ECE 546
Lecture 04
Resistance, Capacitance, Inductance

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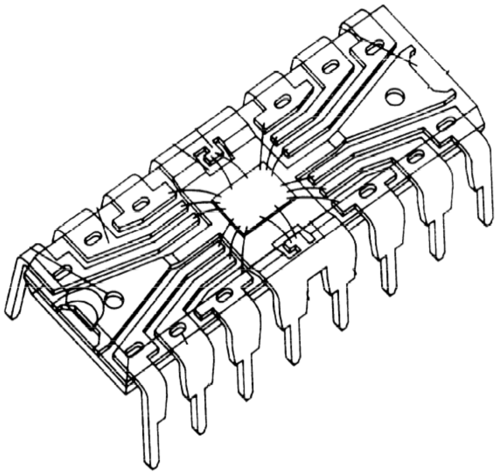
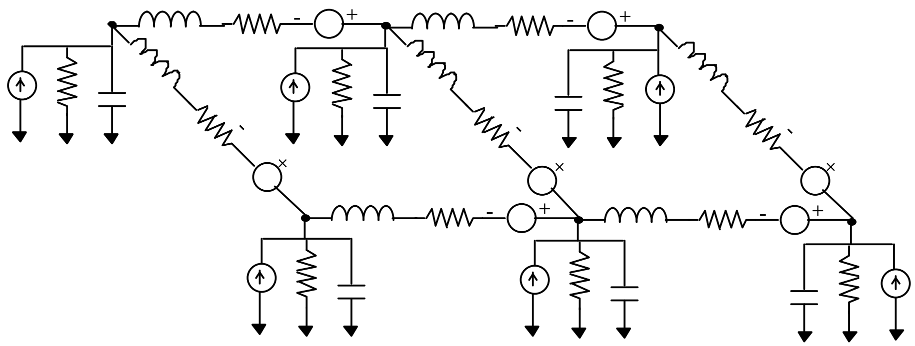
What is Extraction?





Process in which a complex arrangement of conductors and dielectric is converted into a netlist of elements in a form that is amenable to circuit simulation.

Need Field Solvers

Electromagnetic Modeling Tools

We need electromagnetic modeling tools to analyze:

Transmission line propagation
Reflections from discontinuities
Crosstalk between interconnects
Simultaneous switching noise

So we can provide:

Improved design of interconnects
Robust design guidelines
Faster, more cost effective design cycles

Field Solvers – History

♦1960s
Conformal mapping techniques
Finite difference methods (2-D Laplace eq.)
Variational methods

♦1970s
Boundary element method
Finite element method (2-D)
Partial element equivalent circuit (3-D)

♦1980s
Time domain methods (3-D)
Finite element method (3-D)
Moment method (3-D)
rPEEC method (3-D)

♦1990s
Adapting methods to parallel computers
Including methods in CAD tools

♦2000s
Incorporation of Passivity
Incorporation of Causality

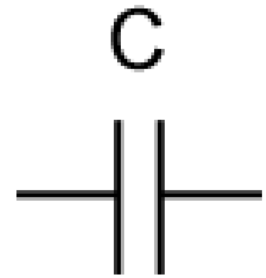
♦2010s
Stochastic Techniques
Multiphysics Tools

Categories of Field Solvers

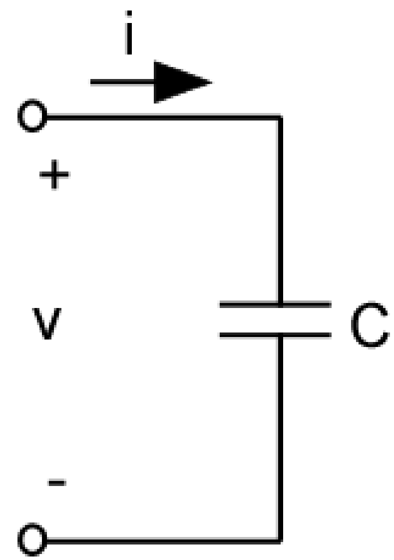
- Method of Moments (MOM)
- Application to 2-D Interconnects
- Closed-Form Green's Function
- Full-Wave and FDTD
- Parallel FDTD
- Applications

Capacitance

Relation: $Q = Cv$



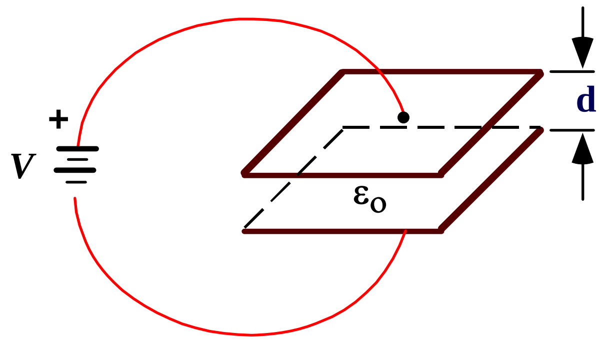
Q: charge stored by capacitor
C: capacitance
v: voltage across capacitor
i: current into capacitor



$$i(t) = C\frac{dv}{dt} = \frac{dQ}{dt}$$

$$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$$

Capacitance



$$C = \frac{\varepsilon_o A}{d}$$

A : area

ε_o : permittivity

For more complex capacitance geometries, need to use numerical methods

Capacitance Calculation

$$\phi(r) = \int g(r,r')\sigma(r')dr'$$

$$\phi(r) = potential\ (known)$$

$$g(r,r') = Green's\ function\ (\text{known})$$

$$\sigma(r') = \text{charge distribution}\ (unknown)$$

Once the charge distribution is known, the total charge Q can be determined. If the potential ϕ=V, we have

Q=CV

To determine the charge distribution, use the moment method

METHOD OF MOMENTS

Operator equation

$$L(f) = g$$

L = integral or differential operator

f = unknown function

g = known function

Expand unknown function f

$$f = \sum_n \alpha_n f_n$$

METHOD OF MOMENTS

in terms of basis functions f_n, with unknown coefficients α_n to get

$$\sum_n \alpha_n L(f_n) = g$$

Finally, take the scalar or ***inner product*** with testing of weighting functions w_m:

$$\sum_n \alpha_n \langle w_m, L f_n \rangle = \langle w_m, g \rangle$$

Matrix equation

$$[l_{mn}][\alpha_n] = [g_m]$$

METHOD OF MOMENTS

$$[l_{mn}] = \begin{bmatrix} \langle w_1, Lf_1 \rangle & \langle w_1, Lf_2 \rangle & \ldots \\ \langle w_2, Lf_1 \rangle & \langle w_2, Lf_2 \rangle & \ldots \\ \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots & \ldots \end{bmatrix}$$

$$[\alpha_n] = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ . \end{bmatrix} \qquad [g_m] = \begin{bmatrix} \langle w_1, g \rangle \\ \langle w_2, g \rangle \\ . \end{bmatrix}$$

Solution for weight coefficients

$$[\alpha_n] = \left[l_{nm}^{-1} \right] [g_m]$$

Moment Method Solution

$$\nabla \cdot D = \rho$$

$$E = -\nabla \phi$$

$$\phi(x,y,z) = \iiint \frac{\rho(x',y',z')}{4\pi\varepsilon R} dx'dy'dz'$$

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon}$$

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

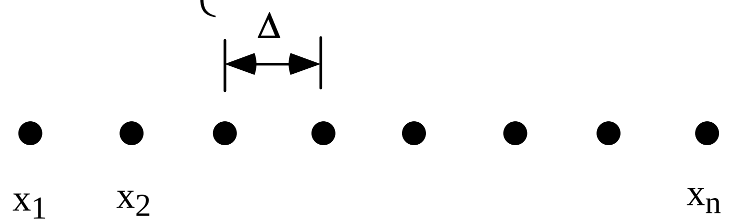
$$L\phi = -\frac{\rho}{\varepsilon}$$

Green's function G: $LG = \delta$

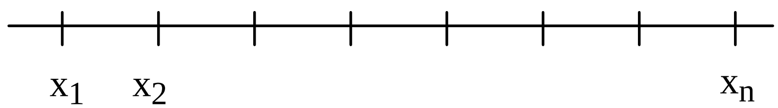
Basis Functions

Subdomain bases

$$P(x_n)=\begin{cases}1 & x_n-\dfrac{\Delta}{2}<x<x_n+\dfrac{\Delta}{2}\\ 0 & otherwise\end{cases}$$

