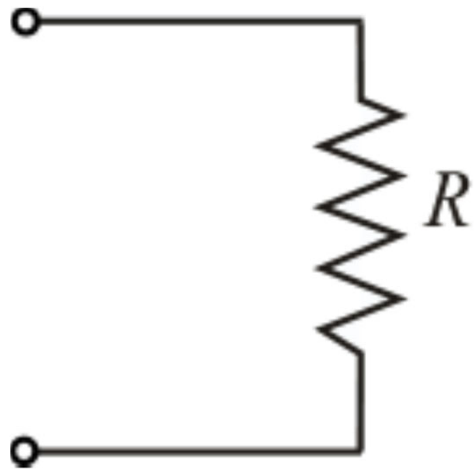
Random fluctuations of voltage and current

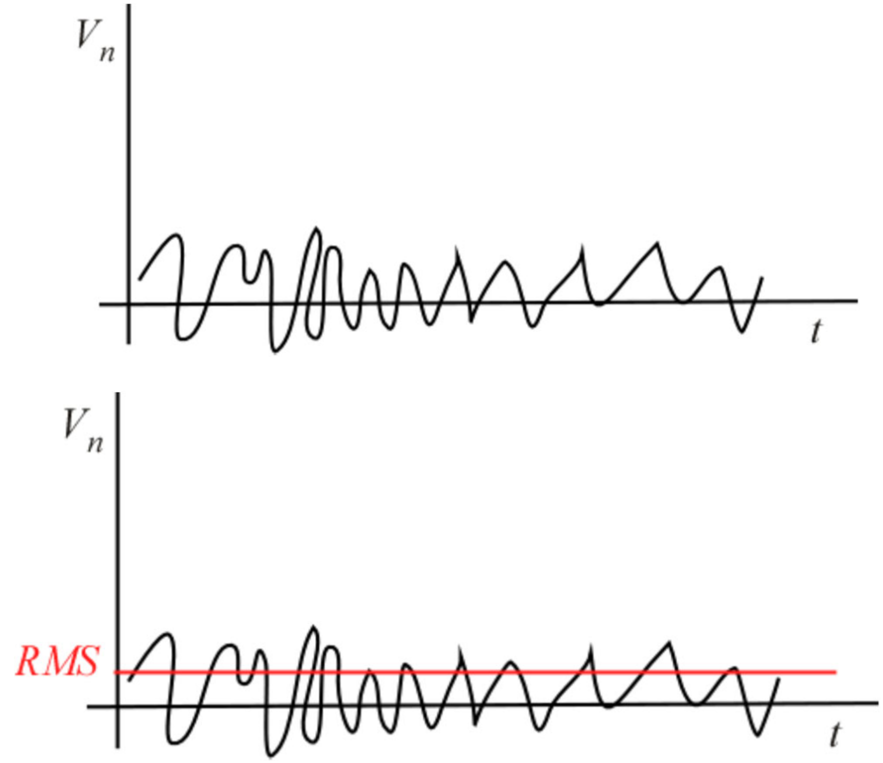
Properties

- Establishes minimum detectable signal
- Sets bounds on range of signals that can be processed
- Comes in different types (thermal, flicker, shot)
- Has frequency and time-domain characteristics
- Not fully understood

Noise from a Resistor

Random fluctuations of charge carriers in a conductor will produce thermal noise





At room temperature a resistor has available noise power ~ -174 dBm in 1 Hz of bandwidth

Nyquist Noise Formula

The spectral density of noise voltage at the terminals of a resistor is given by

$$S_n(f) = 4R\frac{hf}{e^{hf/kT} - 1}\left(\text{Volts}^2 / \text{Hz}\right)$$

f = frequency in Hz
h = Plank's constant = 6.62 x 10^{-34} joules-sec
k = Boltzman's constant =1.38 x 10^{-23} joules/^{o}K
T = absolute temperature ^{o}K
R = resistance in ohms

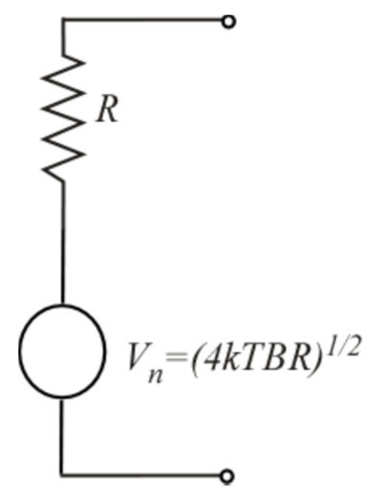
Noise from a Resistor

If $f << kT/h$, then S_n can be approximated as:

$$S_n = 4kTR \text{ (Volts}^2 \text{ / Hz)}$$

Noise voltage at the terminals of a resistor is given by

$$V_n^2 = 4kTBR$$

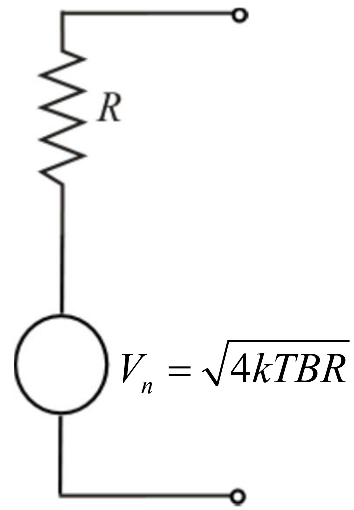


B = bandwidth in Hz

Noise from a Resistor

AC Voltmeter

Input resistance = 10 MΩ
Bandwidth = 40 MHz



$$V_n^2 = 4\times 1.38\times 10^{-23}\times 300\times 4\times 10^{6}\times 10^{7}$$

