

# ECE 453

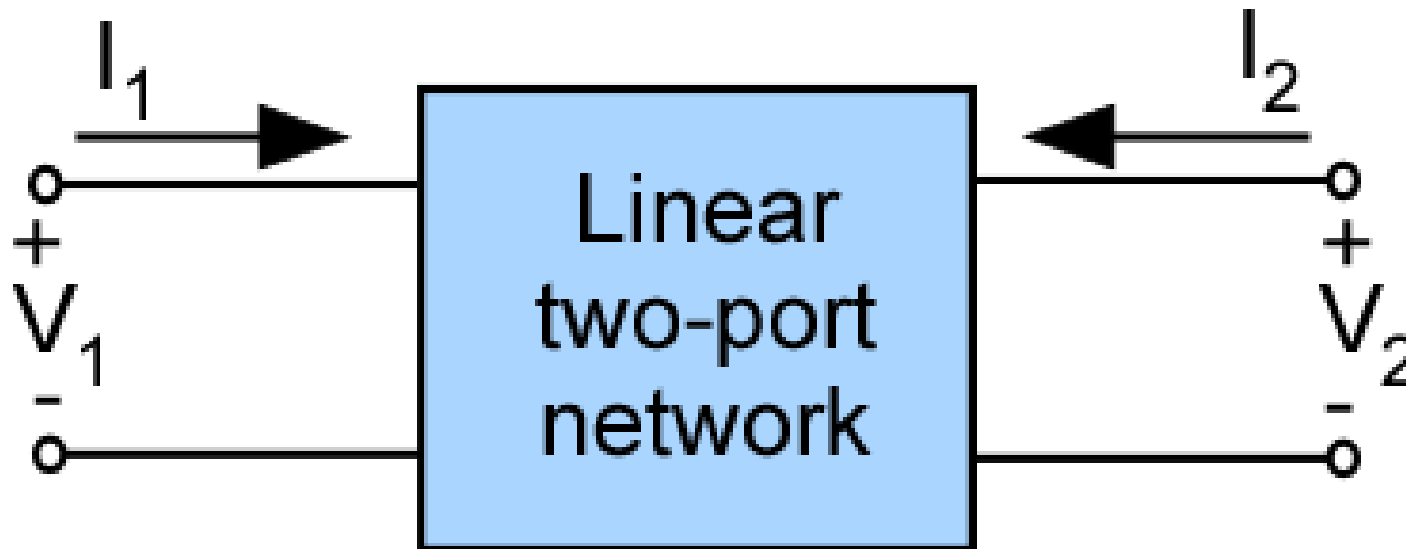
# Wireless Communication Systems

## Network Parameters

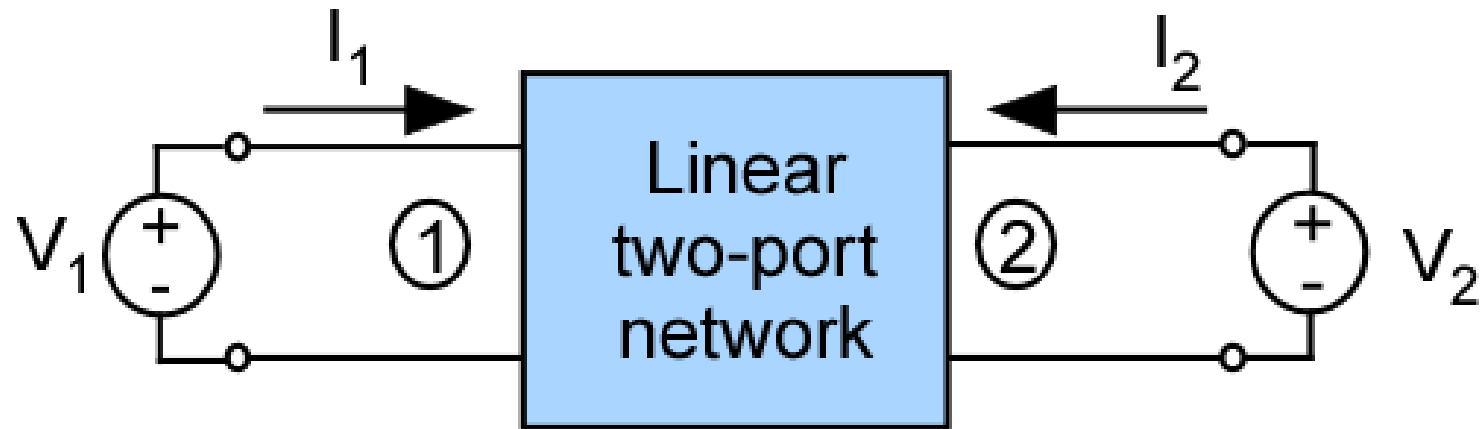
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# Transfer Function Representation