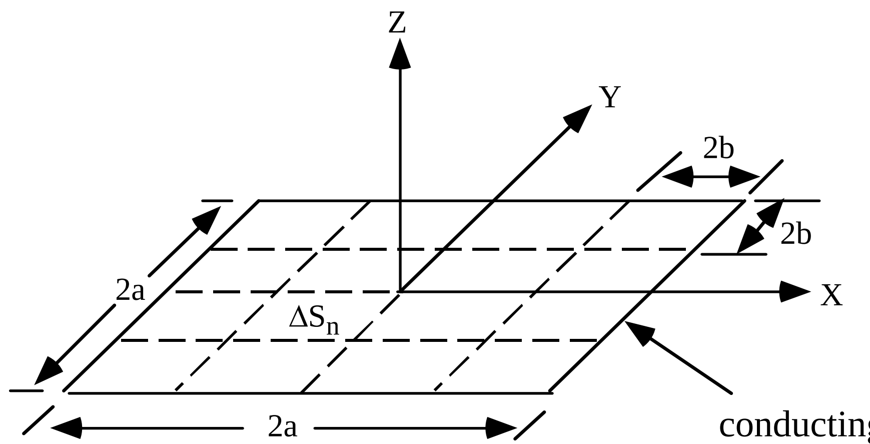


$$T(x_n)=\begin{cases}1-|x| & x_n-\dfrac{\Delta}{2}<x<x_n+\dfrac{\Delta}{2}\\ 0 & otherwise\end{cases}$$



Testing functions often (not always) chosen same as basis function.

Conducting Plate



$$\phi(x,y,z) = \int_{-a}^{a} dx' \int_{-a}^{a} dy' \frac{\sigma(x',y',z')}{4\pi\varepsilon R}$$

σ = charge density on plate

Conducting Plate

Setting ϕ = V on plate

$$R=\sqrt{\left(x-x'\right)^2+\left(y-y'\right)^2}$$

$$V=\int_{-a}^{a}dx'\int_{-a}^{a}dy'\frac{\sigma\left(x',y',z'\right)}{4\pi\varepsilon\sqrt{\left(x-x'\right)^2+\left(y-y'\right)^2}}$$

for |x| < a; |y| < a

Capacitance of plate: $C=\dfrac{q}{V}=\dfrac{1}{V}\displaystyle\int_{-a}^{a}dx'\int_{-a}^{a}dy'\sigma\left(x',y',z'\right)$

Conducting Plate

Basis function P_n

$$P_n\left(x_m, y_n\right) = \begin{cases} 1 & x_m - \dfrac{\Delta s}{2} < x < x_m + \dfrac{\Delta s}{2} \\ 1 & y_n - \dfrac{\Delta s}{2} < y < y_n + \dfrac{\Delta s}{2} \\ 0 & otherwise \end{cases}$$

Representation of unknown charge

$$\sigma\left(x, y\right) = \sum_{n=1}^{N} \alpha_n \mathrm{f}_\mathrm{n}$$

Conducting Plate

Matrix equation:

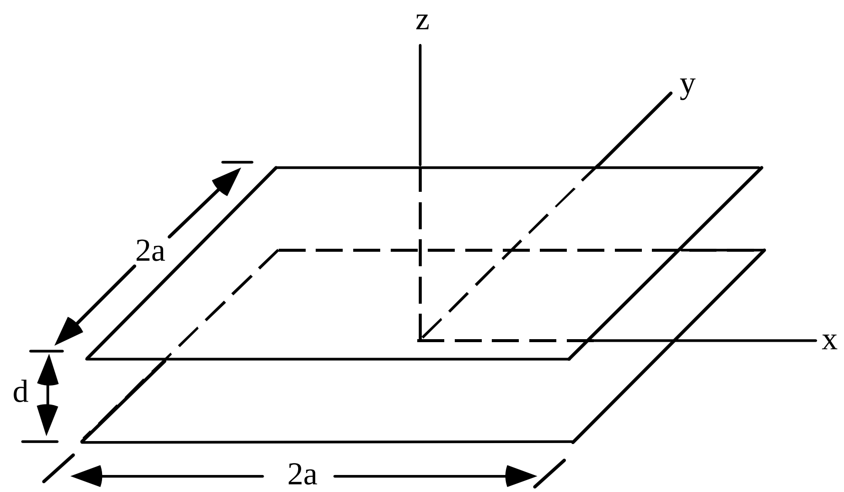
$$V = \sum_{n=1}^{N} l_{mn} f_n$$

Matrix element:

$$l_{mn} = \int_{\Delta x_m} dx' \int_{\Delta y_n} dy' \frac{1}{4\pi\varepsilon\sqrt{\left(x_m - x'\right)^2 + \left(y_n - x'\right)^2}}$$

$$C = \frac{1}{V}\sum_{n=1}^{N} \alpha_n \Delta s_n = \sum_{mn} l_{mn}^{-1} \Delta s_n$$

Parallel Plates



Using N unknowns per plate, we get 2N × 2N matrix equation:

$$[l]=\begin{bmatrix}[l^{tt}] & [l^{tb}] \\ [l^{bt}] & [l^{bb}]\end{bmatrix}$$

Subscript ‘t’ for top and ‘b’ for bottom plate, respectively.

Parallel Plates

Matrix equation becomes $\left[l_{mn}^{tt} - l_{mn}^{tb} \right] \left[\alpha_n^t \right] = \left[g_m^t \right]$

Solution: $\left[\alpha_m^t \right] = \left[\left(l^{tt} - l^{tb} \right)_{mn}^{-1} \right] \left[g_n^t \right]$

Capacitance $C = \dfrac{\text{charge on top plate}}{2\text{V}}$

$$= \frac{1}{2V} \sum_{top} \alpha_n^t \Delta s_n$$

Using $\Delta s = 4b^2$ and all elements of $\left[g^t \right] = V$

$$C = 2b^2 \sum_{mn} \left(l^{tt} - l^{tb} \right)_{mn}^{-1}$$

Inductance

Relation: $\Psi = Li$

Ψ: flux stored by inductor

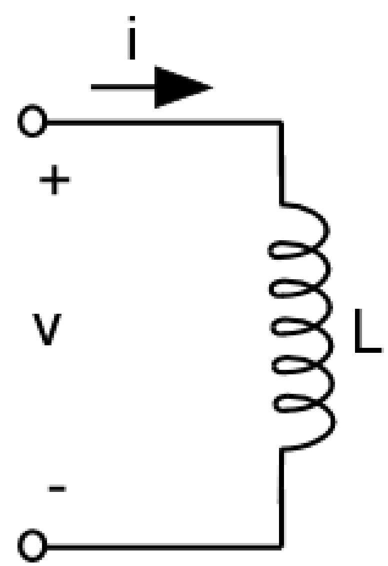
L: inductance

i: current through inductor

v: voltage across inductor

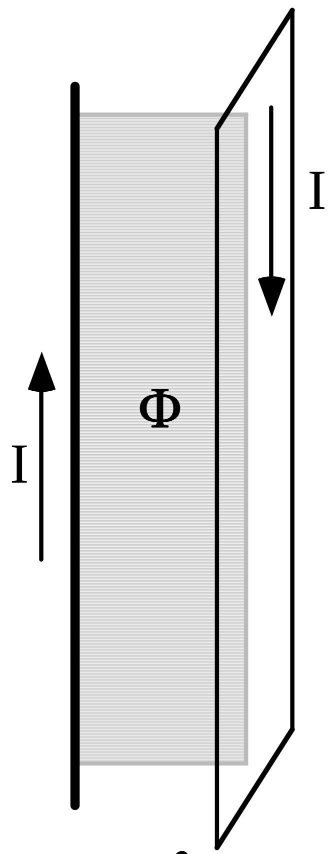
$$v(t) = L\frac{di}{dt} = \frac{d\Psi}{dt}$$