$$V_n^2 = 66.24\times 10^{-8}$$

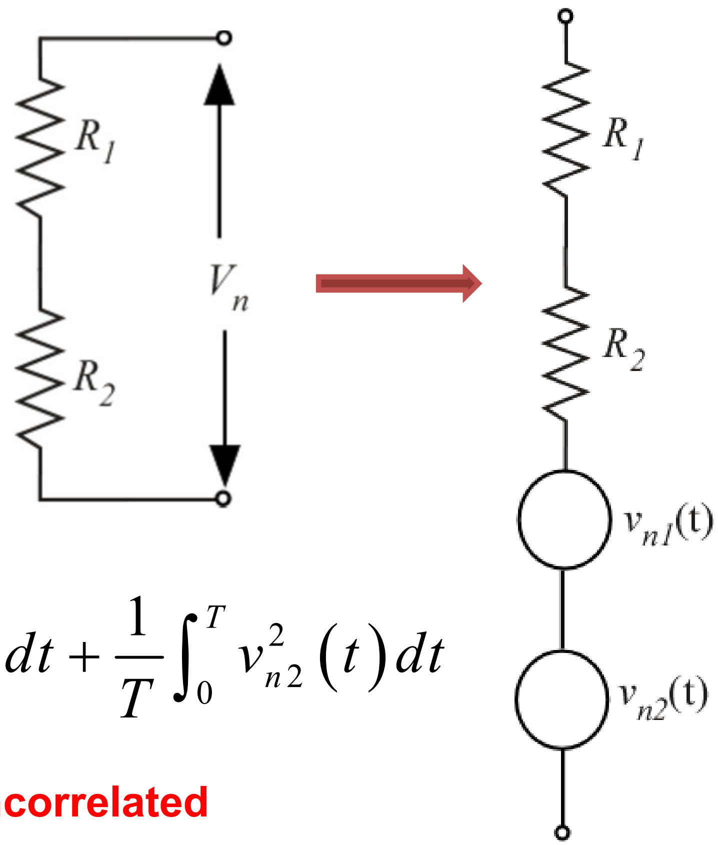
$$V_n \simeq 8\times 10^{-4} = 0.8\, mV$$

Resistors in Series

$$V_{n1}^2 = \frac{1}{T}\int_0^T v_{n1}^2(t)\,dt$$

$$V_{n2}^2 = \frac{1}{T}\int_0^T v_{n2}^2(t)\,dt$$

$$V_{nT}^2 = \frac{1}{T}\int_0^T \left[v_{n1}(t) + v_{n2}(t)\right]^2 dt$$

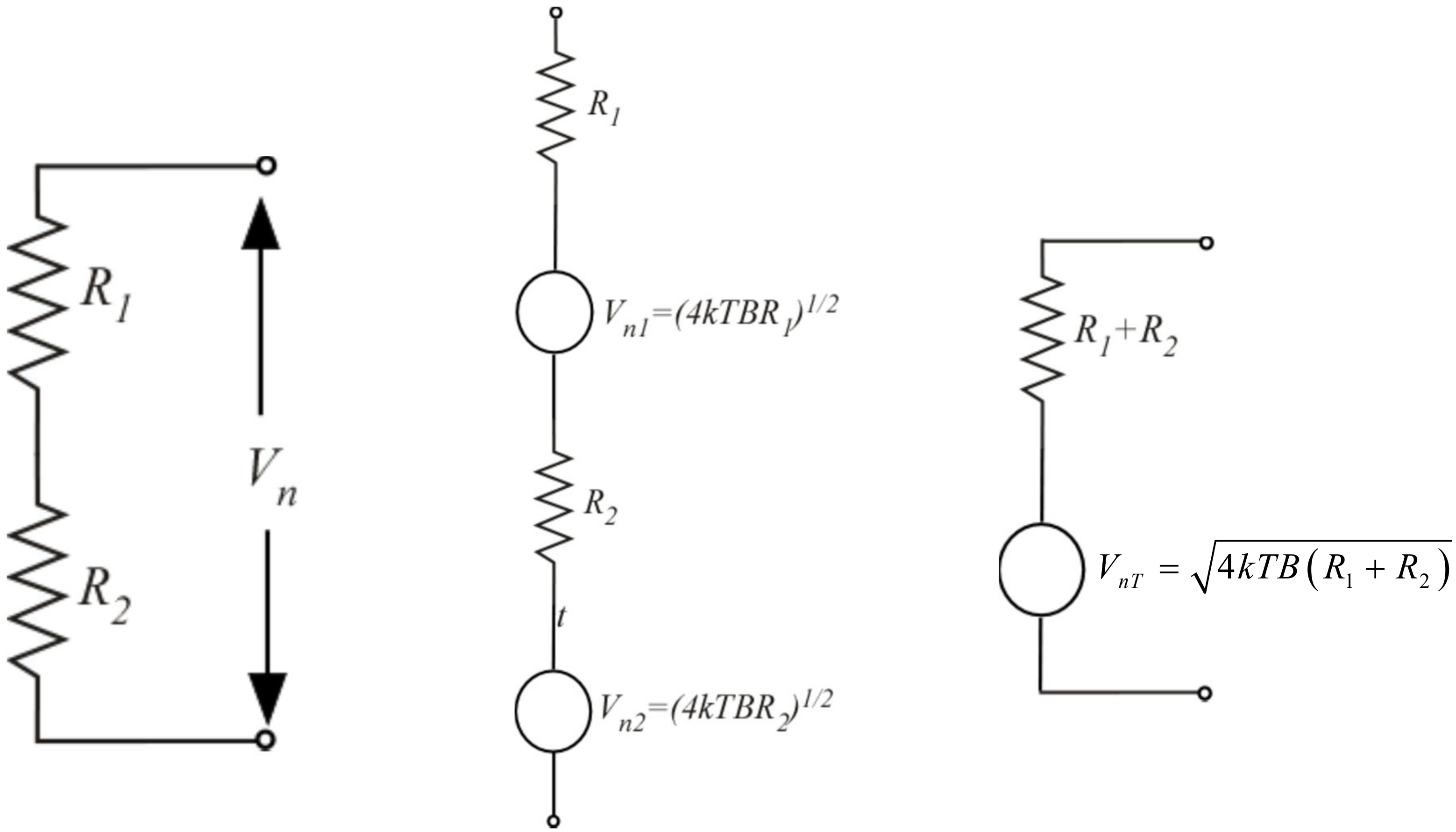


$$V_{nT}^2 = \frac{1}{T}\int_0^T v_{n1}^2(t)\,dt + \frac{1}{T}\int_0^T 2v_{n1}(t)v_{n2}(t)\,dt + \frac{1}{T}\int_0^T v_{n2}^2(t)\,dt$$