Use a two-terminal representation of system for input and output

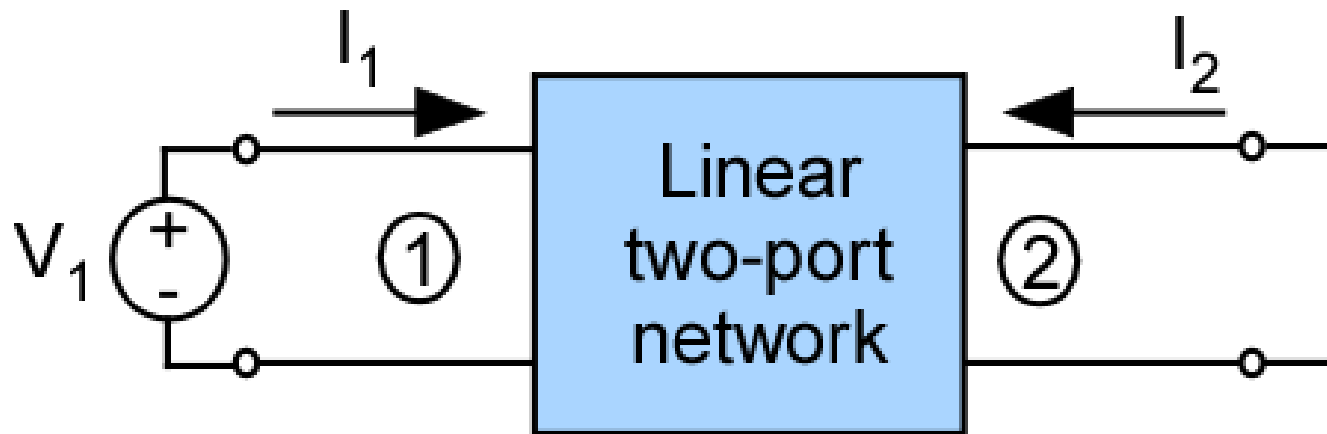
# Y-parameter Representation



$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

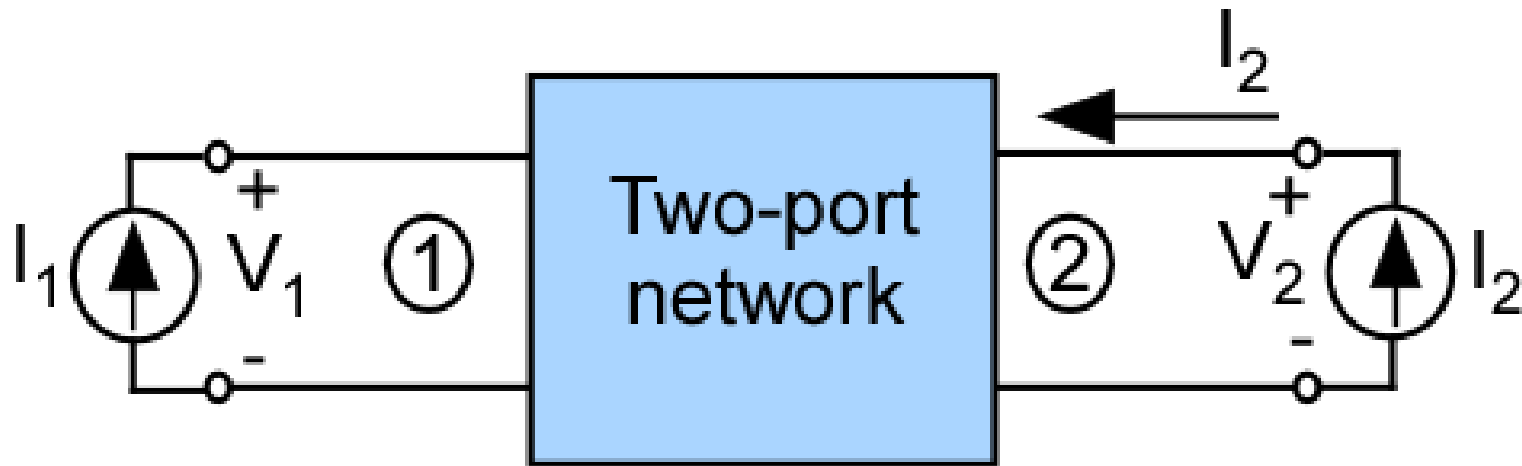
# Y Parameter Calculations



$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \quad y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}$$

