ECE 453
Wireless Communication Systems

Network Parameters

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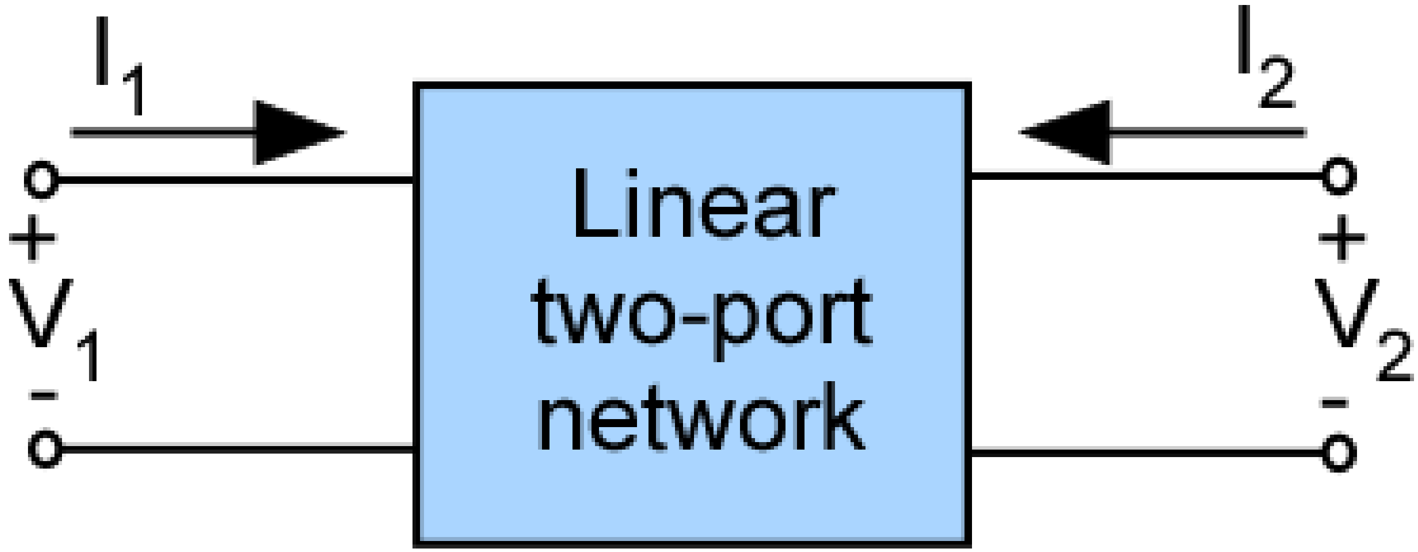
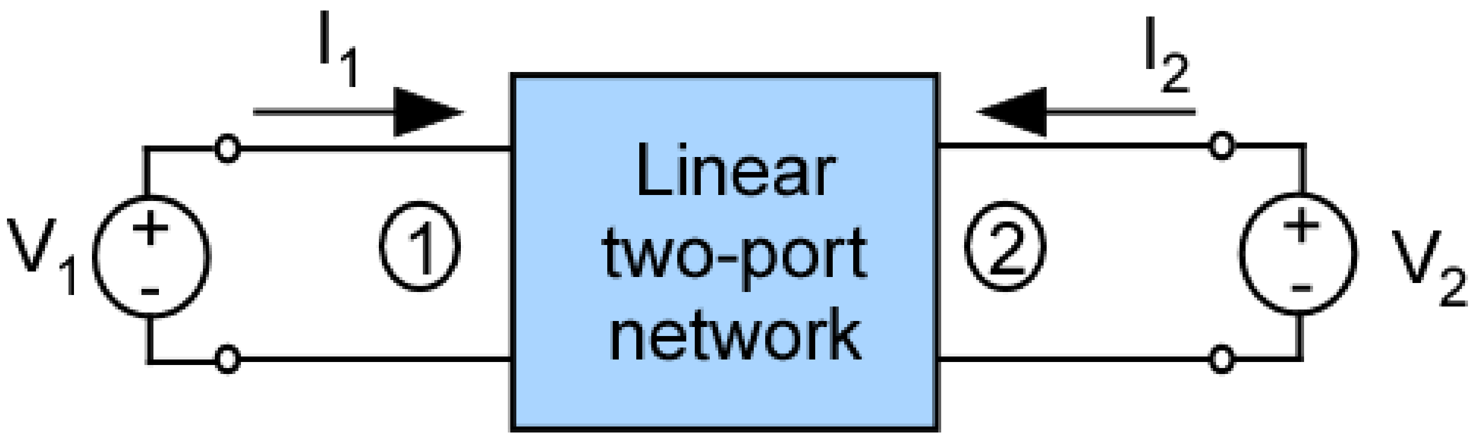
Transfer Function Representation