0 if v_{n1} and v_{n2} are uncorrelated

$$V_{nT}^2 = V_{n1}^2 + V_{n2}^2 = 4kTBR_1 + 4kTBR_2$$

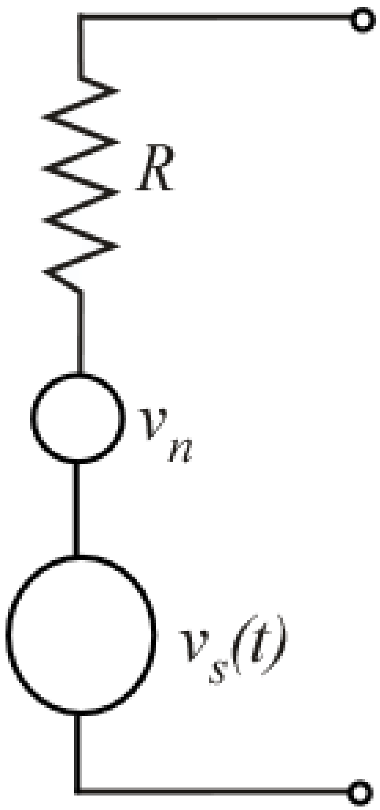
Resistors in Series



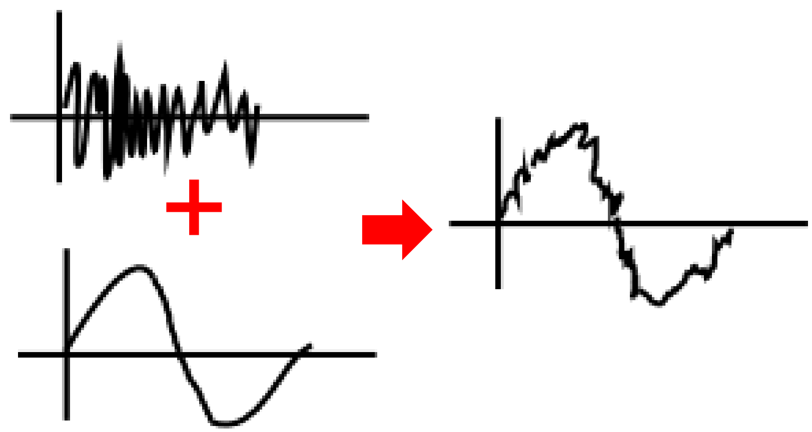
$$V_{nT} = \sqrt{4kTB\left(R_1 + R_2\right)}$$