To make  $V_2=0$ , place a short at port 2

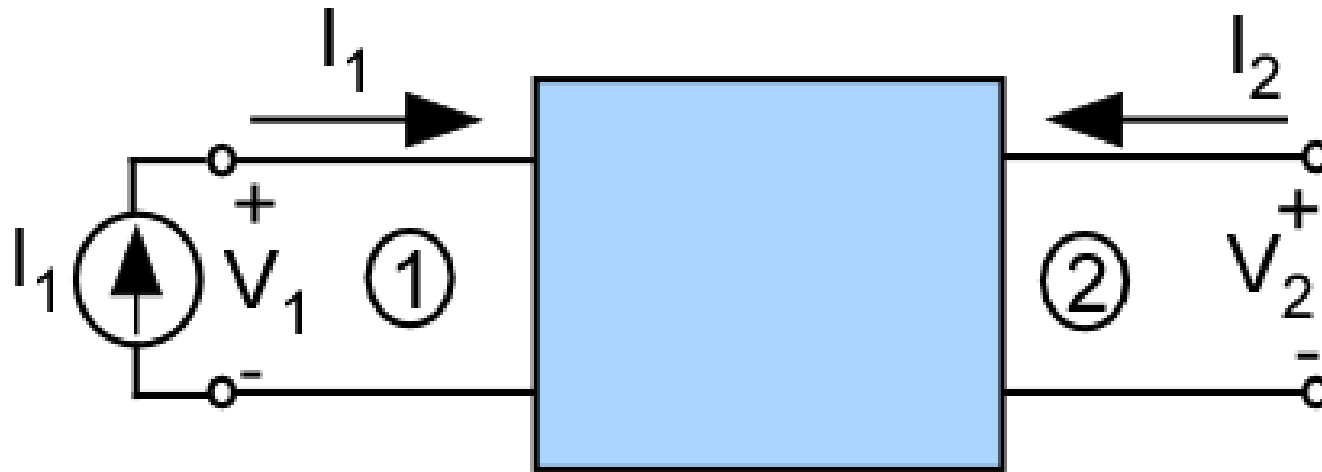
# Z Parameters



$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

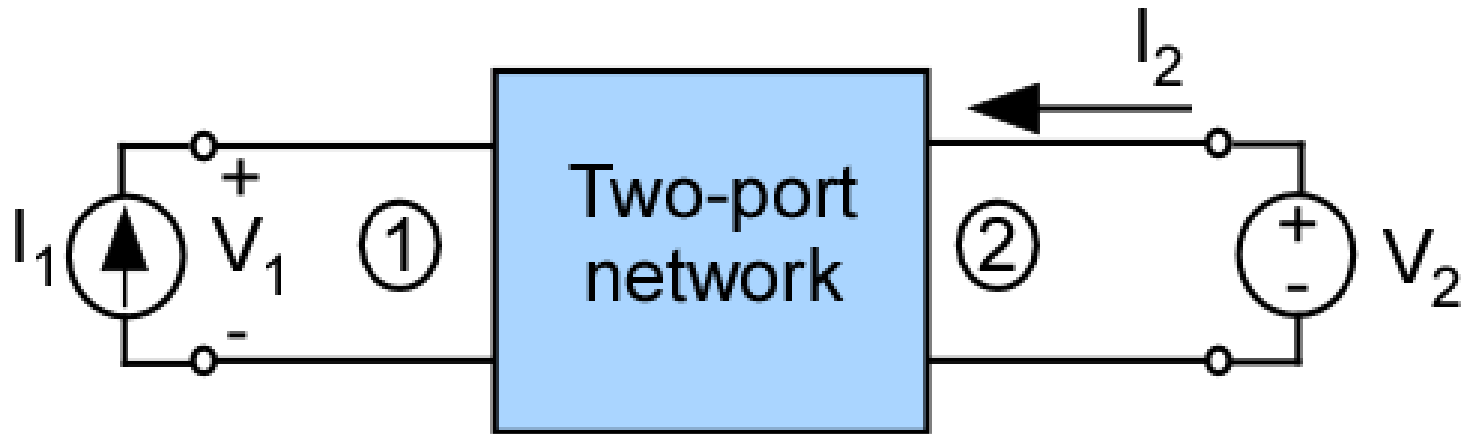