Use a two-terminal representation of system for input and output

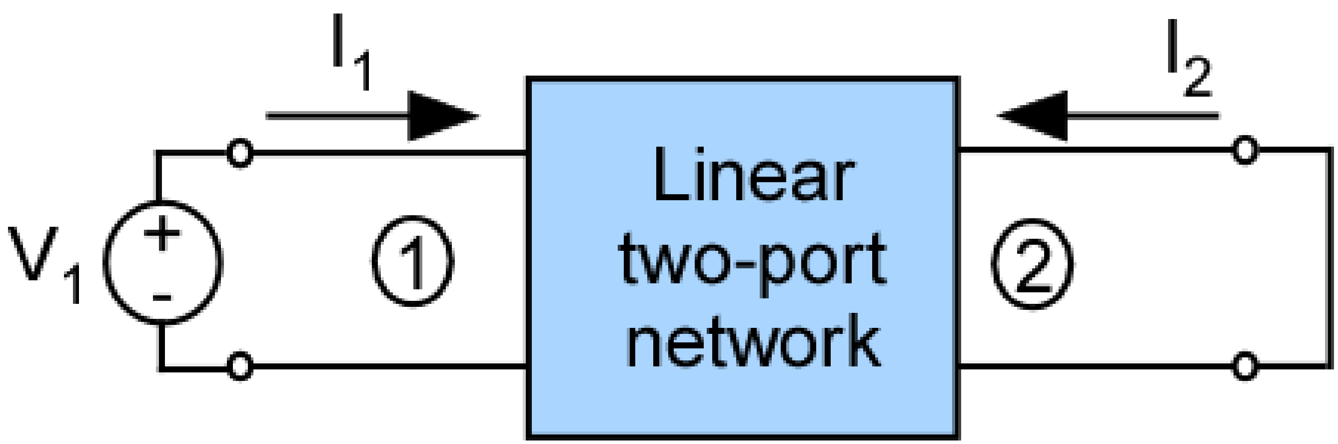
Y-parameter Representation



$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

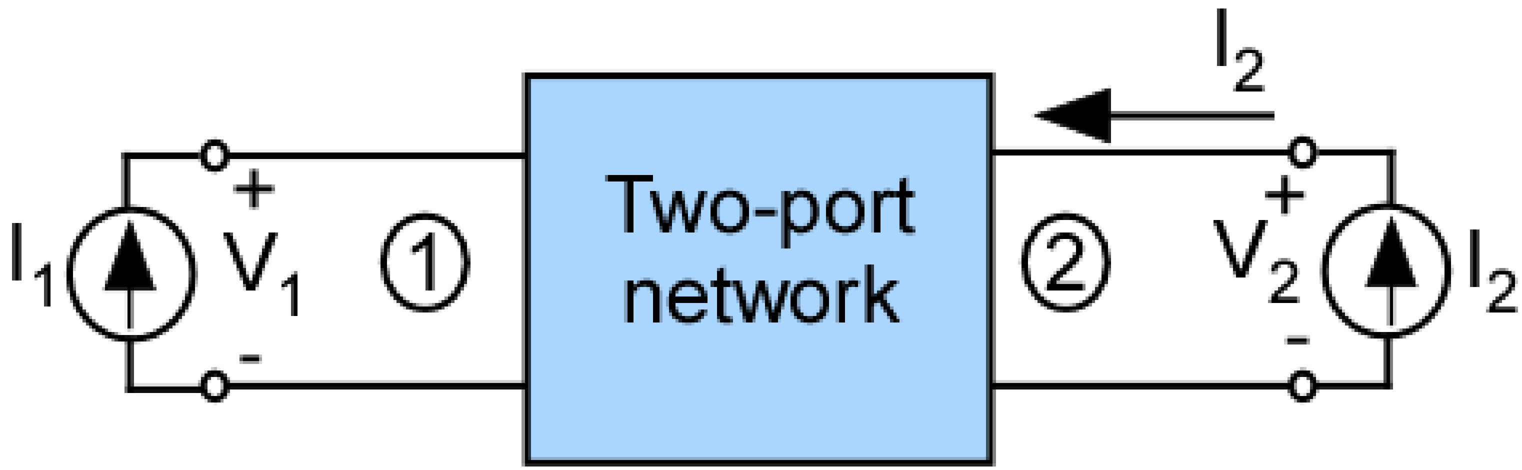
Y Parameter Calculations



$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \qquad y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}$$