$$V_{nT}^2 = V_{n1}^2 + V_{n2}^2 \;\; for\ 2\ uncorrelated\ noise\ sources$$

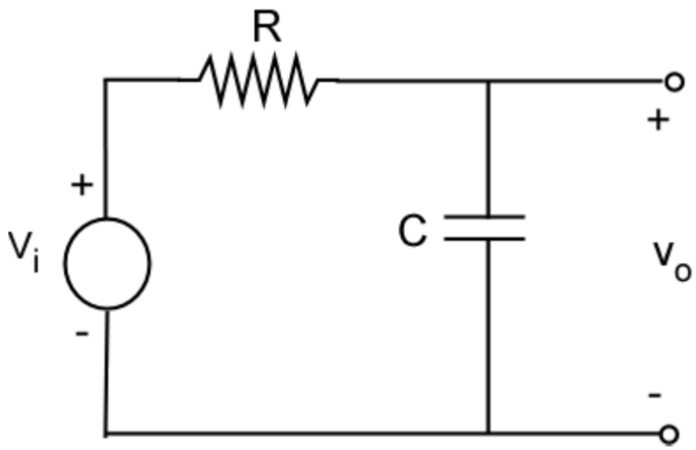
Effective Value of Noise Sources

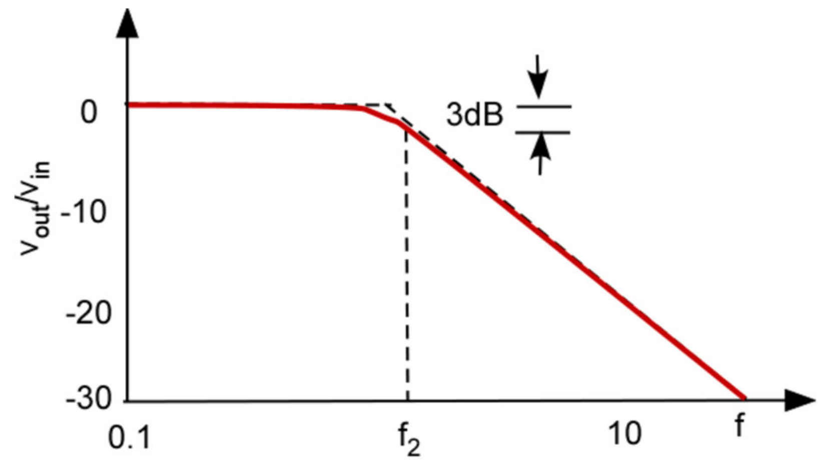


$$V_{nT}^2 = V_n^2 + V_S^2$$



Bandwidth





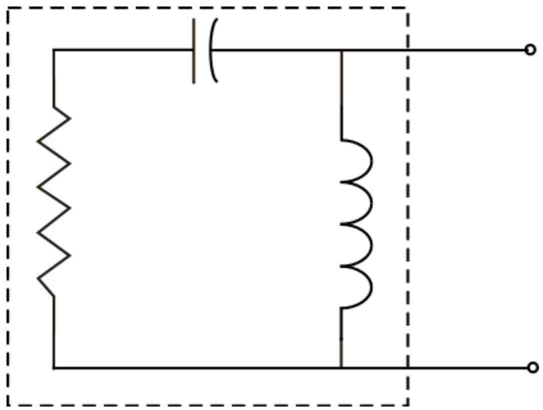
$$\frac{V_o}{V_{in}} = \frac{1}{1 + j\omega RC}$$

The output voltage drops to 0.707 of its low-frequency value when $\omega RC = 1$

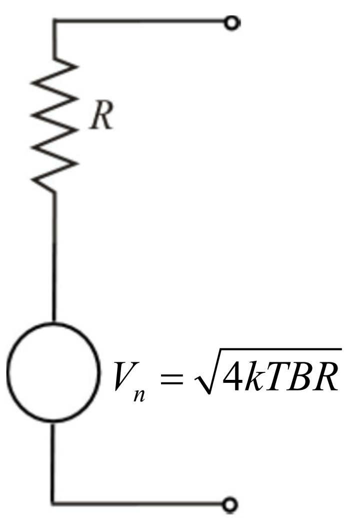
The half-power frequency is given by $2\pi f_2 RC = 1$

➔ $f_2 = 1/2\pi RC$

Noise from a Circuit



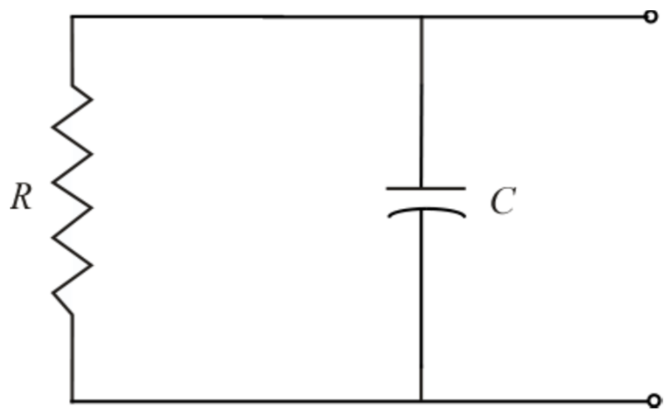
$$V_n^2 = 4kT\left[\int_0^\infty R(f)df = BR\right]$$

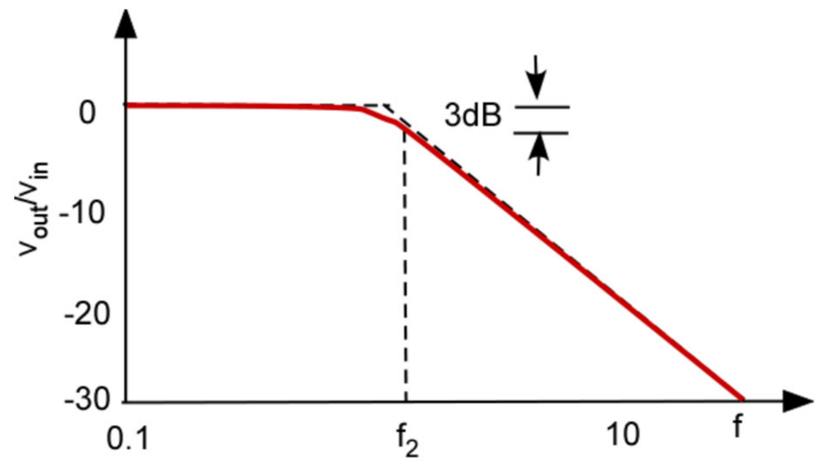


R(f) is the real part of the impedance at each frequency

Noise from a Circuit

For an RC circuit





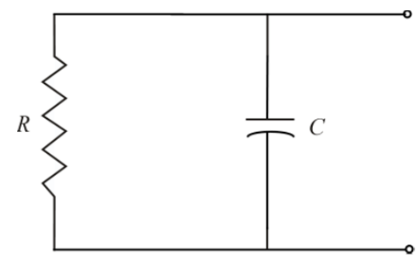
For coherent signals, the bandwidth of an *RC* circuit is

$$f_2 = \frac{1}{2\pi RC}$$

$|Z|$ is *0.707R* when $\omega_2 R = 1/R$ ➔ $2\pi f_2 = 1/RC$

RC Circuit

The noise from the *RC* circuit is given by



$$V_n^2 = 4kT\int_0^\infty R(f)\,df$$

$$R(f) = Re(Z) = \frac{G}{G^2+\omega^2 C^2}$$

$$Z = \frac{G - j\omega C}{G + j\omega C}$$

In terms of $\omega_2 = 1/RC$,

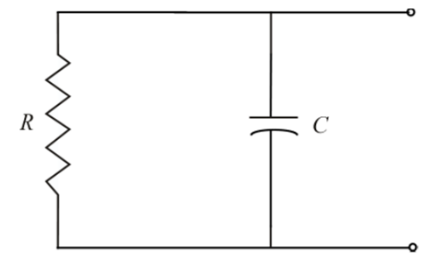
$$R(f) = \frac{1}{R\left[\dfrac{1}{R^2} + \dfrac{\omega^2}{\omega_2^2 R^2}\right]}$$

$$R(f) = \frac{R}{\left[1 + \dfrac{\omega^2}{\omega_2^2}\right]}$$

RC Circuit

The noise from the *RC* circuit is given by

$$V_n^2 = 4kT\int_0^\infty R(f)\,df = \frac{4kT}{2\pi}\int_0^\infty \frac{R}{1+\frac{\omega^2}{\omega_2^2}}\,d\omega$$



