ECE 453
Wireless Communication Systems

Scattering Parameters

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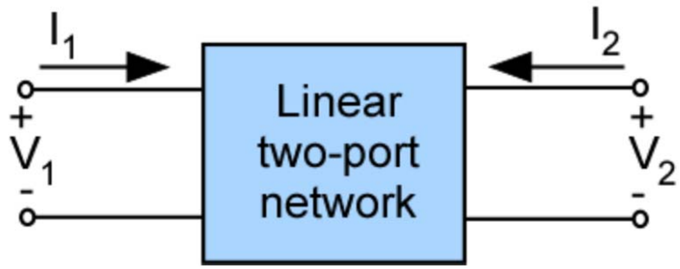
TWO-PORT NETWORK REPRESENTATION



Z Parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Y Parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

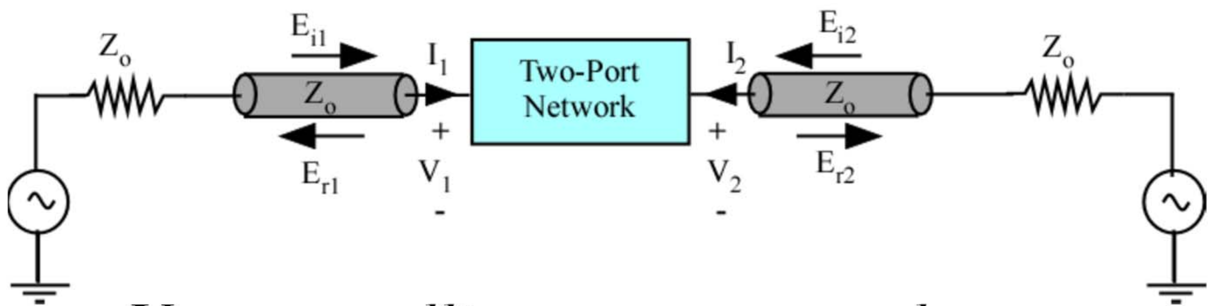
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

- At microwave frequencies, it is more difficult to measure total voltages and currents.

- Short and open circuits are difficult to achieve at high frequencies.

- Most active devices are not short- or open-circuit stable.

Wave Approach



Use a travelling wave approach

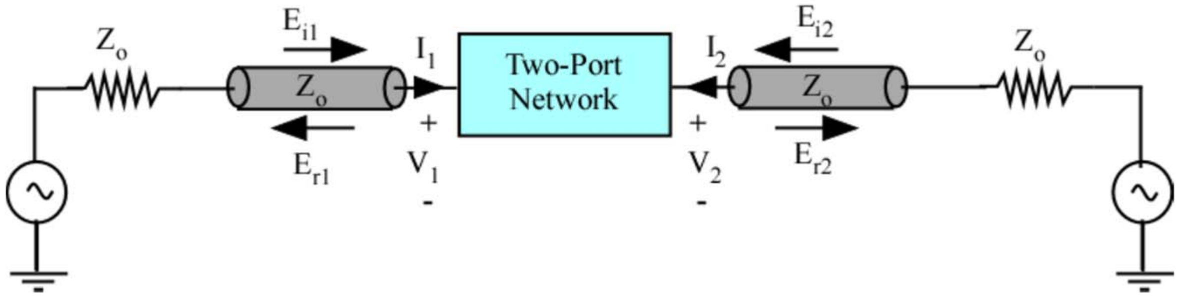
$$V_1 = E_{i1} + E_{r1} \qquad V_2 = E_{i2} + E_{r2}$$

$$I_1 = \frac{E_{i1} - E_{r1}}{Z_o} \qquad I_2 = \frac{E_{i2} - E_{r2}}{Z_o}$$

- **Total voltage and current are made up of sums of forward and backward traveling waves.**