To make $V_2 = 0$, place a short at port 2

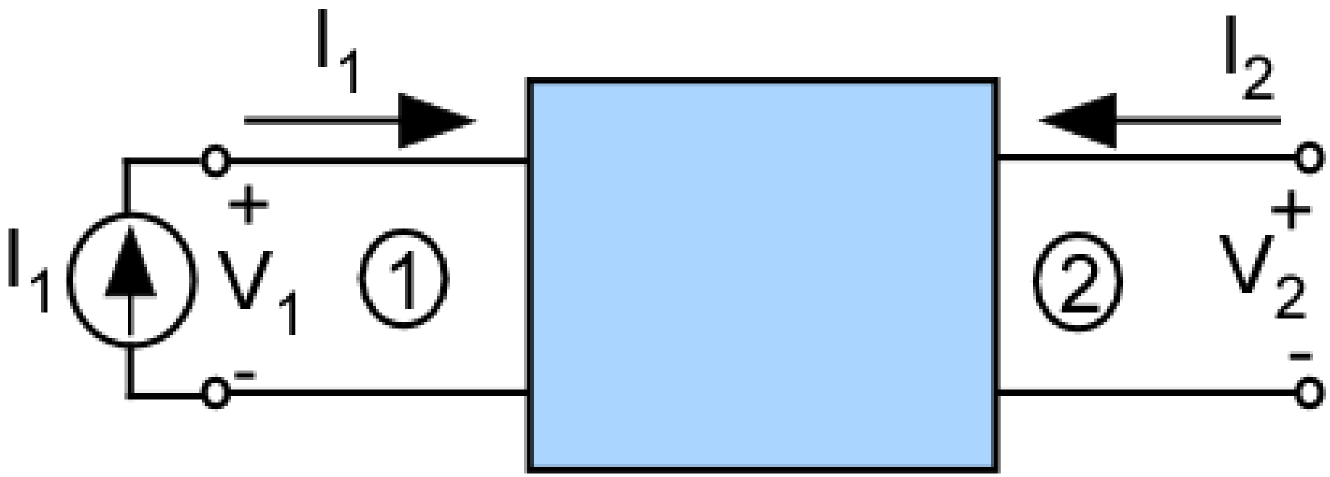
Z Parameters



$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

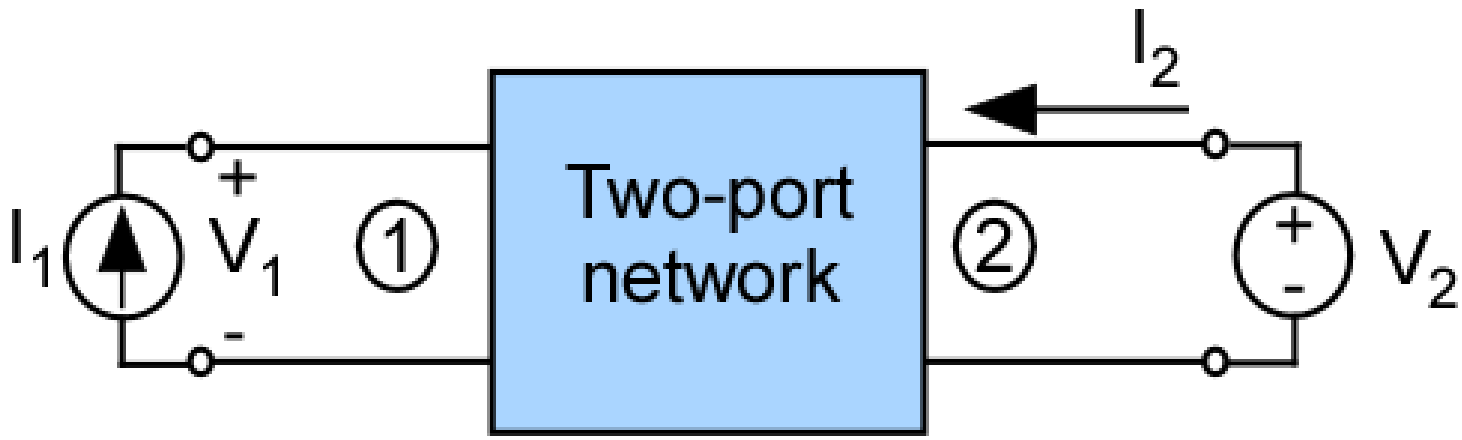
Z-parameter Calculations



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

To make $I_2 = 0$, place an open at port 2

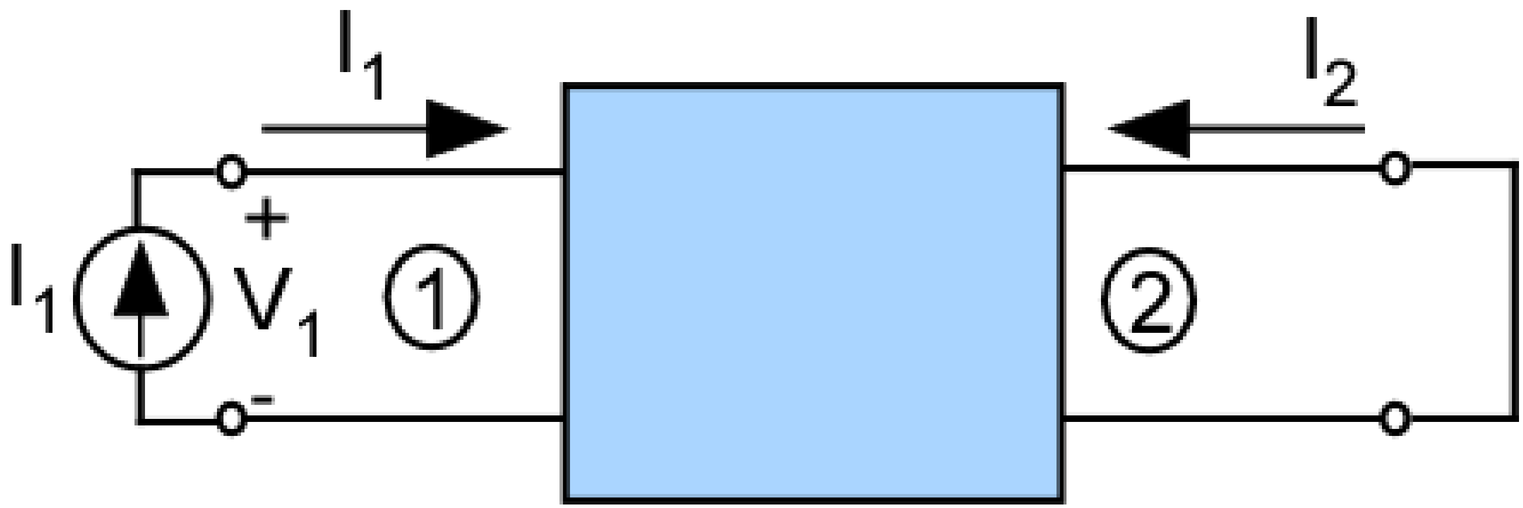
H Parameters



$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