# Z-parameter Calculations



$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}$$

To make  $I_2=0$ , place an open at port 2

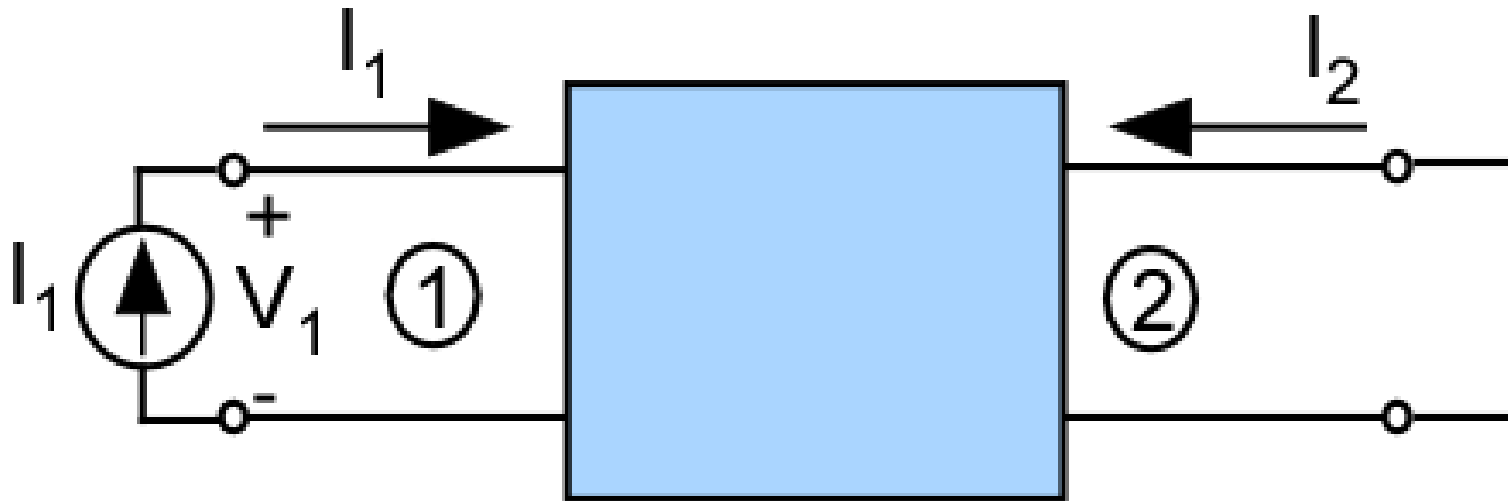
# H Parameters



$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