- **Traveling waves can be determined from standing-wave ratio.**

Wave Approach



$$a_1 = \frac{E_{i1}}{\sqrt{Z_o}} \qquad a_2 = \frac{E_{i2}}{\sqrt{Z_o}}$$

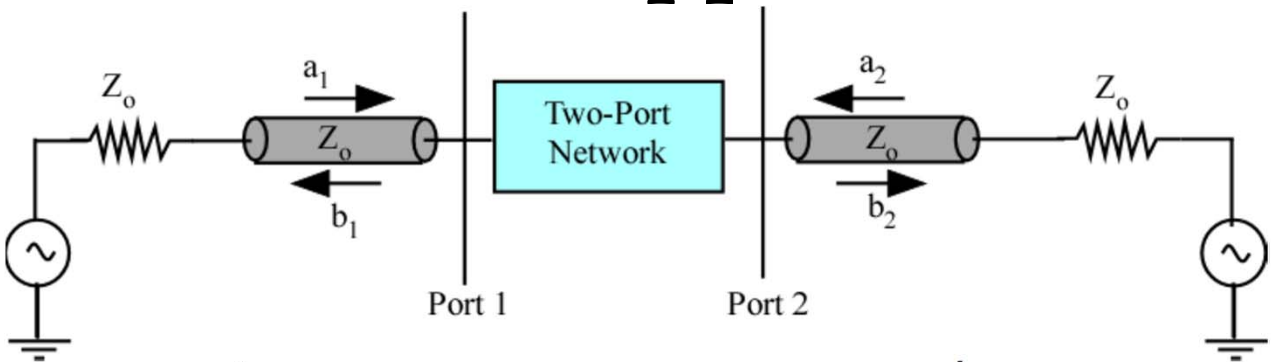
$$b_1 = \frac{E_{r1}}{\sqrt{Z_o}} \qquad b_2 = \frac{E_{r2}}{\sqrt{Z_o}}$$

Z_o is the reference impedance of the system

$$\mathbf{b_1 = S_{11}\, a_1 + S_{12}\, a_2}$$

$$\mathbf{b_2 = S_{21}\, a_1 + S_{22}\, a_2}$$

Wave Approach



$$S_{11} = \frac{b_1}{a_1}\Big|_{a2=0} \qquad S_{21} = \frac{b_2}{a_1}\Big|_{a2=0}$$

$$S_{12} = \frac{b_1}{a_2}\Big|_{a1=0} \qquad S_{22} = \frac{b_2}{a_2}\Big|_{a1=0}$$

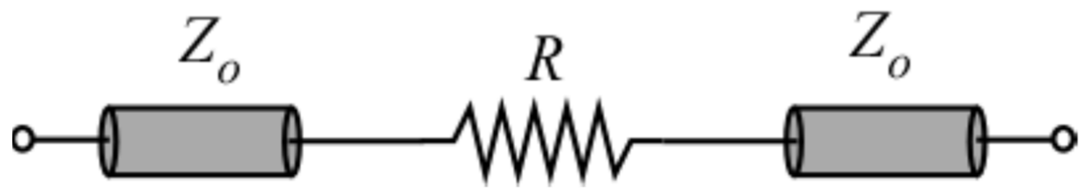
To make $a_i = 0$

1) Provide no excitation at port i

2) Match port i to the characteristic impedance of the reference lines.

CAUTION : a_i and b_i are the traveling waves in the <u>reference lines</u>.