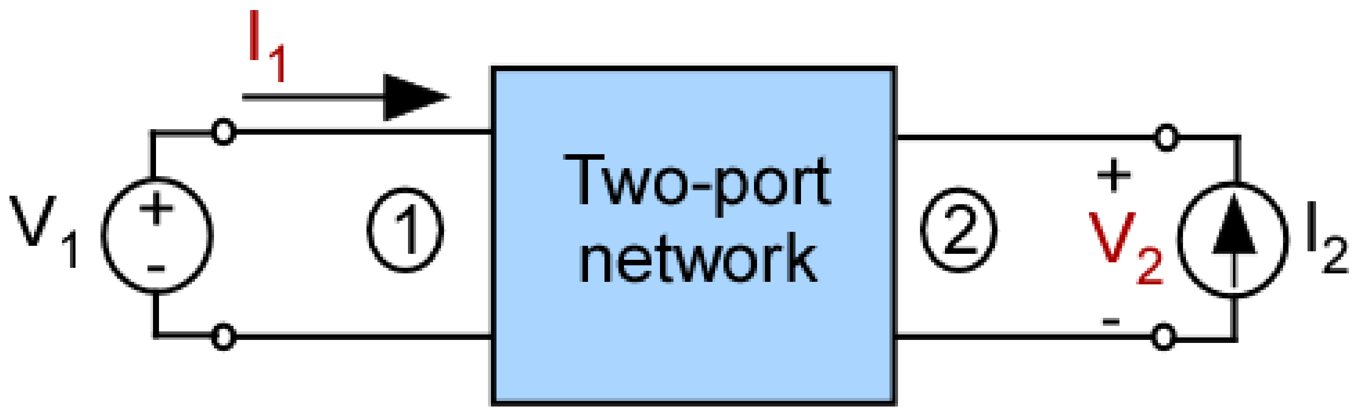
H Parameter Calculations



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

To make *V_2= 0*, place a short at port 2

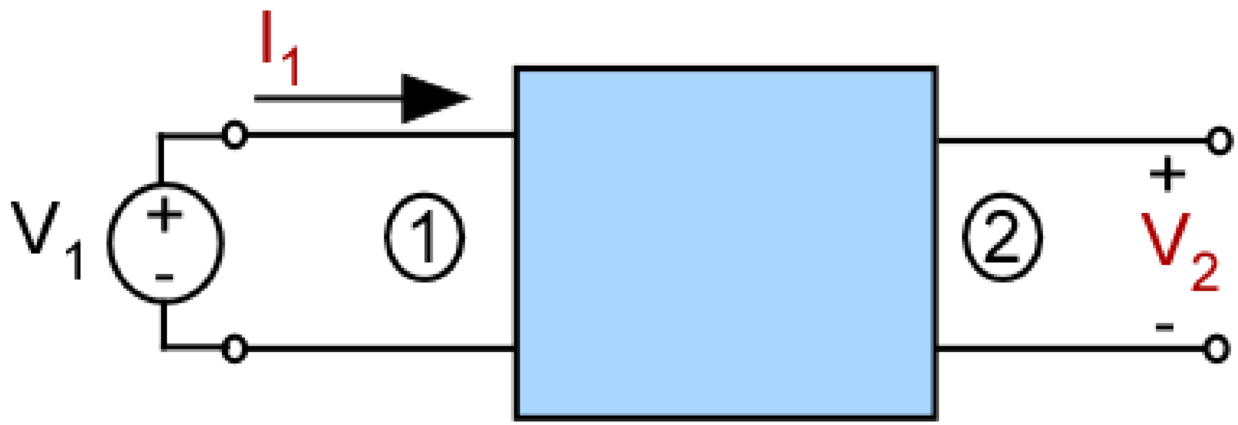
G Parameters



$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

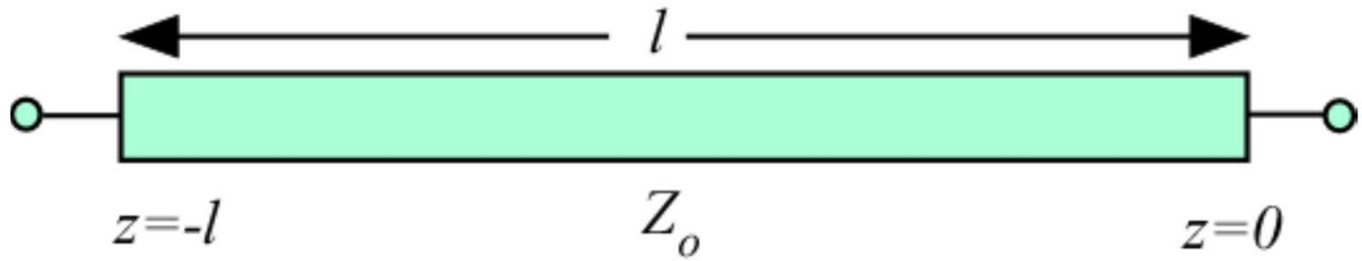
G-Parameter Calculations



$$g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0} \qquad g_{21} = \left. \frac{V_2}{V_1} \right|_{I_2=0}$$