# H Parameter Calculations

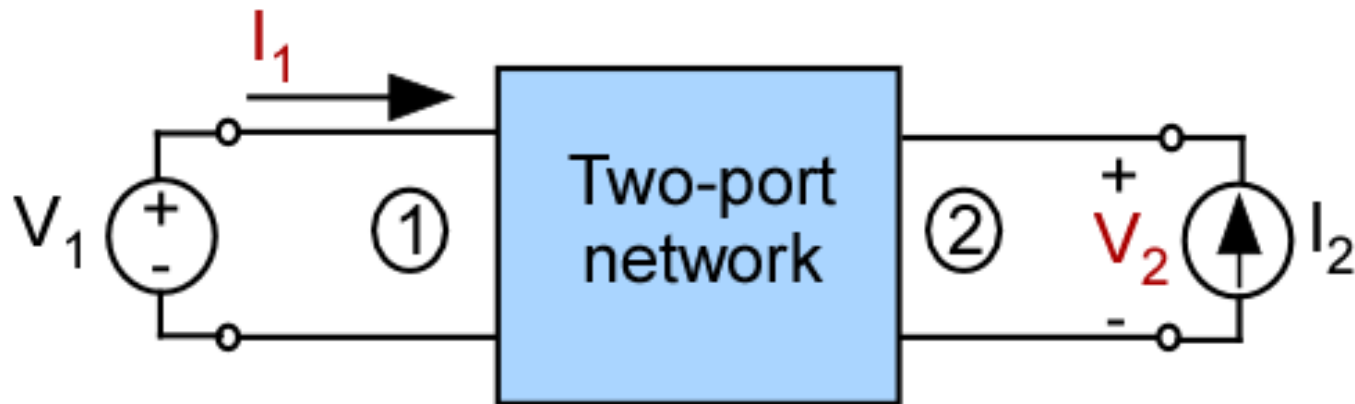


$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} \quad h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0}$$