Using $\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab}tan^{-1}\frac{bx}{a}$

$$V_n^2 = \frac{4kTR}{2\pi}\omega_2\frac{\pi}{2} = 4kTRf_2\frac{\pi}{2}$$

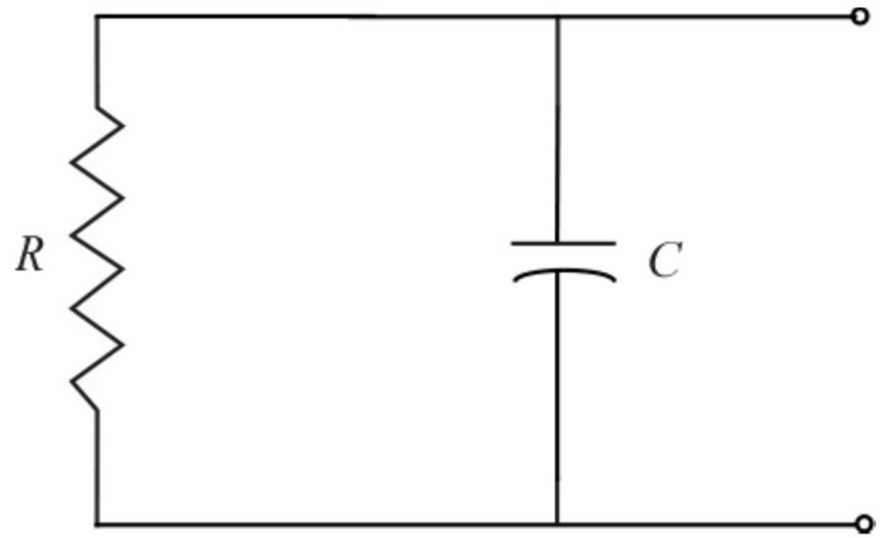
The coherent signal bandwidth is from 0 to f_2

RC Circuit

For an *RC* circuit,

$$V_n^2 = \frac{4kTR}{2\pi}\omega_2\frac{\pi}{2} = 4kTRf_2\frac{\pi}{2}$$

where $$f_2 = \frac{1}{2\pi RC}$$

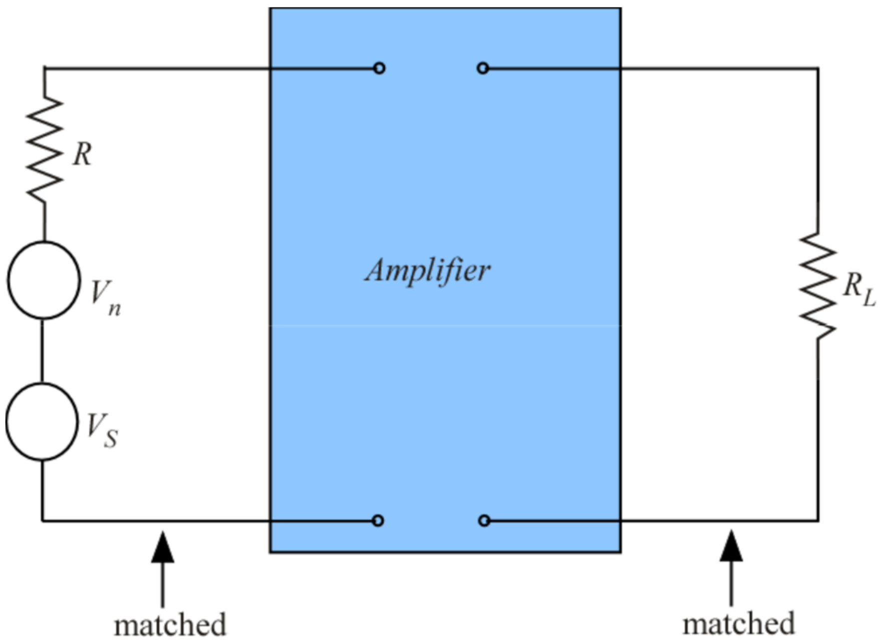


Consequently,

$$V_n^2 = 4kTR\frac{\pi}{2\pi RC2} = \frac{kT}{C}$$

Independent of resistance!

Noise Factor

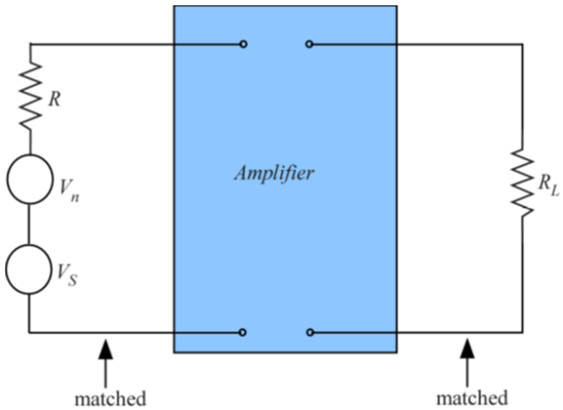


The noise at the input N_{in} is given by

$$N_{in} = \frac{V_n^2}{R_{in}} = \frac{4kTBR}{(2)^2 R} = kTB$$

S_{in} – Signal power into the amplifier
N_{in} – Noise power into the amplifier
S_o – Signal power at the output
N_o – Noise power at the output

Noise Factor



Let N_{int} be the noise power generated in the amplifier referenced to the amplifier input

The total noise power at the amplifier output is:

$$N_o = N_{in}G + N_{int}G$$

where G is the power gain. We define the noise factor as:

$$Noise\ Factor = F = \frac{S_{in}/N_{in}}{S_o/N_o} > 1$$ if noise is generated by the amplifier