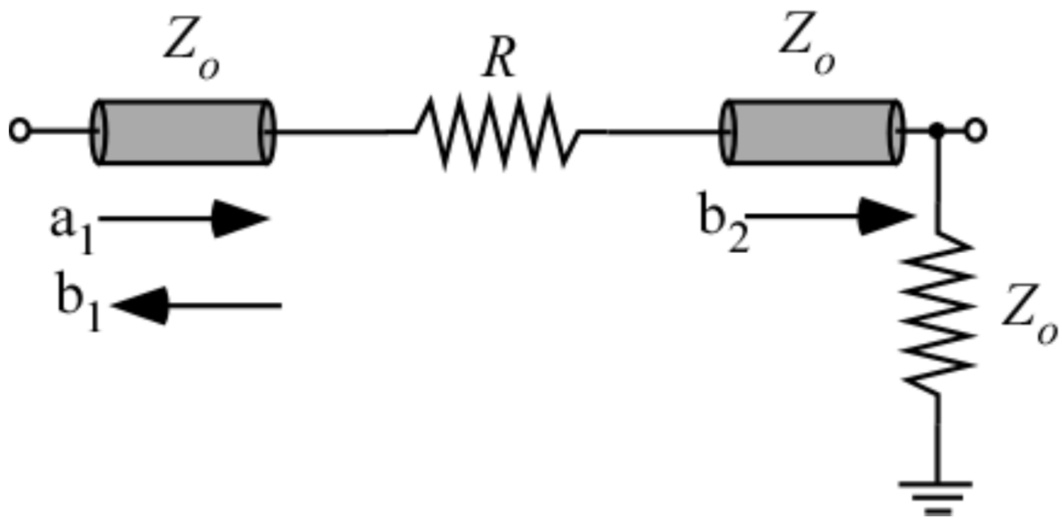
S-Parameters of Resistor



Determine S-Parameter of 2-port resistance

- Insert R between two reference TL
- Provide excitation at port 1 for S_{11} and S_{21}
- Provide excitation at port 2 for S_{12} and S_{22}
- Can use symmetry and reciprocity

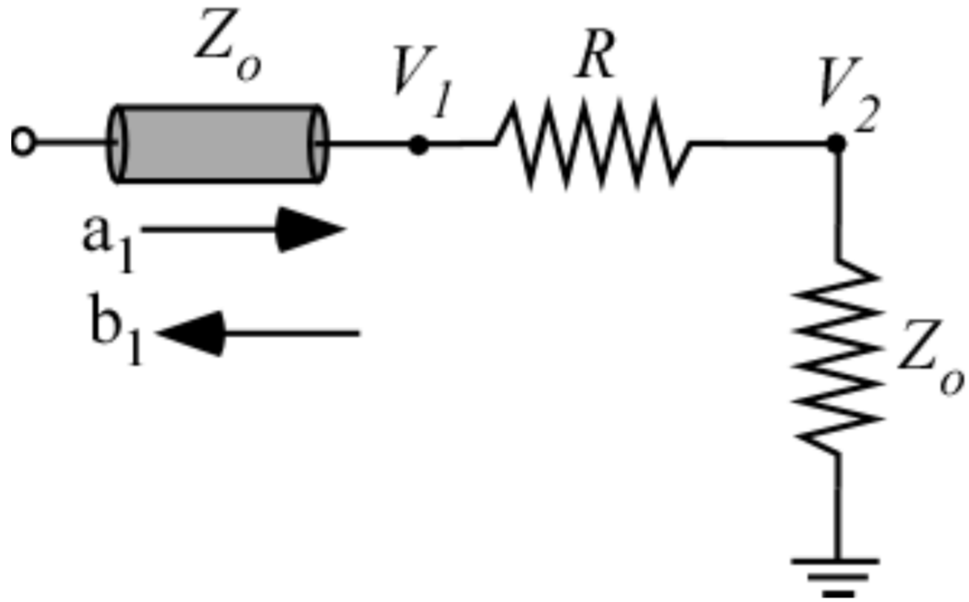
S-Parameters of Resistor



$$S_{11} = \frac{b_1}{a_1} = \Gamma = \frac{(R+Z_o)-Z_o}{(R+Z_o)+Z_o} = \frac{R}{R+2Z_o}$$