To make $I_2 = 0$, place an open at port 2

Y-Parameters of TL



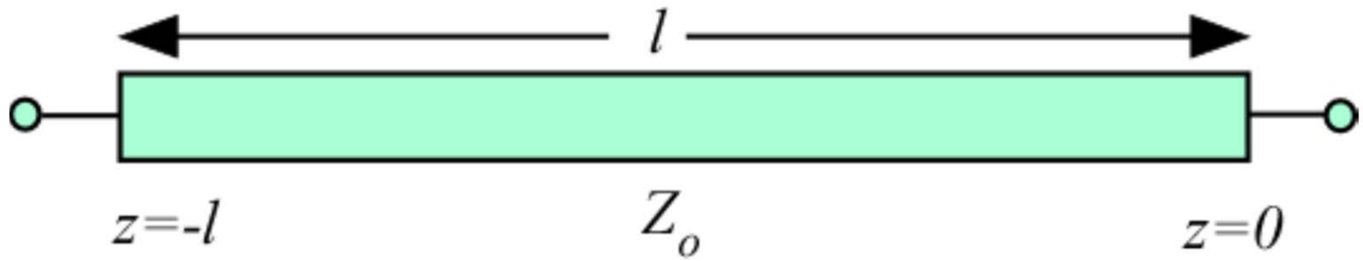
Find the Y-parameters of a lossless transmission line with propagation constant β and characteristic impedance Z_o (admittance Y_o)

$$V(z) = V_+ e^{-j\beta z} + V_- e^{+j\beta z}$$

$$I(z) = Y_o\left(V_+ e^{-j\beta z} - V_- e^{+j\beta z}\right)$$

Let port 1 be at $z=-l$ and port 2 at $z=0$

Y-Parameters of TL



at port 1

$$V_1 = V_+ e^{+j\beta l} + V_- e^{-j\beta l}$$

$$I_1 = Y_o\left(V_+ e^{+j\beta l} - V_- e^{-j\beta l}\right)$$

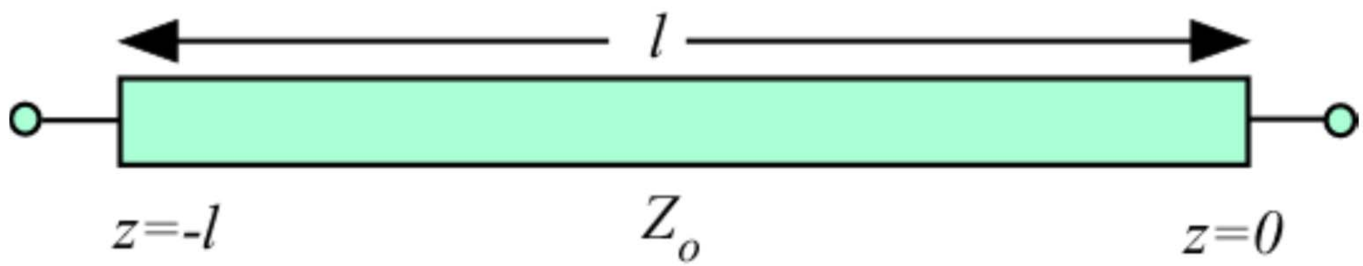
at port 2 (*z* = *0*)

$$V_2 = V_+ + V_-$$

$$I_2 = -Y_o\left(V_+ - V_-\right)$$

$$V_+ = \frac{V_2 - Z_o I_2}{2} \quad \text{and} \quad V_- = \frac{V_2 + Z_o I_2}{2}$$

Y-Parameters of TL



So that

$$V_1 = \left(\frac{V_2 - Z_o I_2}{2}\right)e^{+j\beta l} + \left(\frac{V_2 + Z_o I_2}{2}\right)e^{-j\beta l}$$

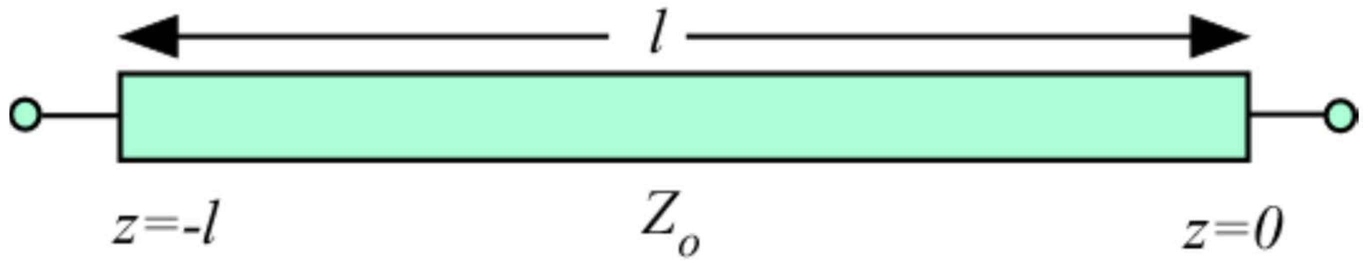
$$I_1 = Y_o\left(\frac{V_2 - Z_o I_2}{2}\right)e^{+j\beta l} - Y_o\left(\frac{V_2 + Z_o I_2}{2}\right)e^{-j\beta l}$$

and

$$V_1 = V_2 \cos\beta l - Z_o I_2 j \sin\beta l$$

$$I_1 = +Y_o V_2 j \sin\beta l - I_2 \cos\beta l$$

Y-Parameters of TL



Using definitions for Y_{11}

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{-I_2 \cos \beta l}{-jZ_o I_2 \sin \beta l} = \frac{-jY_o \cos \beta l}{\sin \beta l}$$