To make  $V_2=0$ , place a short at port 2



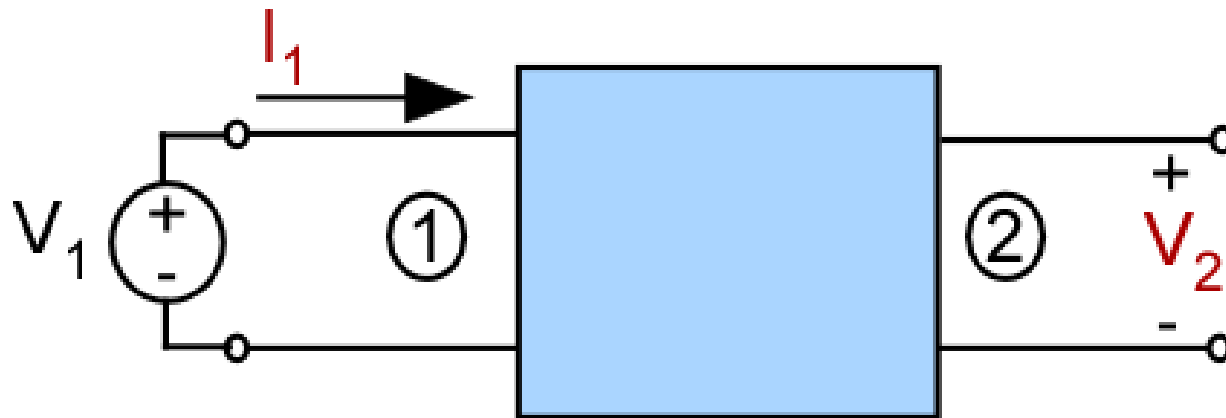
# G Parameters



$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

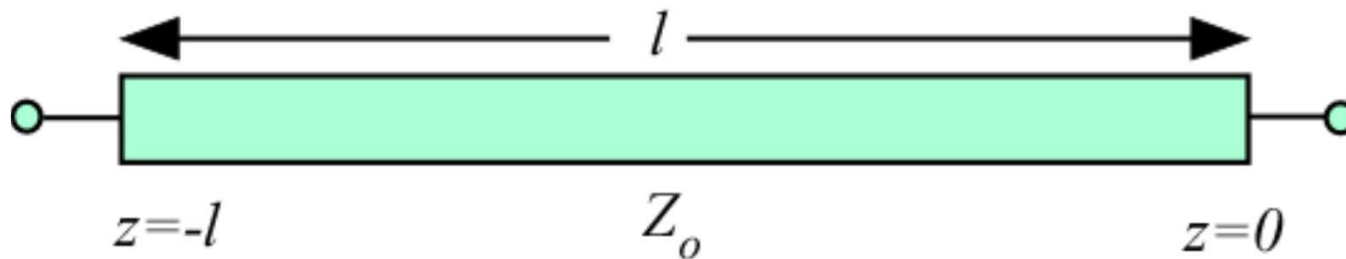
# G-Parameter Calculations



$$g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0} \quad g_{21} = \left. \frac{V_2}{V_1} \right|_{I_2=0}$$

To make  $I_2=0$ , place an open at port 2

# Y-Parameters of TL



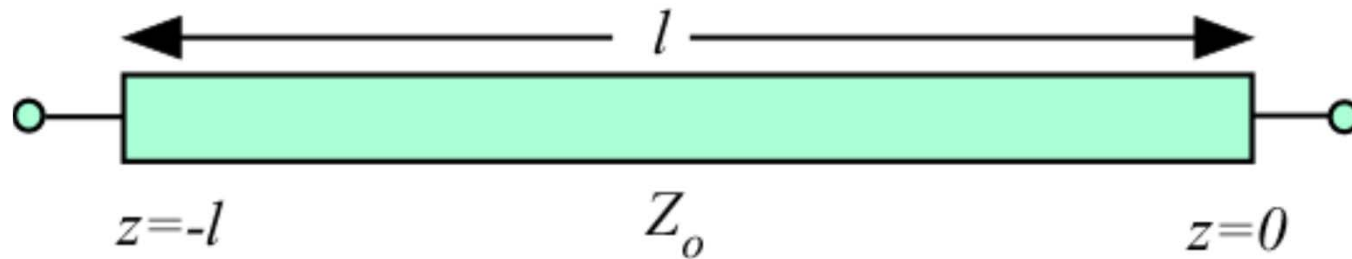
Find the Y-parameters of a lossless transmission line with propagation constant  $\beta$  and characteristic impedance  $Z_o$  (admittance  $Y_o$ )