Noise Figure

$$F = \frac{S_{in} / N_{in}}{S_o / N_o} = \frac{S_{in}}{N_{in}} \times \frac{N_o}{S_o} = \frac{S_{in}}{N_{in}} \times \frac{N_o}{S_{in} G}$$

$$F = \frac{N_o}{kTBG} = \frac{N_{in} G + N_{int} G}{kTBG}$$

$$F\left(kTB\right) = kTB + N_{int}$$

$$N_{int} = \left(F - 1\right) kTB$$

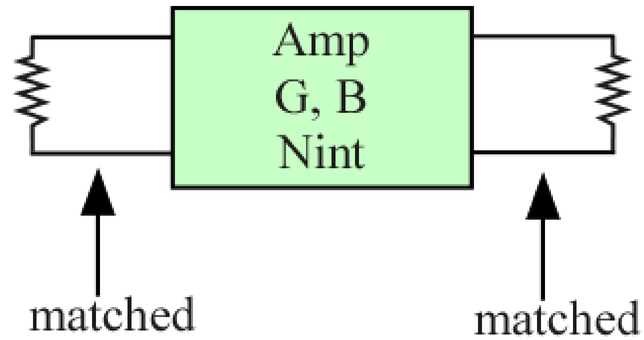
The *Noise Figure* is the noise factor expressed in decibels

$$NF = 10 \log(F)$$

Noise Figure

$$F = \frac{S_{in}/N_{in}}{S_o/N_o} = \frac{S_{in}}{S_o} \times \frac{N_o}{N_{in}} = \frac{1}{G} \times \left[\frac{\left(N_{in} + N_{int}\right)G}{N_{in}} \right]$$

$$F = \frac{1}{G} \times \left[\frac{\left(kTB + N_{int}\right)G}{kTB} \right]$$



$N_{in} = kTB$ if matched

To have a standard value for F, we use $T = 290^{o}$ for the temperature of the resistor giving the N_{in}

$$F\left(G\right)kTB = \left(kTB + N_{int}\right)G$$

$$\left(F - 1\right)kTB = N_{int}$$

Noise Figure - Example

Receiver

***NF*=9 dB, *BW*=3 kHz, R_o = 500 Ω, R_{in} = 50 Ω, $G = 10^8$**

$$NF = 10\,log_{10}\frac{S_{in}/N_{in}}{S_o/N_o}$$

What input signal voltage do we need to get $S_o/N_o = 10$?

$$S_o = GS_{in} \qquad N_o = G\,N_o\big|_{in}$$

$$N_o = kTBG + N_{int}G = kTBG + (F-1)kTBG$$

$$N_o\big|_{in} = F(kTB) = 8\times 1.38\times 10^{-23}\times 290\times 3\times 10^{3} = 9.936\times 10^{-17}\ watts$$

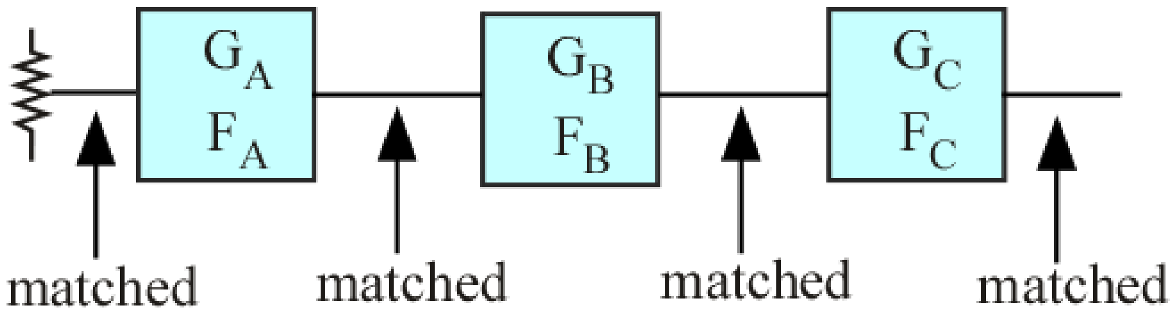
Noise Figure - Example

For $S_o/N_o = 10$, we need

$$S_{in} = 10N_o\Big|_{in} = \frac{V_{in}^2}{R_{in}} = \frac{V_{in}^2}{50\Omega} = 10\times 9.93\times 10^{-17}\ watts$$

$V_{in} = 0.223\ \mu V$

Cascaded Stages



What is the overall Noise Factor?

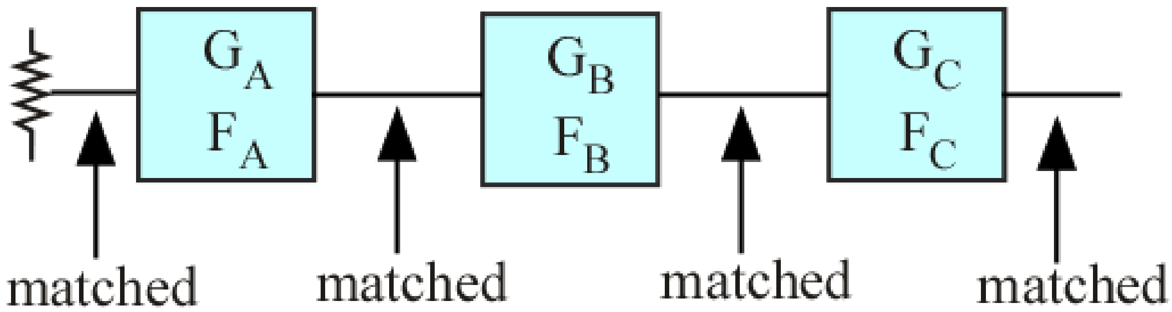
$$N_{output} = kTBG_A G_B G_C + (F_A - 1)kTBG_A G_B G_C$$
$$+(F_B - 1)kTBG_B G_C + (F_C - 1)kTBG_C$$