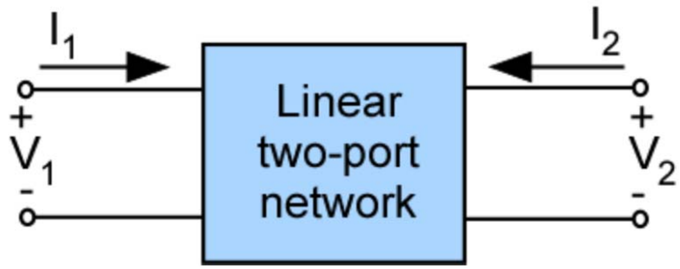
and

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-I_2}{-jZ_o I_2 \sin \beta l} = \frac{+jY_o}{\sin \beta l}$$

$Y_{22} = Y_{11}$ by symmetry

$Y_{12} = Y_{21}$ by reciprocity

TWO-PORT NETWORK REPRESENTATION



<u>Z Parameters</u>

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

<u>Y Parameters</u>

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

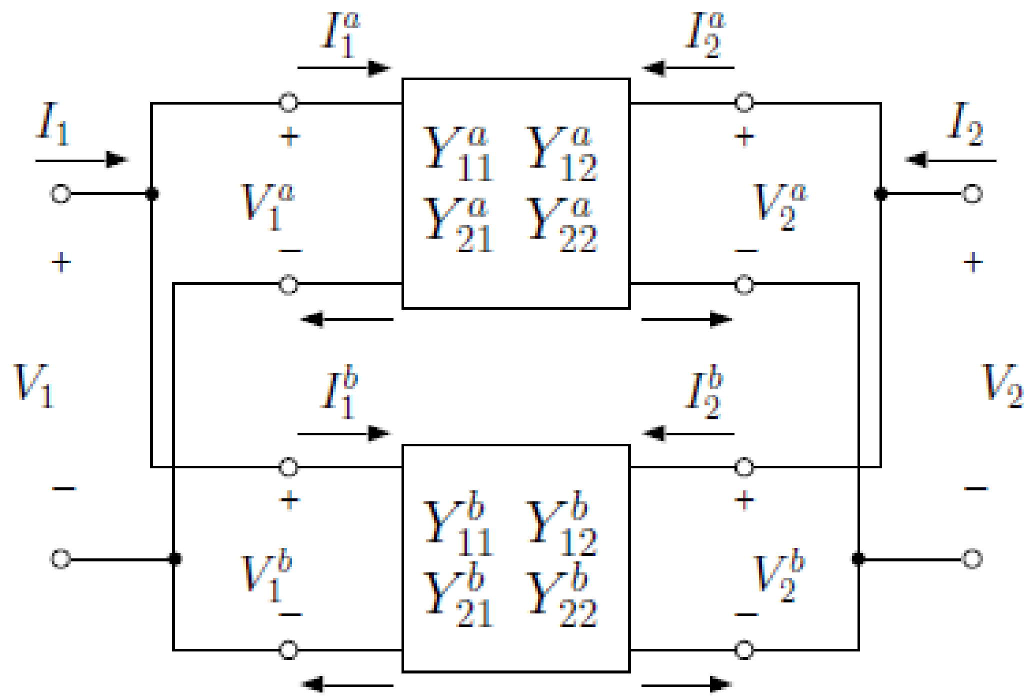
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

- At microwave frequencies, it is more difficult to measure total voltages and currents.

- Short and open circuits are difficult to achieve at high frequencies.

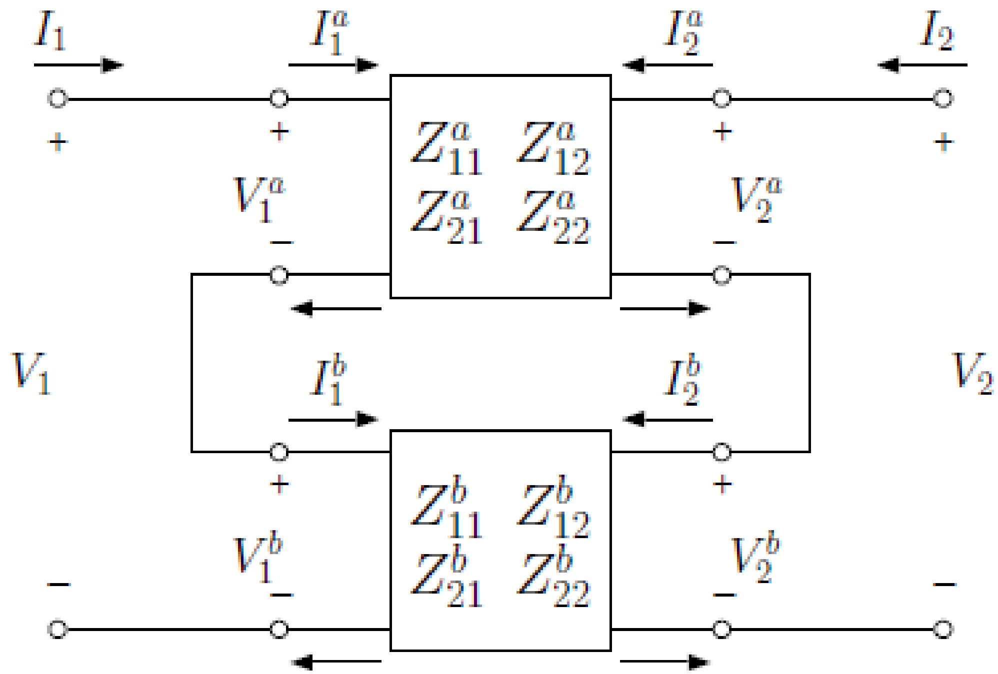
- Most active devices are not short- or open-circuit stable.