$$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$$

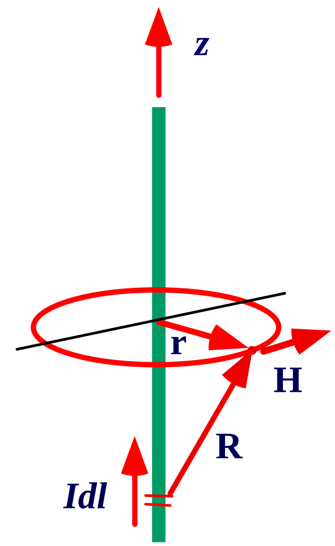


Inductance

Magnetic Flux

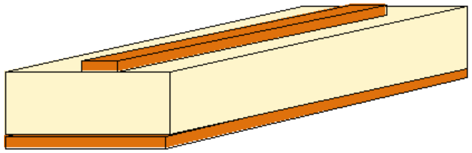


$$\Phi = \int_S \vec{B} \cdot d\vec{S}$$



$$\text{Inductance} = \frac{\text{Total flux linked}}{\text{Current}}$$

2-D Isomorphism



Electrostatics

$$\left(\hat{z}\times\hat{n}\right)\cdot\nabla_t V_i = 0$$

$$\hat{n}\cdot\left(\varepsilon_{ri}\nabla_t V_i\right) = -\frac{q_s}{\varepsilon_o}$$

$$CV = Q$$

Magnetostatics

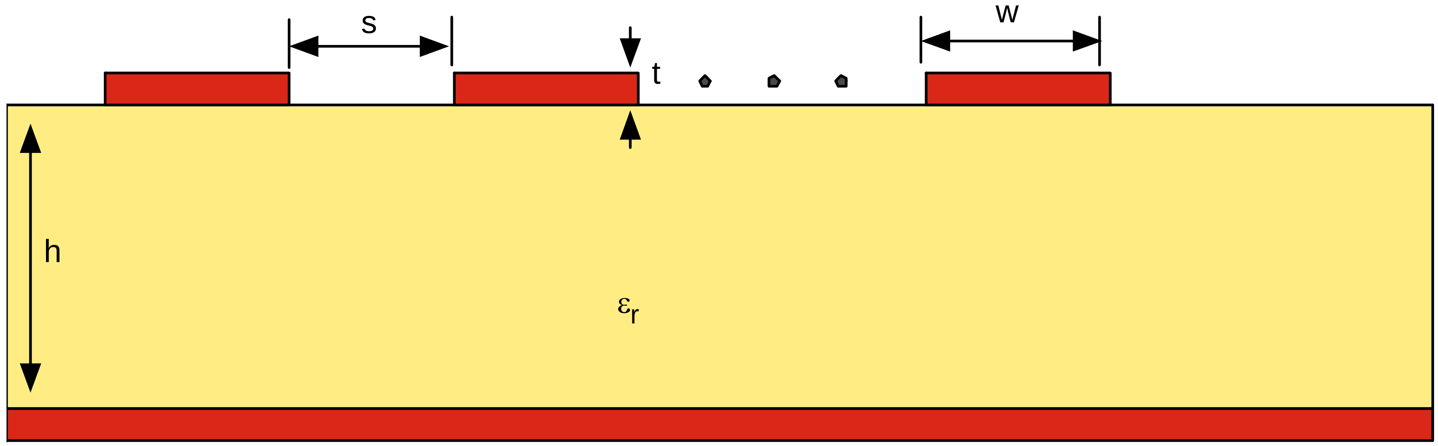
$$\left(\hat{z}\times\hat{n}\right)\cdot\nabla_t A_{zi} = 0$$

$$\hat{n}\cdot\left(\frac{1}{\mu_{ri}}\nabla_t A_{zi}\right) = -\mu_o J_z$$

$$LI = \psi$$

Consequence: 2D inductance can be calculated from 2D capacitance formulas

2-D N-line LC Extractor using MOM



- Symmetric signal traces
- Uniform spacing
- Lossless lines
- Uses MOM for solution

Output from MoM Extractor

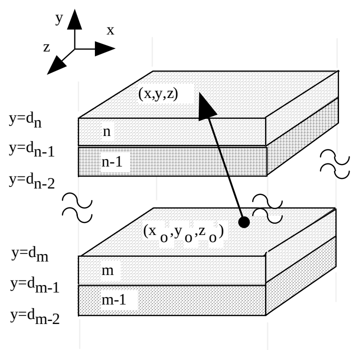
Capacitance (pF/m)

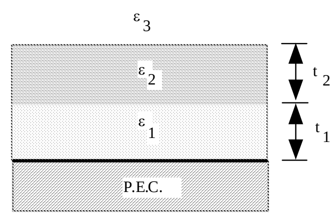
118.02299	-8.86533	-0.03030	-0.00011	-0.00000
-8.86533	119.04875	-8.86185	-0.03029	-0.00011
-0.03030	-8.86185	119.04876	-8.86185	-0.03030
-0.00011	-0.03029	-8.86185	119.04875	-8.86533
-0.00000	-0.00011	-0.03030	-8.86533	118.02299

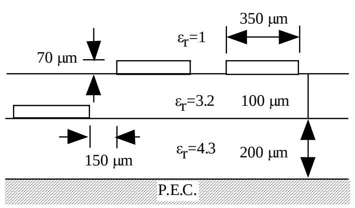
Inductance (nH/m)

312.71680	23.42397	1.83394	0.14361	0.01128
23.42397	311.76042	23.34917	1.82812	0.14361
1.83394	23.34917	311.75461	23.34917	1.83394
0.14361	1.82812	23.34917	311.76042	23.42397
0.01128	0.14361	1.83394	23.42397	312.71680

RLGC: Formulation Method







Closed-Form Spatial Green's Function

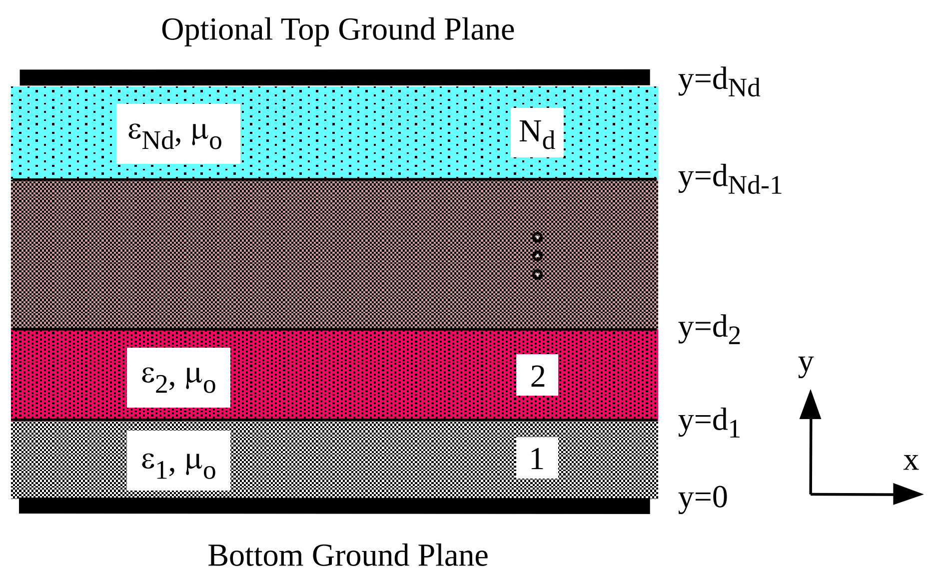
*** Computes 2-D and 3-D capacitance matrix in multilayered dielectric**