F of 3 stages

$$F = \frac{N_o}{GN_{in}} = \frac{N_o}{kTBG_A G_B G_C}$$

$$= 1 + (F_A - 1) + \frac{(F_B - 1)}{G_A} + \frac{(F_C - 1)}{G_A G_B}$$

Cascaded Stages



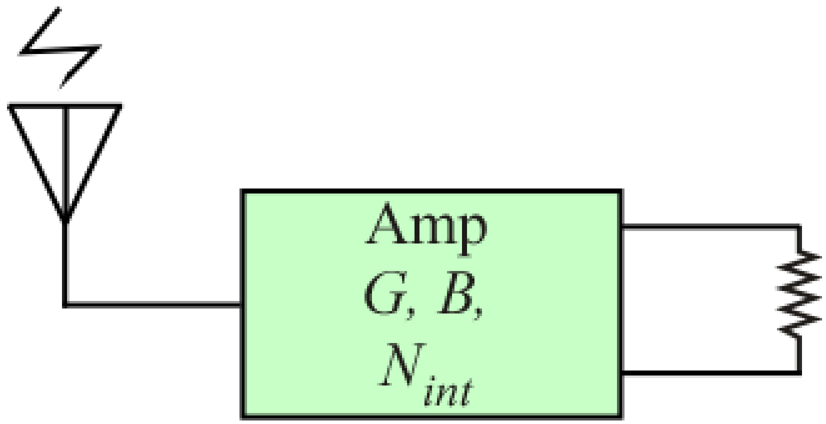
Noise Factor of 3 stages

$$F = F_A + \frac{(F_B - 1)}{G_A} + \frac{(F_C - 1)}{G_A G_B}$$

Frii's formula for *m* cascaded stages

$$F^T = F_1 + \sum_{n=2}^{m} \frac{F_n - 1}{\prod_{i=2}^{n} G_{i-1}}$$

Antenna



$$N_o = N_{int}G + N_A G$$

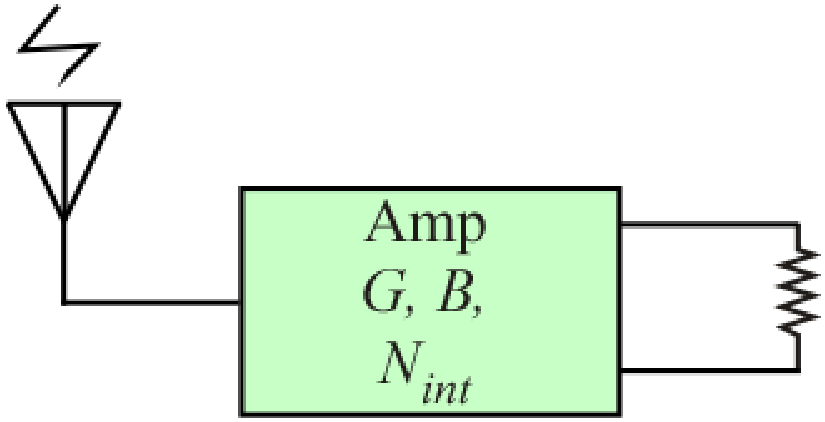
noise from antenna

We can write:

$$N_A = kT_A B$$

T_A = equivalent temperature of the antenna

Antenna



200-Ω antenna exhibiting an rms noise voltage of 0.1 μV with B=10^4 Hz

$$V^2 = 4kT_A RB$$

$$T_A = \frac{V^2}{4kRB} = \frac{10^{-14}}{1.38\times 10^{-23}\times 200\times 10^4} = 90.6K$$

Noise equivalent to that of a 200-Ω resistor at temperature of 90.6K

Antenna

Antenna temperatures

- T_A looking at earth = 300 K
- T_A looking at the moon = 450 K
- T_A looking at the sun = 3000 K

$$N_o = kT_{eq}BG$$

where T_{eq} is the temperature of the system referred to the input of the amplifier

$$N_o = kT_A BG + N_{int}G$$

$$kT_{eq}BG = kT_A BG + \left(F-1\right)kT_{stan}BG$$

Antenna

$$T_{eq} = T_A + \underbrace{(NF - 1)T_{290}BG}_{T_{rec}}$$

$$F = \frac{S_{in} / N_{int}}{S_o / N_o}$$

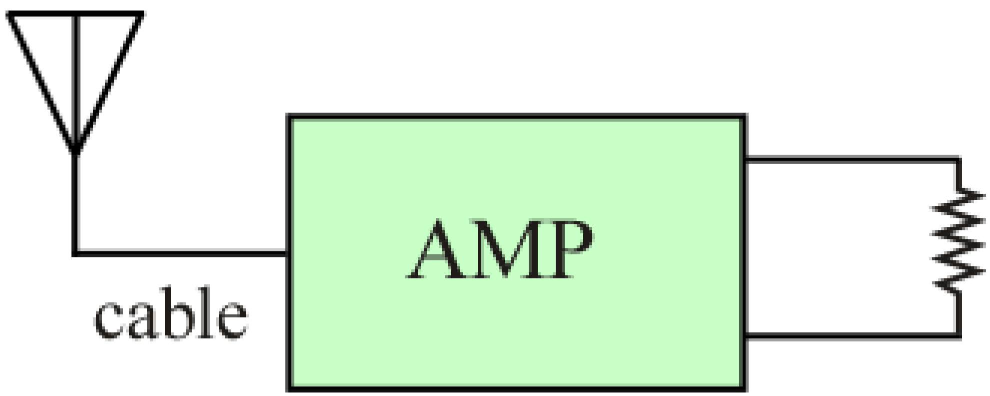
For $F = 1$, the amplifier is noiseless and $T_{rec} = 0$