$$S_{11} = \frac{R}{R+2Z_o}$$ **and by symmetry,** $$S_{22} = \frac{R}{R+2Z_o}$$

Calculating S_{21} of Resistor

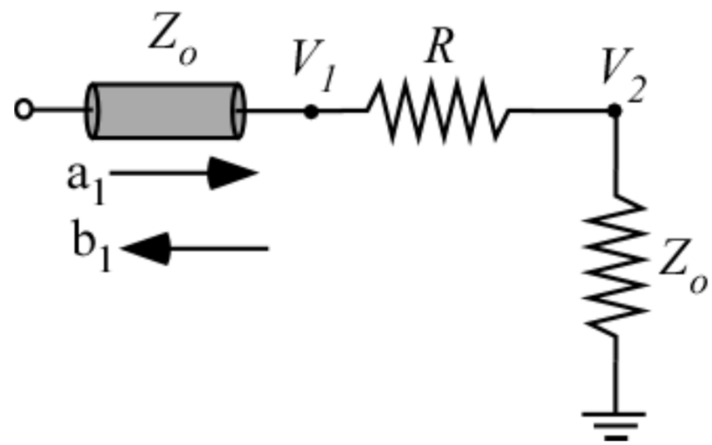


Since a_2=0, the total voltage in port 2 is: $V_2 = b_2\sqrt{Z_o}$

$$V_2 = \frac{V_1 Z_o}{R_1 + Z_o} = \frac{\sqrt{Z_o}\left(a_1 + b_1\right)Z_o}{R_1 + Z_o} = \frac{\sqrt{Z_o}\left(a_1 + S_{11}a_1\right)Z_o}{R_1 + Z_o}$$

S-Parameters of Resistor

$$V_2 = \frac{Z_o\sqrt{Z_o}\left(1+S_{11}\right)a_1}{R_1+Z_o} = \frac{2Z_o a_1\sqrt{Z_o}}{R_1+2Z_o}$$



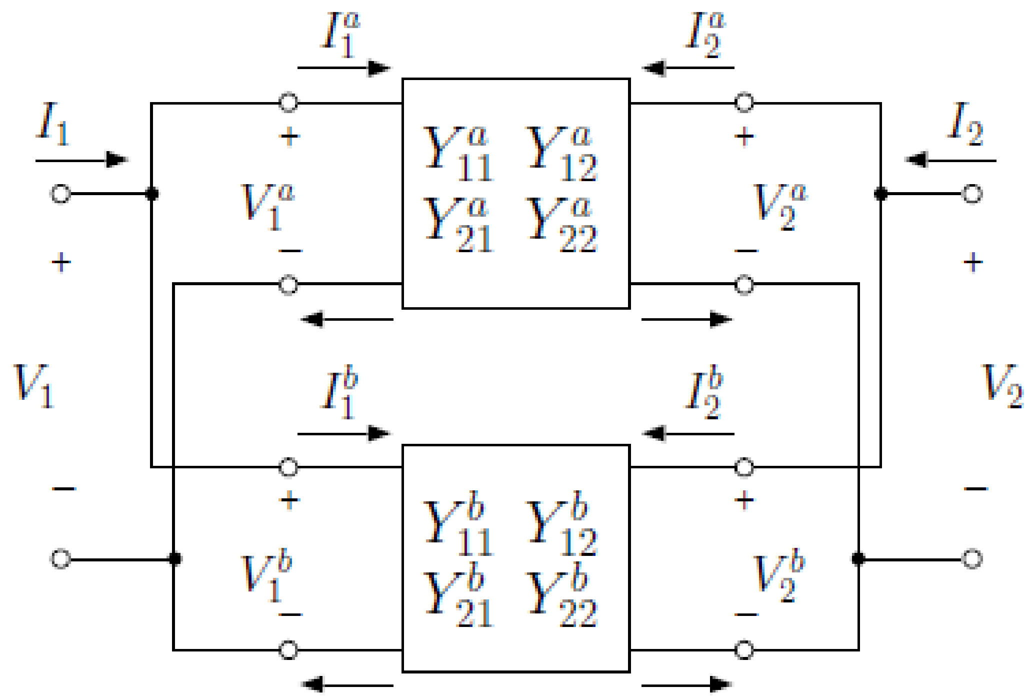
$$S_{21} = \frac{b_2}{a_1} = \frac{V_2}{\sqrt{Z_o}}\frac{1}{a_1} = \frac{2Z_o}{R+2Z_o}$$

$$S_{21} = \frac{2Z_o}{R+2Z_o}$$ **and by reciprocity,** $$S_{12} = \frac{2Z_o}{R+2Z_o}$$