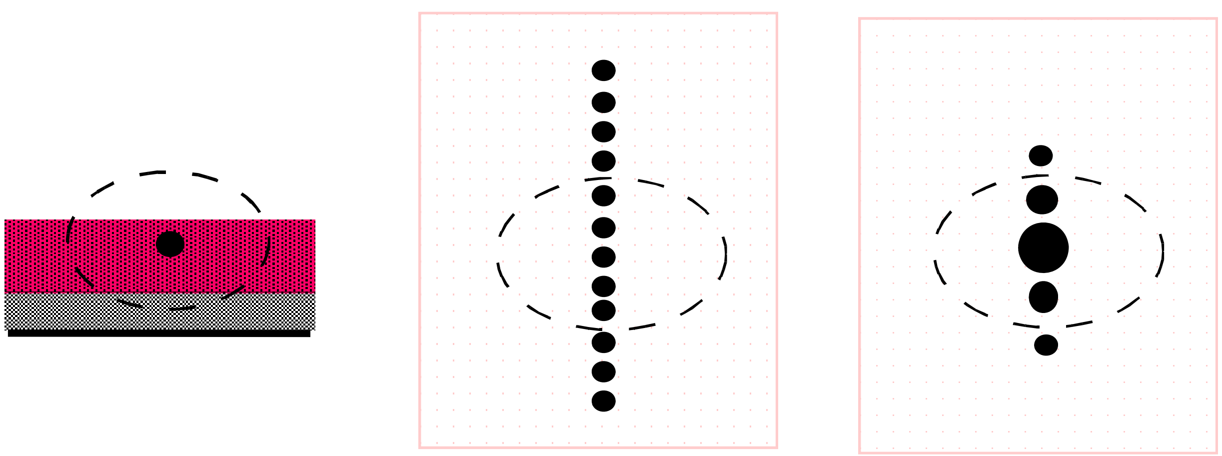
*** Method is applicable to arbitrary polygon-shaped conductors**

*** Computationally efficient**

- **Reference**
  - K. S. Oh, D. B. Kuznetsov and J. E. Schutt-Aine, "Capacitance Computations in a Multilayered Dielectric Medium Using Closed-Form Spatial Green's Functions," IEEE Trans. Microwave Theory Tech., vol. MTT-42, pp. 1443-1453, August 1994.

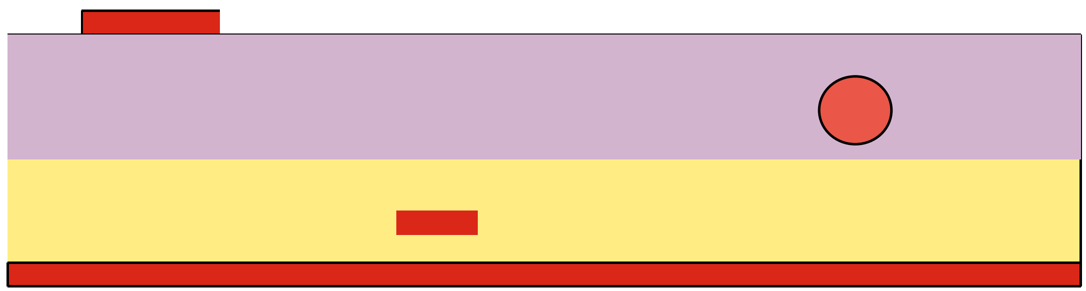
Multilayer Green's Function





Extraction Program: RLGC

RLGC computes the four transmission line parameters, viz., the capacitance matrix **C**, the inductance matrix **L**, the conductance matrix **G**, and the resistance matrix **R**, of a multiconductor transmission line in a multilayered dielectric medium. **RLGC** features the following list of functions:

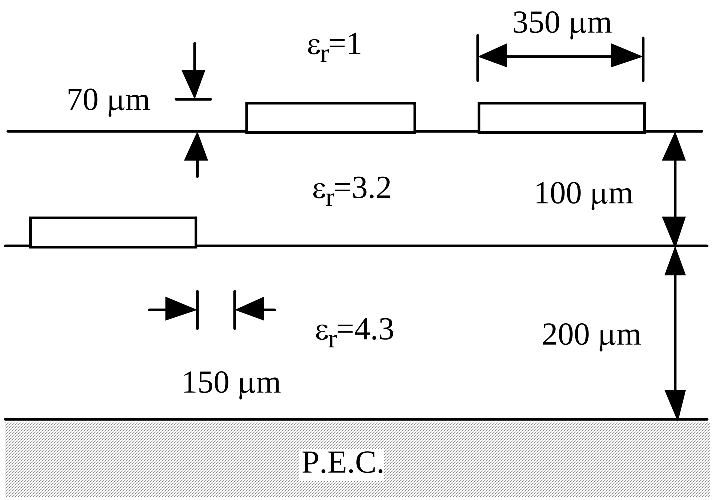


RLGC – Multilayer Extractor

- **Features**
  - Handling of dielectric layers with no ground plane, either top or bottom ground plane (microstrip cases), or both top and bottom ground planes (stripline cases)
  - Static solutions are obtained using the Method of Moment (MoM) in conjunction with the recently-developed closed-form Green's functions: one of the most accurate and efficient methods for static analysis
  - Modeling of dielectric losses as well as conductor losses (including ground plane losses
  - The resistance matrix R is computed based on the current distribution - more accurate than the use of any closed-form formulae
  - Both the proximity effect and the skin effect are modeled in the resistance matrix R.
  - Computes the potential distribution
  - Handling of an arbitrary number of dielectric layers as well as an arbitrary number of conductors.
  - The cross section of a conductor can be arbitrary or even be infinitely thin

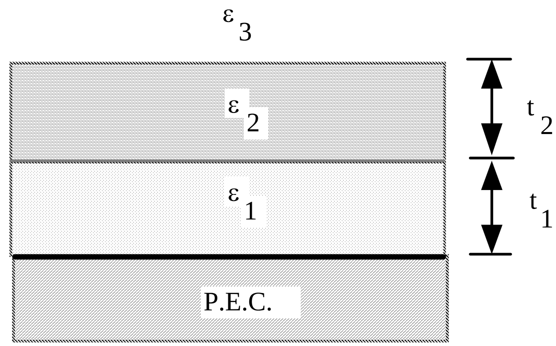
- **Reference**
  - K. S. Oh, D. B. Kuznetsov and J. E. Schutt-Aine, "Capacitance Computations in a Multilayered Dielectric Medium Using Closed-Form Spatial Green's Functions," IEEE Trans. Microwave Theory Tech., vol. MTT-42, pp. 1443-1453, August 1994.

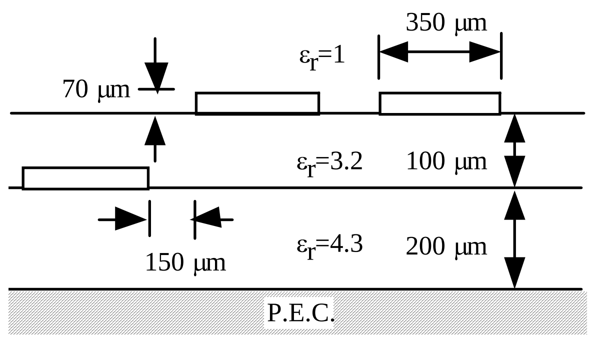
RLGC – General Topology



Three conductors in a layered medium. All conductor dimensions and spacing are identical. The loss tangents of the lower and upper dielectric layers are 0.004 and 0.001respectively, the conductivity of each line is 5.8e7 S/m, and the operating frequency is 1 GHz

3-Line Capacitance Results



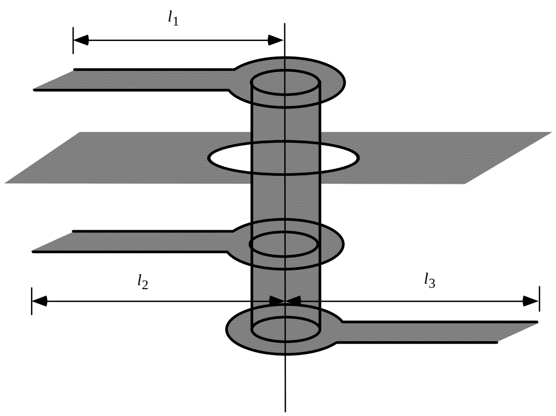


Capacitance Matrix (pF/m)