Cable

RG8v cable given 3dB loss per 100 ft at 200 MHz

Cable

- Attenuates signal
- Attenuates noise from antenna
- Generates noise



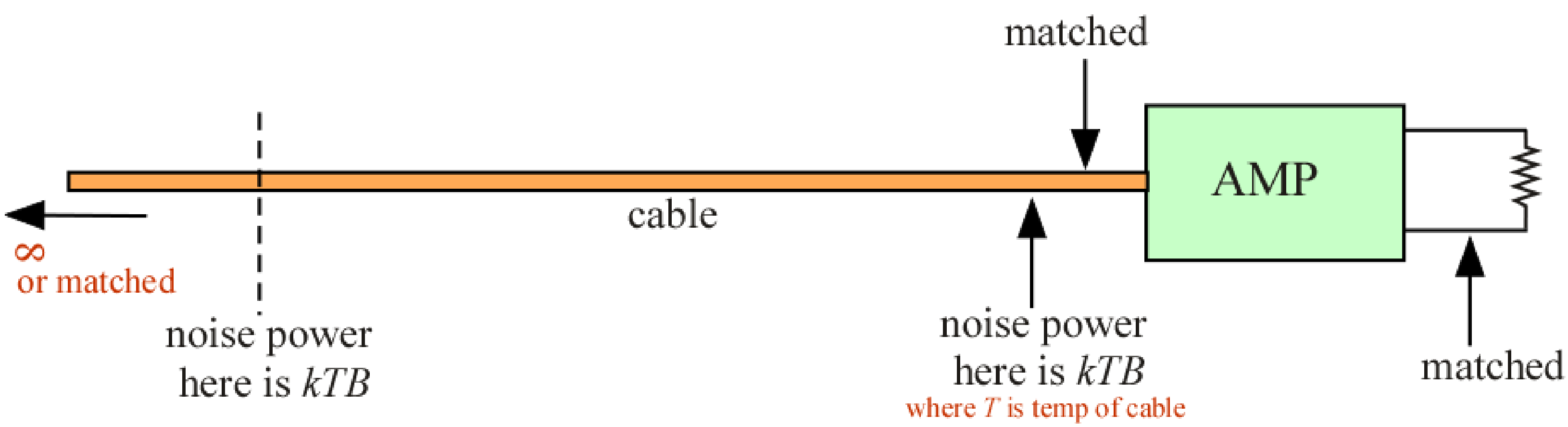
Cable

Let L = loss factor of cable $\quad L = \dfrac{P_{in}}{P_{out}}$

For 6-dB loss $L = 4$

N into the amplifier $= kT_{cable}B$

Also, N into the amplifier $= \dfrac{kT_{cable}B}{L} +$ Noise generated in the finite length of cable



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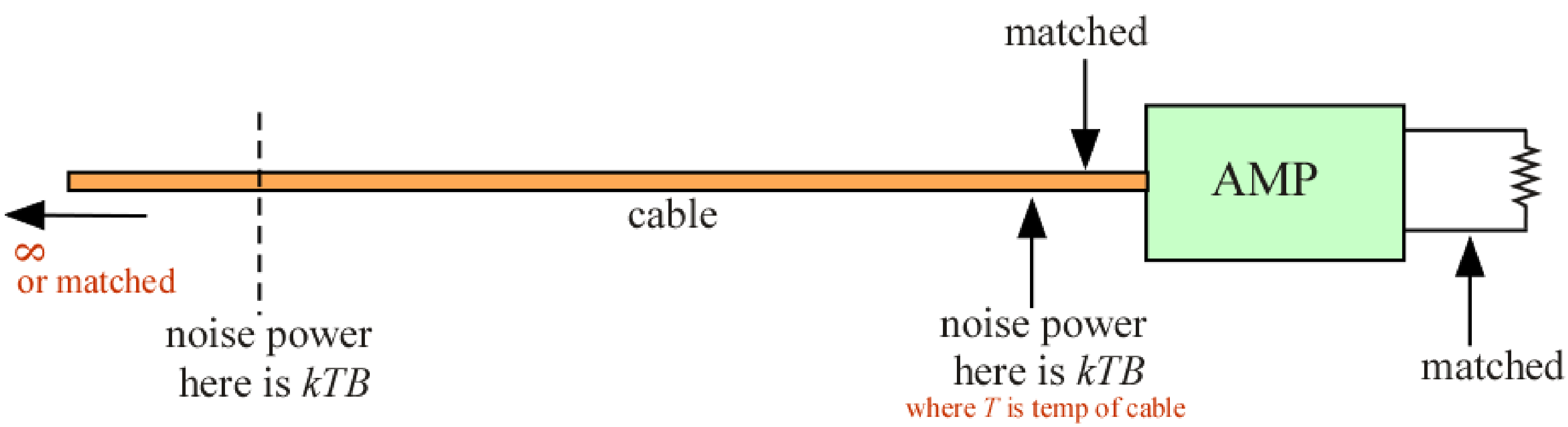
Also, N into the amplifier $= \dfrac{kT_{cable}B}{L} +$ **Noise generated in the finite length of cable**



Cable

Noise generated in the finite length of cable $= kT_{cable}B - \frac{kT_{cable}B}{L} = kT_{cable}\left[1 - \frac{1}{L}\right]B$

The equivalent noise temperature of a finite length of cable = $T_{cable}\left[1 - \frac{1}{L}\right]$

- For a short cable $L = 1$ and T_{eq} of cable =0
- For a very long cable $L \rightarrow inf$ and T_{eq} of cable = T_{cable}

$$\underbrace{T_{system}}_{\substack{referred\ to \\ amp\ input}} = \frac{T_A}{L} + T_{cable}\left[1 - \frac{1}{L}\right] + T_{amp}$$

$$N_o\Big|_{input} = kT_{system}B$$