S parameters of resistor R

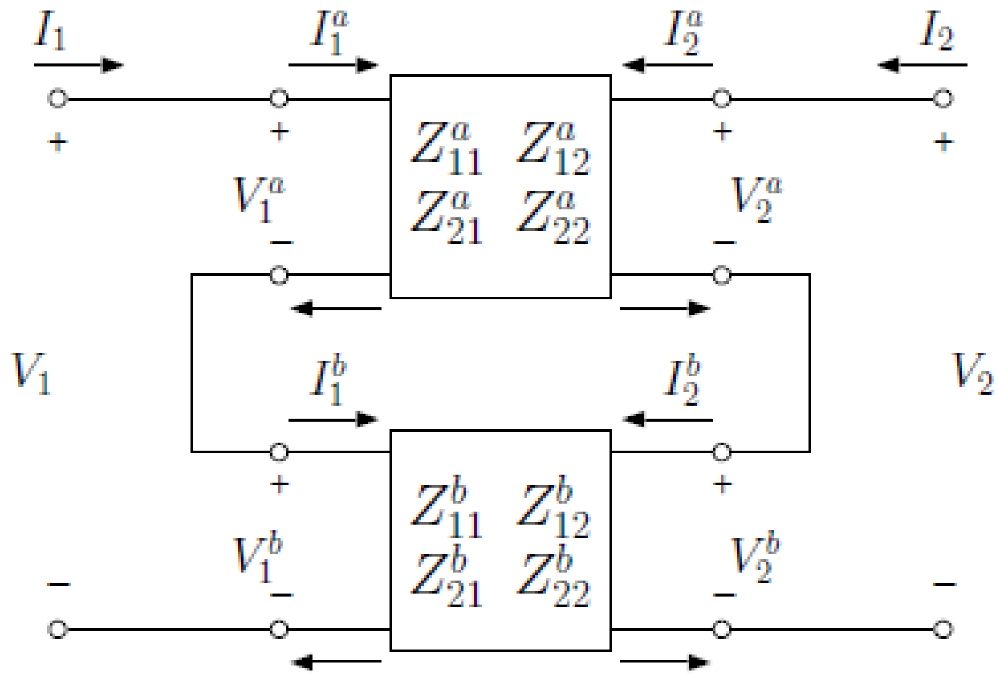
$$S = \begin{bmatrix} \frac{R}{R+2Z_o} & \frac{2Z_o}{R+2Z_o} \\ \frac{2Z_o}{R+2Z_o} & \frac{R}{R+2Z_o} \end{bmatrix}$$

Two-Ports in Parallel



$$\mathbf{Y} = \mathbf{Y^a} + \mathbf{Y^b}$$

Two-Ports in Series



$$\mathbf{Z} = \mathbf{Z}^{\mathbf{a}} + \mathbf{Z}^{\mathbf{b}}$$

Scattering Transfer Parameters

In T-Parameters, traveling waves at the input are related to those at the output

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$b_1 = T_{11}a_2 + T_{12}b_2$$
$$a_1 = T_{21}a_2 + T_{22}b_2$$

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} T_{12}T_{22}^{-1} & T_{11} - T_{12}T_{21}T_{22}^{-1} \\ T_{22}^{-1} & -T_{21}T_{22}^{-1} \end{pmatrix}$$

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} S_{12} - S_{11}S_{22}S_{21}^{-1} & S_{11}S_{21}^{-1} \\ -S_{22}S_{21}^{-1} & S_{21}^{-1} \end{pmatrix}$$

T parameters can be cascaded $\mathbf{T} = \mathbf{T_A} \cdot \mathbf{T_B}$

Parameter Conversion

7.7.1 Converting to Y-parameters

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{D_Z} & -\frac{Z_{12}}{D_Z} \\ -\frac{Z_{21}}{D_Z} & \frac{Z_{11}}{D_Z} \end{bmatrix} = \begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{D_h}{h_{11}} \end{bmatrix} = \begin{bmatrix} \frac{D}{B} & -\frac{D_{ABCD}}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$$