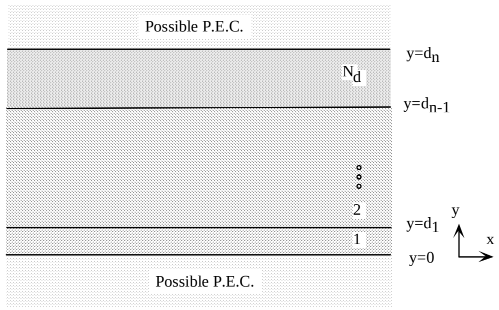
$$\begin{bmatrix} 142.09 & -21.765 & -0.8920 \\ -21.733 & 93.529 & -18.098 \\ -0.8900 & -18.097 & 87.962 \end{bmatrix} \qquad \begin{bmatrix} 145.33 & -23.630 & -1.4124 \\ -22.512 & 93.774 & -17.870 \\ -1.3244 & -17.876 & 87.876 \end{bmatrix}$$

Delabare et al. RLGC Method

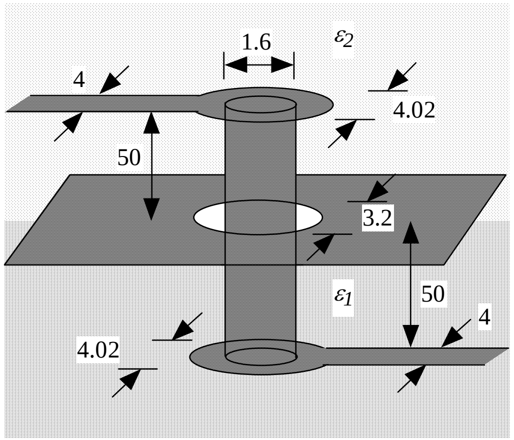
Modeling Vias



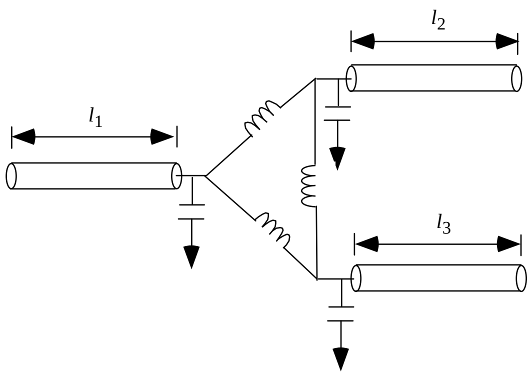
Via



Medium

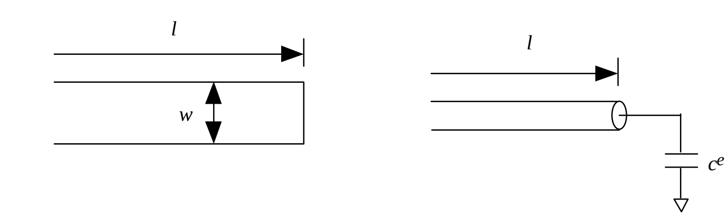


Via in multilayer medium

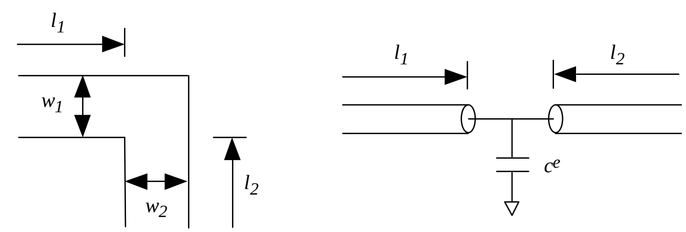


Equivalent circuit

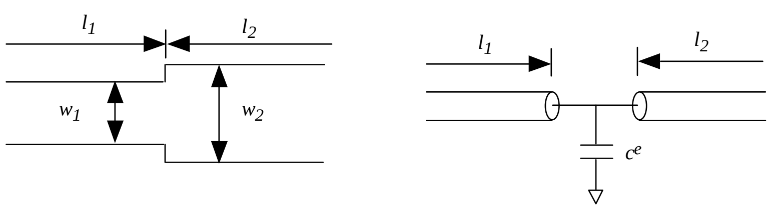
Modeling Discontinuities



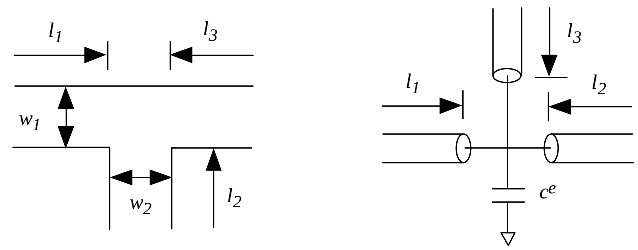
Open



Bend



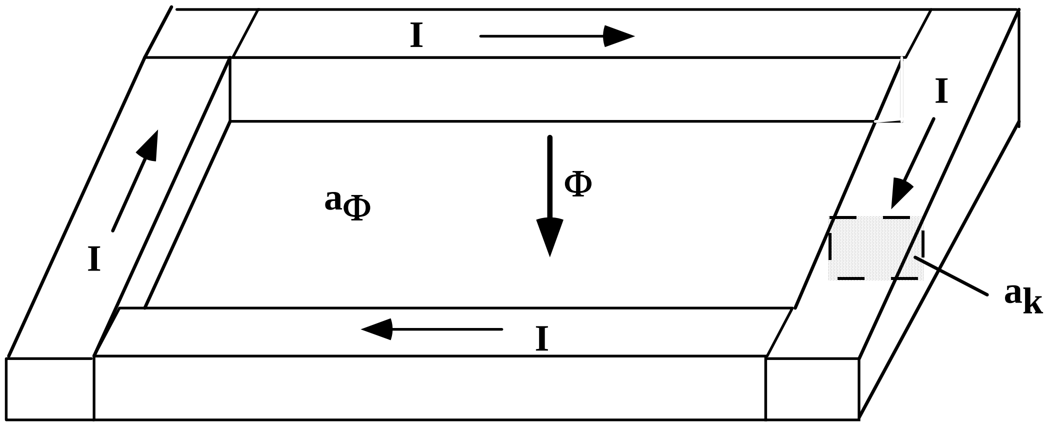
Step



T-Junction

3D Inductance Calculation

Loop Inductance

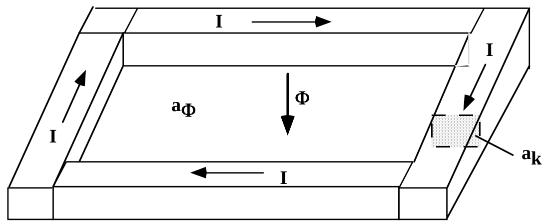


$$L_{loop} = \frac{\Phi}{I} = \frac{1}{I}\int_{a_\Phi} \vec{B}\cdot d\vec{a} = \frac{1}{I}\int_{a_\Phi} \left(\nabla\times\vec{A}\right)\cdot d\vec{a}$$

QUESTION: Can we associate inductance with piece of conductor rather than a loop? ➔PEEC Method

Partial Inductance (PEEC) Approach

QUESTION: Can we associate inductance with piece of conductor rather than a loop?



$$L_{loop} = \sum_{i=1}^{4} \sum_{j=1}^{4} \frac{1}{a_i a_j} \int_{a_i} \int_{a_j} \int_{l_i} \int_{l_j} \frac{\mu}{4\pi} \frac{d\vec{l}_i \cdot d\vec{l}_j}{\left|\vec{r}_i - \vec{r}_j\right|} da_i da_j$$

DEFINITION OF PARTIAL INDUCTANCE

$$L_{pij} = \frac{1}{a_i a_j} \frac{\mu}{4\pi} \int_{a_i} \int_{a_j} \int_{l_i} \int_{l_j} \frac{d\vec{l}_i \cdot d\vec{l}_j}{\left|\vec{r}_i - \vec{r}_j\right|} da_i da_j$$