$$V(z) = V_+ e^{-j\beta z} + V_- e^{+j\beta z}$$

$$I(z) = Y_o (V_+ e^{-j\beta z} - V_- e^{+j\beta z})$$

Let port 1 be at  $z=-l$  and port 2 at  $z=0$

# Y-Parameters of TL



at port 1

$$V_1 = V_+ e^{+j\beta l} + V_- e^{-j\beta l}$$

$$I_1 = Y_o (V_+ e^{+j\beta l} - V_- e^{-j\beta l})$$

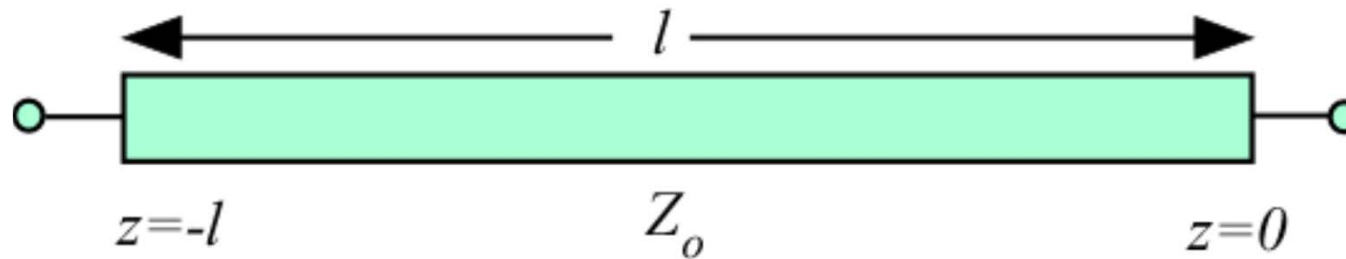
at port 2 ( $z = 0$ )

$$V_2 = V_+ + V_-$$

$$I_2 = -Y_o (V_+ - V_-)$$

$$V_+ = \frac{V_2 - Z_o I_2}{2} \quad \text{and} \quad V_- = \frac{V_2 + Z_o I_2}{2}$$

# Y-Parameters of TL



So that

$$V_1 = \left( \frac{V_2 - Z_o I_2}{2} \right) e^{+j\beta l} + \left( \frac{V_2 + Z_o I_2}{2} \right) e^{-j\beta l}$$

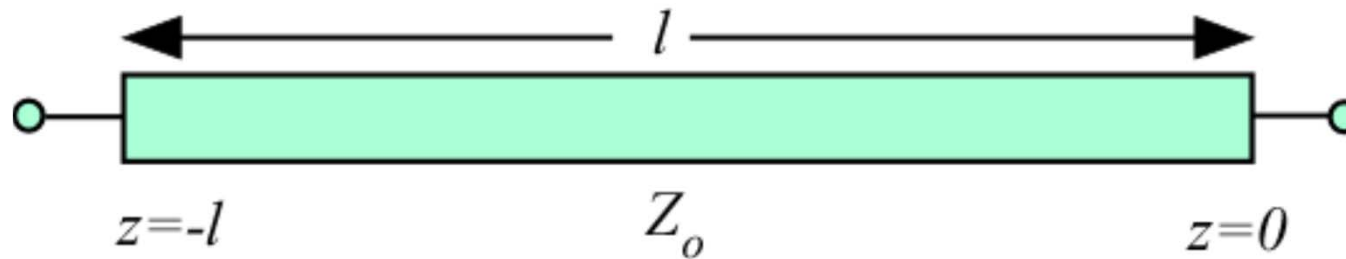
$$I_1 = Y_o \left( \frac{V_2 - Z_o I_2}{2} \right) e^{+j\beta l} - Y_o \left( \frac{V_2 + Z_o I_2}{2} \right) e^{-j\beta l}$$

and

$$V_1 = V_2 \cos \beta l - Z_o I_2 j \sin \beta l$$

$$I_1 = +Y_o V_2 j \sin \beta l - I_2 \cos \beta l$$

# Y-Parameters of TL



Using definitions for  $Y_{11}$

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \frac{-I_2 \cos \beta l}{-jZ_o I_2 \sin \beta l} = \frac{-jY_o \cos \beta l}{\sin \beta l}$$

and

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = \frac{-I_2}{-jZ_o I_2 \sin \beta l} = \frac{+jY_o}{\sin \beta l}$$

$$Y_{22} = Y_{11} \text{ by symmetry}$$

$$Y_{12} = Y_{21} \text{ by reciprocity}$$