7.7.2 Converting to Z-parameters

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{D_Y} & -\frac{Y_{12}}{D_Y} \\ -\frac{Y_{21}}{D_Y} & \frac{Y_{11}}{D_Y} \end{bmatrix} = \begin{bmatrix} \frac{D_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix} = \begin{bmatrix} \frac{A}{C} & \frac{D_{ABCD}}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$$

7.7.3 Converting to h-parameters

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{D_Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix} = \begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{D_Y}{Y_{11}} \end{bmatrix} = \begin{bmatrix} \frac{B}{D} & \frac{D_{ABCD}}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$$

7.7.4 Converting to ABCD-parameters

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{D_Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix} = \begin{bmatrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{D_Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{bmatrix} = \begin{bmatrix} -\frac{D_h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{bmatrix}$$

N-Port S Parameters

$$\begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdot & \cdot \\ S_{21} & S_{22} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ a_n \end{bmatrix}$$

$$\mathbf{b} = \mathbf{Sa}$$

If $b_i = 0$, then no reflected wave on port i ➔ port is matched

$$a_i = \frac{V_i^+}{\sqrt{Z_{oi}}}$$

$$b_i = \frac{V_i^-}{\sqrt{Z_{oi}}}$$

V_i^+ **: incident voltage wave in port *i***

V_i^- **: reflected voltage wave in port *i***

Z_{oi} **: impedance in port *i***

N-Port S Parameters

$\mathbf{v} = \sqrt{Z_o}(\mathbf{a}+\mathbf{b})$ (1) $\quad \mathbf{i} = \frac{1}{\sqrt{Z_o}}(\mathbf{a}-\mathbf{b})$ (2) $\quad \mathbf{v} = \mathbf{Zi}$ (3)

Substitute (1) and (2) into (3)

$$\sqrt{Z_o}(\mathbf{a}+\mathbf{b}) = \mathbf{Z}\frac{1}{\sqrt{Z_o}}(\mathbf{a}-\mathbf{b})$$

Defining S such that b = Sa and substituting for b

$$Z_o(\mathbf{U}+\mathbf{S})\mathbf{a} = Z_o(\mathbf{U}-\mathbf{S})\mathbf{a}$$

U : unit matrix

S→Z

$$\mathbf{Z} = Z_o(\mathbf{U}+\mathbf{S})(\mathbf{U}-\mathbf{S})^{-1}$$

Z→S

$$\mathbf{S} = (\mathbf{Z}+Z_o\mathbf{U})^{-1}(\mathbf{Z}-Z_o\mathbf{U})$$

N-Port S Parameters

If the port reference impedances are different, we define k as

$$\mathbf{k} = \begin{bmatrix} \sqrt{Z_{o1}} & & & \\ & \sqrt{Z_{o2}} & & \\ & & \cdot & \\ & & & \sqrt{Z_{on}} \end{bmatrix}$$

$\mathbf{v} = \mathbf{k}(\mathbf{a}+\mathbf{b})$ and $\mathbf{i} = \mathbf{k}^{-1}(\mathbf{a}-\mathbf{b})$ and $\mathbf{k}(\mathbf{a}+\mathbf{b}) = \mathbf{Z}\mathbf{k}^{-1}(\mathbf{a}-\mathbf{b})$

Z→S

$$\mathbf{S} = \left(\mathbf{Z}\mathbf{k}^{-1}+\mathbf{k}\right)\left(\mathbf{Z}\mathbf{k}^{-1}-\mathbf{k}\right)$$

S→Z

$$\mathbf{Z} = \mathbf{k}\left(\mathbf{U}+\mathbf{S}\right)\left(\mathbf{U}-\mathbf{S}\right)^{-1}\mathbf{k}$$

Normalization

Assume original S parameters as S_1 with system k_1. Then the representation S_2 on system k_2 is given by

Transformation Equation

$$\mathbf{S_2 = \left[k_1(U+S_1)(U-S_1)^{-1}k_1k_2 + k_2\right]^{-1}\left[k_1(U+S_1)(U-S_1)^{-1}k_1k_2 - k_2\right]}$$

If Z is symmetric, S is also symmetric