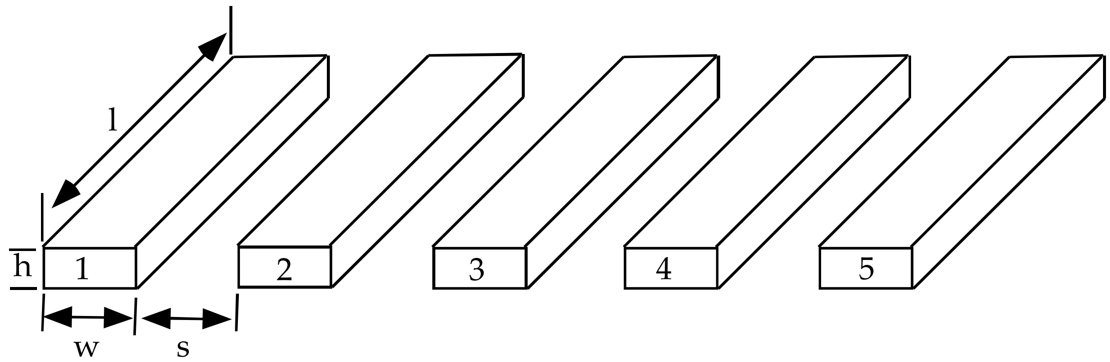
$$L_{loop} = \sum_{i=1}^{4} \sum_{j=1}^{4} s_{ij} L_{pij}$$

Circuit Element K

$$[K]=[L]^{-1}$$

- **Better locality property**
- **Leads to sparser matrix**
- **Diagonally dominant**
- **Allows truncation of far off-diagonal elements**
- **Better suited for on-chip inductance analysis**

Locality of K Matrix



$$[L] = \begin{bmatrix} 11.4 & 4.26 & 2.54 & 1.79 & 1.38 \\ 4.26 & 11.4 & 4.26 & 2.54 & 1.79 \\ 2.54 & 4.26 & 11.4 & 4.26 & 2.54 \\ 1.79 & 2.54 & 4.26 & 11.4 & 4.26 \\ 1.38 & 1.79 & 2.54 & 4.26 & 11.4 \end{bmatrix} \qquad [K] = \begin{bmatrix} 103 & -34.1 & -7.80 & -4.31 & -3.76 \\ -34.1 & 114 & -31.6 & -6.67 & -4.31 \\ -7.80 & -31.6 & 115 & -31.6 & -7.80 \\ -4.31 & -6.67 & -31.6 & 114 & -34.1 \\ -3.76 & -4.31 & -7.80 & -34.1 & 103 \end{bmatrix}$$

Package Inductance & Capacitance

Component	Capacitance (pF)	Inductance (nH)
68 pin plastic DIP pin†	4	35
68 pin ceramic DIP pin††	7	20
68 pin SMT chip carrier †	2	7

† No ground plane; capacitance is dominated by wire-to-wire component.

†† With ground plane; capacitance and inductance are determined by the distance between the lead frame and the ground plane, and the lead length.

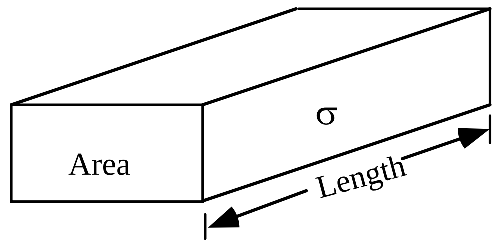
Package Inductance & Capacitance

Component	Capacitance (pF)	Inductance (nH)
68 pin PGA pin††	2	7
256 pin PGA pin††	5	15
Wire bond	1	1
Solder bump	0.5	0.1

† No ground plane; capacitance is dominated by wire-to-wire component.

†† With ground plane; capacitance and inductance are determined by the distance between the lead frame and the ground plane, and the lead length.

Metallic Conductors



Resistance : R

$$R = \frac{Length}{\sigma \cdot Area}$$

Package level:

W=3 mils

R=0.0045 Ω/mm

Submicron level:

W=0.25 microns

R=422 Ω/mm

Metallic Conductors

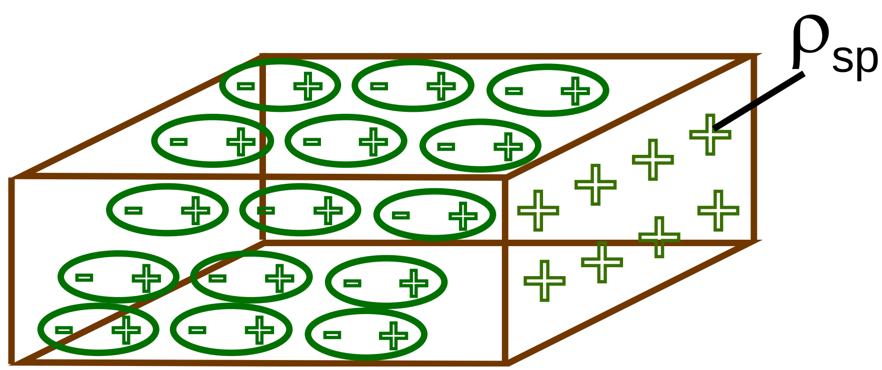
Metal	Conductivity σ (Ω^{-1} m^{-1} $\times 10^{-7}$)
Silver	6.1
Copper	5.8
Gold	3.5
Aluminum	1.8
Tungsten	1.8
Brass	1.5
Solder	0.7
Lead	0.5
Mercury	0.1

Dielectrics

- Dielectrics contain charges that are tightly bound to the nuclei
- Charges can move a fraction of an atomic distance away from equilibrium position
- Electron orbits can be distorted when an electric field is applied

Dielectrics

- Charge density within volume is zero
- Surface charge density is nonzero



$$\mathbf{D}=\varepsilon_o(1+\chi_e)\mathbf{E}=\varepsilon\mathbf{E}$$

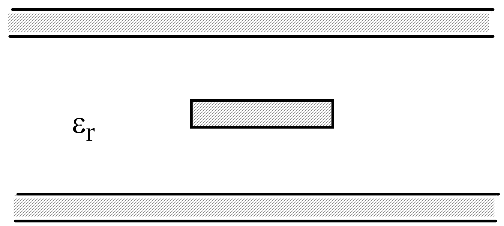
Dielectric Materials

$$v = \sqrt{\frac{1}{LC}}$$

Material	Conductivity (Ω^{-1}-m^{-1})
Germanium	2.2
Silicon	0.0016
Glass	10^{-10}-10^{-14}
Quartz	0.5×10^{-17}

$$\tan\delta = \frac{\sigma}{\omega\varepsilon}$$

Dielectric Materials



$$v = \sqrt{\frac{1}{LC}}$$

Material	ε_r	v(cm/s)
Polyimide	2.5-3.5	16-19
Silicon dioxide	3.9	15
Epoxy glass (FR4)	5.0	13
Alumina (ceramic)	9.5	10

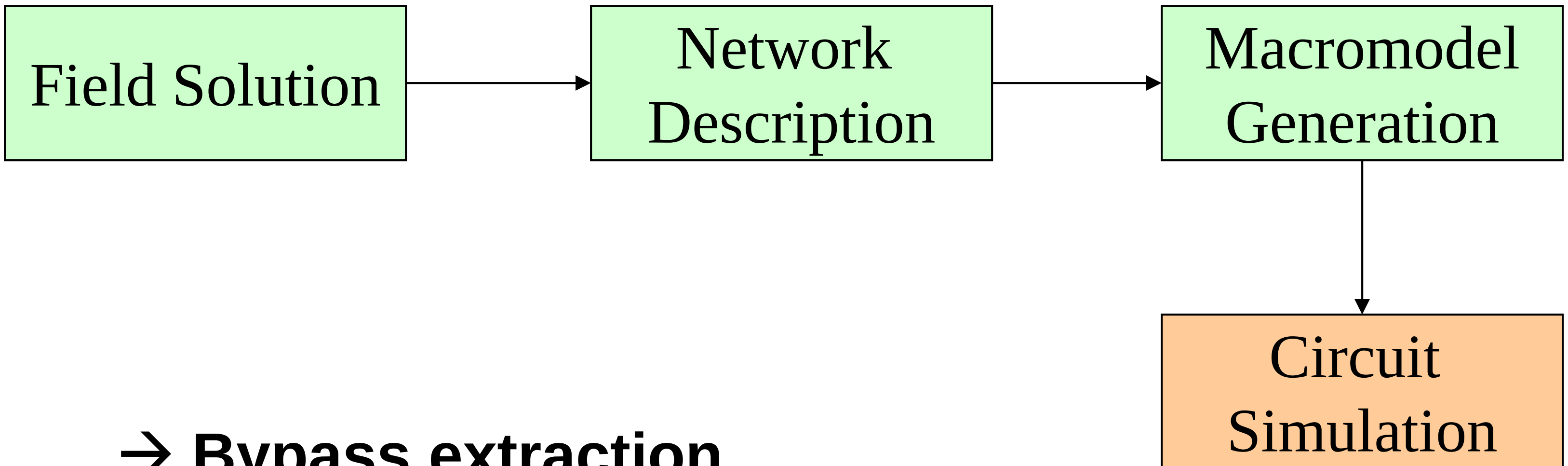
Conductivity of Dielectric Materials

$$\varepsilon = \varepsilon_r + j\,\varepsilon_i$$

Material	Conductivity ($\Omega^{-1}\ m^{-1}$)
Germanium	2.2
Silicon	0.0016
Glass	10^{-10} - 10^{-14}
Quartz	0.5×10^{-17}

$$\text{Loss TANGENT : } \tan\delta = \frac{\sigma}{\omega\,\varepsilon}$$

Combining Field and Circuit Solutions



→ **Bypass extraction procedure through the use of Y, Z, or S parameters (frequency domain)**