Two-Ports in Parallel



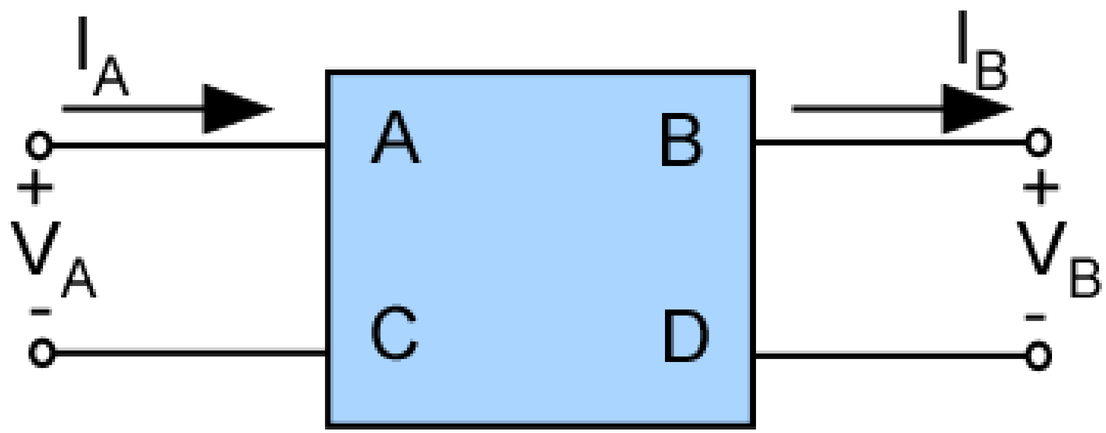
$$\mathbf{Y} = \mathbf{Y}^{\mathbf{a}} + \mathbf{Y}^{\mathbf{b}}$$

Two-Ports in Series



$$\mathbf{Z} = \mathbf{Z^a} + \mathbf{Z^b}$$

ABCD -Parameters



$$V_A = AV_B + BI_B$$

$$I_A = CV_B + DI_B$$

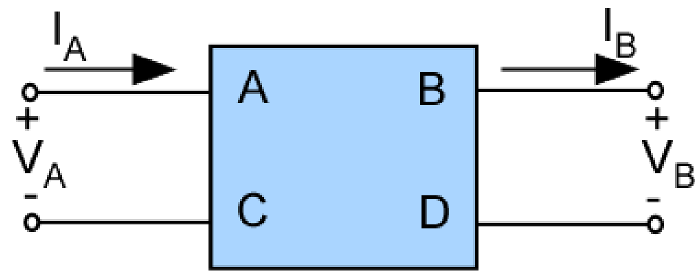
ABCD -Parameters

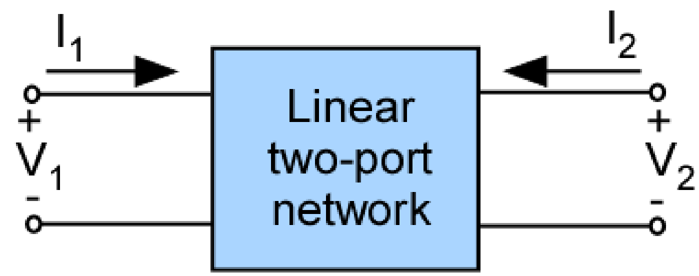
$$V_A = V_1$$

$$V_B = V_2$$

$$I_A = I_1$$

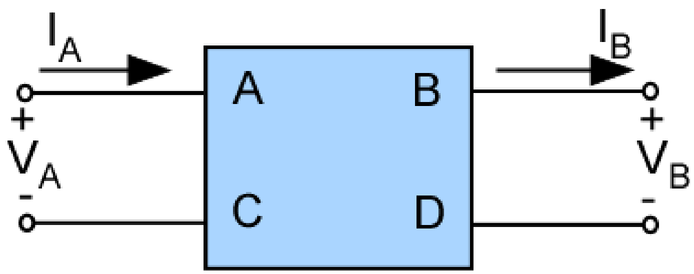
$$I_B = -I_2$$

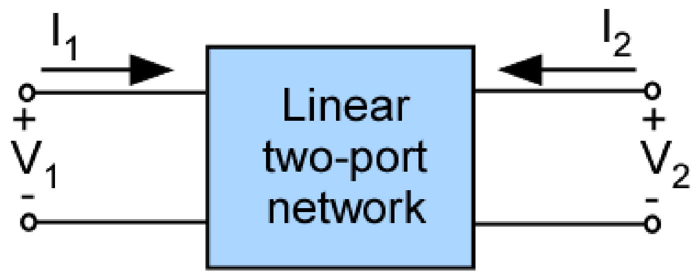




Relationship with Z parameters is obtained by first expressing ABCD parameters in terms of Z parameters

ABCD -Parameters





From

$$V_A = Z_{11} I_A - Z_{12} I_B$$

$$V_B = Z_{21} I_A - Z_{22} I_B$$

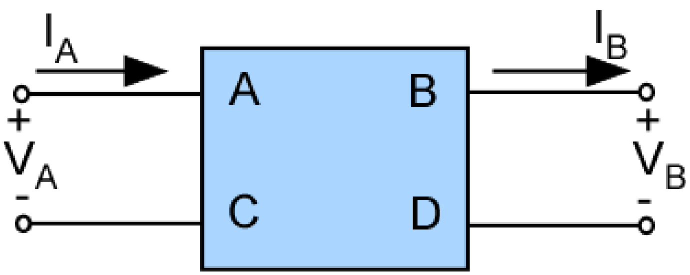
We get

$$A = \frac{Z_{11}}{Z_{21}} \qquad B = \frac{\Delta}{Z_{21}}$$

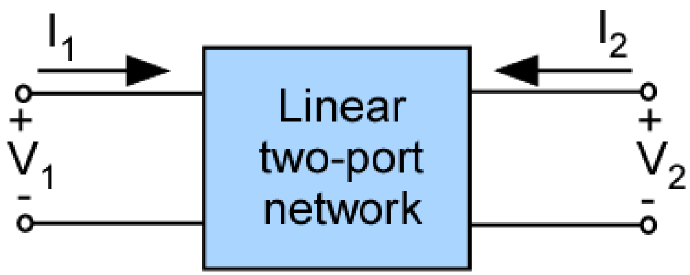
$$C = \frac{1}{Z_{21}} \qquad D = \frac{Z_{22}}{Z_{21}}$$

$$\Delta = Z_{11} Z_{22} - Z_{12} Z_{21}$$

ABCD -Parameters



$$Z_{11} = \frac{A}{C} \qquad Z_{12} = \frac{(AD - BC)}{C}$$



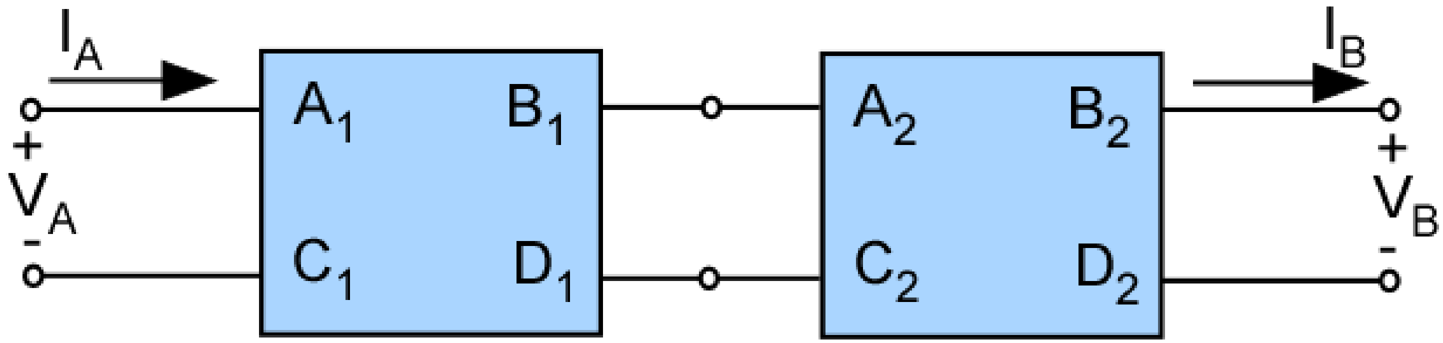
$$Z_{21} = \frac{1}{C} \qquad Z_{22} = \frac{D}{C}$$

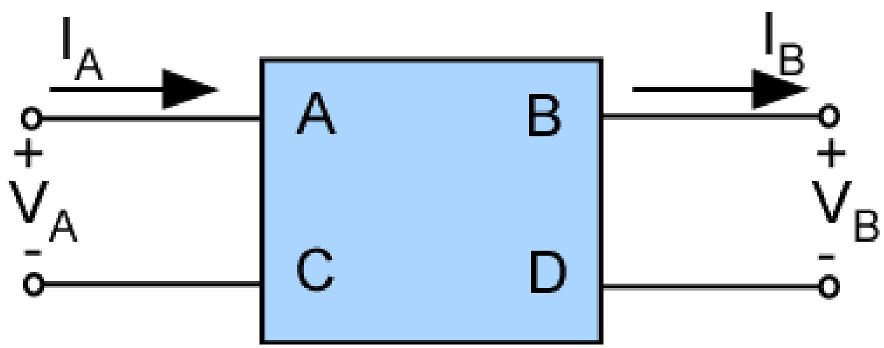
For a reciprocal network, $Z_{21} = Z_{12}$, therefore

$$AD - BC = 1$$

Reciprocity condition for ABCD parameters

ABCD -Parameters





When cascading two-ports, it is best to use ABCD parameters. Put voltage and currents in cascadable form with the input variables in terms of the output variables

$$\mathbf{ABCD} = \left(\mathbf{ABCD}\right)_1 \cdot \left(\mathbf{ABCD}\right)_2$$