Full-Wave Methods

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ **Faraday's Law of Induction**

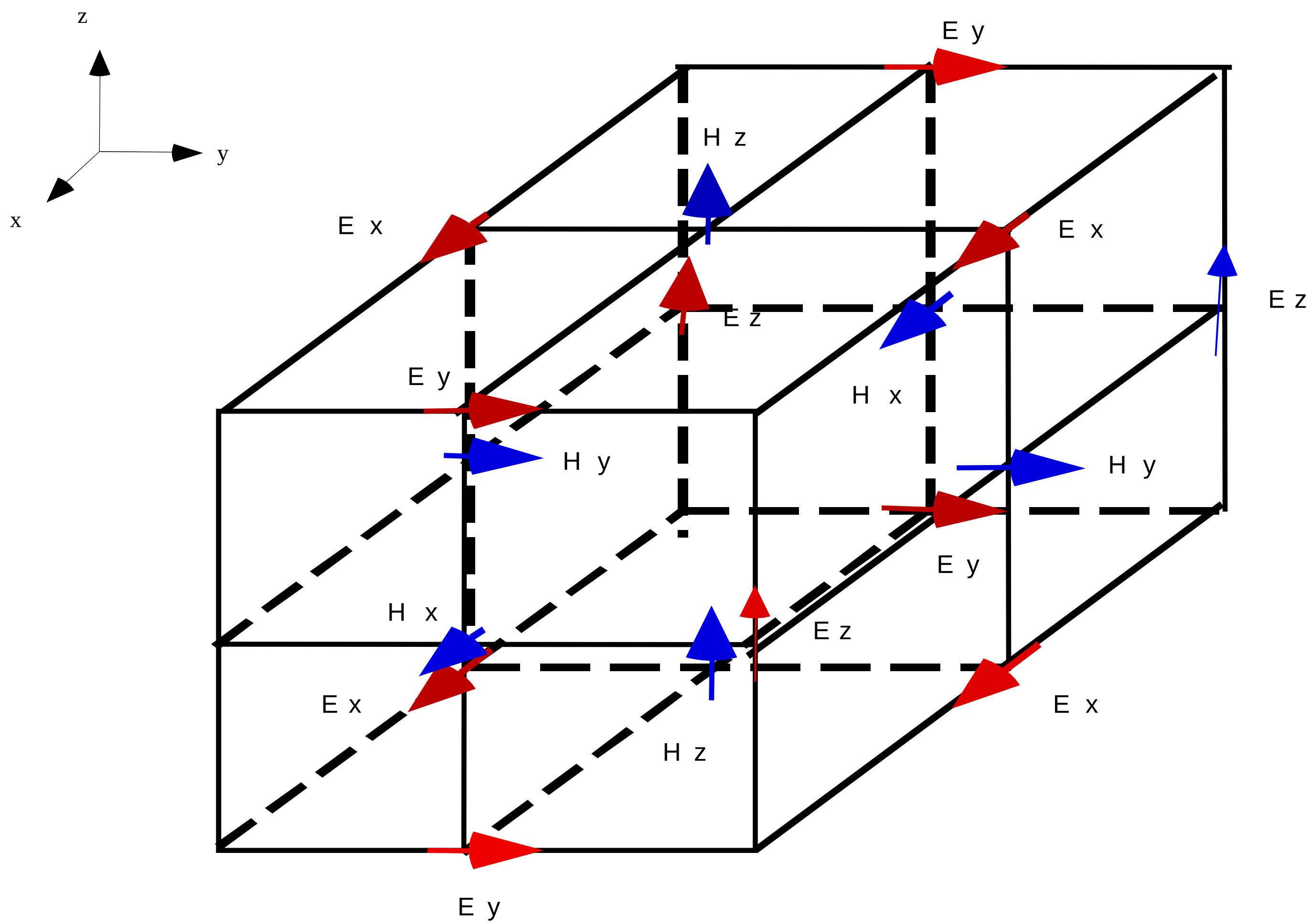
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$ **Ampère's Law**

$$\nabla \cdot \vec{D} = \rho$$ **Gauss' Law for electric field**

$$\nabla \cdot \vec{B} = 0$$ **Gauss' Law for magnetic field**

FDTD: Discretize equations and solve with appropriate boundary conditions

Finite Difference Time Domain (FDTD)

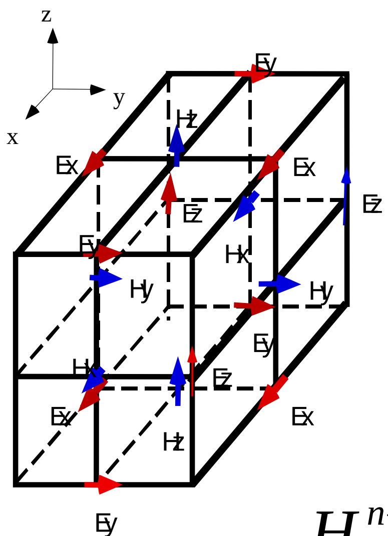


Space Discretization

FDTD – Yee Algorithm

$$E_x^n(i,j,k) = E_x^{n-1} + \frac{c}{\varepsilon}\frac{\Delta t}{\Delta y}\left(H_z^{n-1/2}(i,j,k) - H_z^{n-1/2}(i,j-1,k)\right)$$

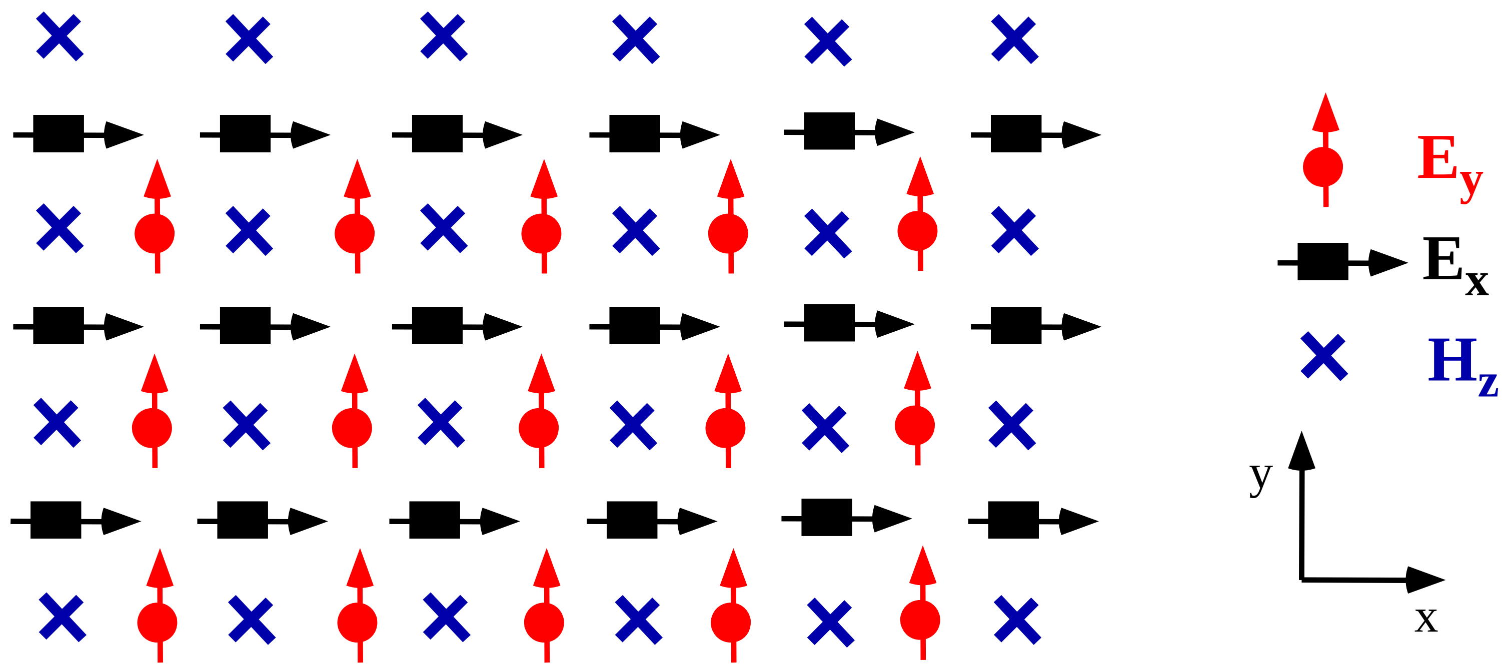
$$-\frac{c}{\varepsilon}\frac{\Delta t}{\Delta z}\left(H_y^{n-1/2}(i,j,k) - H_y^{n-1/2}(i,j,k-1)\right)$$



$$H_x^{n+1/2}(i,j,k) = H_x^{n-1/2} + \frac{c}{\mu}\frac{\Delta t}{\Delta y}\left(E_z^n(i,j+1,k) - E_z^n(i,j,k)\right)$$

$$+\frac{c}{\mu}\frac{\Delta t}{\Delta z}\left(E_y^{n-1/2}(i,j,k+1) - E_y^n(i,j,k)\right)$$

2D-FDTD



$$E_x^n\left(i+\frac{1}{2},j\right)=E_x^{n-1}\left(i+\frac{1}{2},j\right)+\frac{\Delta t}{\varepsilon_o\Delta y}\left[H_z^{n-1/2}\left(i+\frac{1}{2},j+\frac{1}{2}\right)-H_z^{n-1/2}\left(i+\frac{1}{2},j-\frac{1}{2}\right)\right]$$

$$E_y^n\left(i,j+\frac{1}{2}\right)=E_y^{n-1}\left(i,j+\frac{1}{2}\right)-\frac{\Delta t}{\varepsilon_o\Delta x}\left[H_z^{n-1/2}\left(i+\frac{1}{2},j+\frac{1}{2}\right)-H_z^{n-1/2}\left(i-\frac{1}{2},j+\frac{1}{2}\right)\right]$$

$$H_z^{n+1/2}\left(i+\frac{1}{2},j+\frac{1}{2}\right)=H_z^{n-1/2}\left(i+\frac{1}{2},j+\frac{1}{2}\right)+\frac{\Delta t}{\mu_o\Delta y}\left[E_x^n\left(i+\frac{1}{2},j+1\right)-E_x^n\left(i+\frac{1}{2},j\right)\right]$$

$$-\frac{\Delta t}{\mu_o\Delta x}\left[E_y^n\left(i+1,j+\frac{1}{2}\right)-E_x^n\left(i,j+\frac{1}{2}\right)\right]$$

Absorbing Boundary Condition: 2D-PML Formulation

Simulation Medium

$$\varepsilon_o \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y}$$

$$\varepsilon_o \frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial x}$$

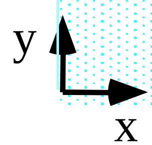
$$\mu_o \frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}$$

PML Medium

$$\varepsilon_o \frac{\partial E_x}{\partial t} + \sigma E_x = \frac{\partial H_z}{\partial y}$$

$$\varepsilon_o \frac{\partial E_y}{\partial t} + \sigma E_y = -\frac{\partial H_z}{\partial x}$$

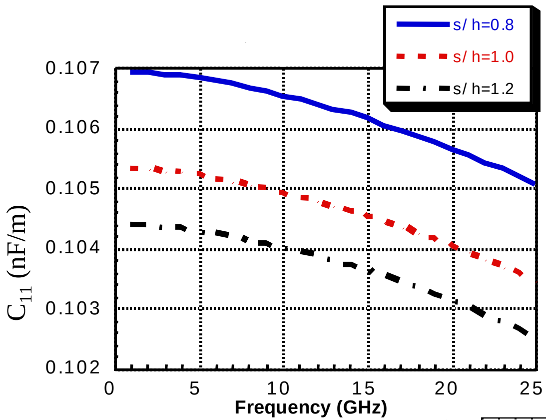
$$\mu_o \frac{\partial H_z}{\partial t} + \sigma^* H_z = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}$$

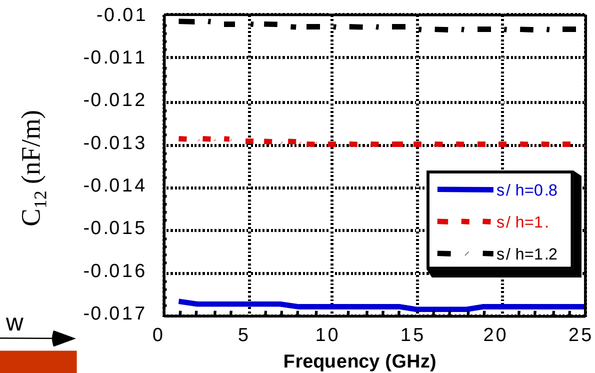


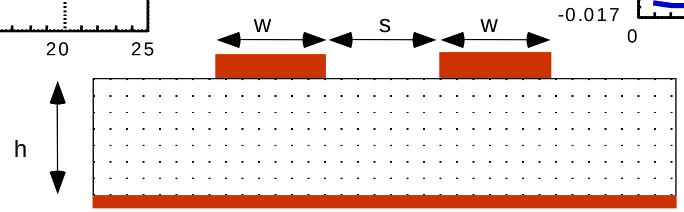
$$\frac{\sigma}{\varepsilon_o} = \frac{\sigma^*}{\mu_o}$$

No reflection from PML interface

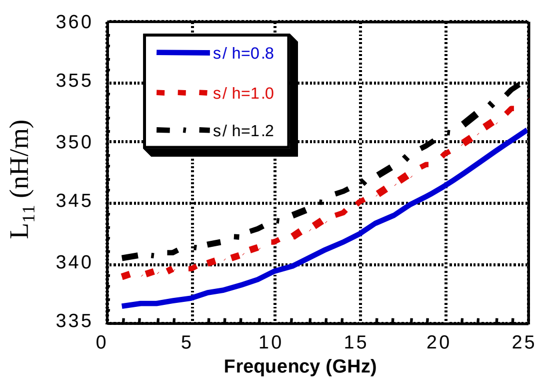
FDTD – Coupled Microstrips

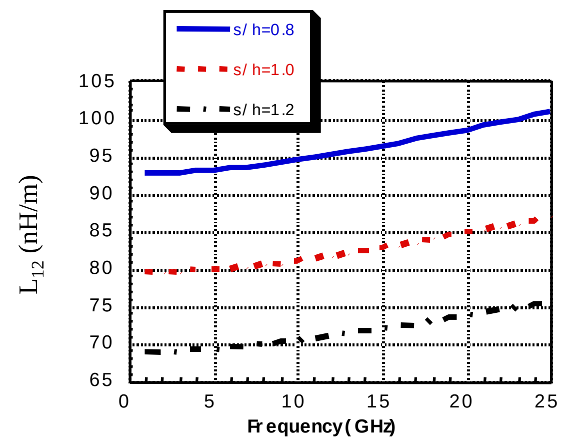






Parameters: w=0.3mm, h=0.25mm. Dielectric constant is 4.5.





Parallel-FDTD - Motivations

PCB interconnect EM problems

1. Interconnect coupling / crosstalk
2. Signal distortion
3. Radiation (EMI)
4. etc

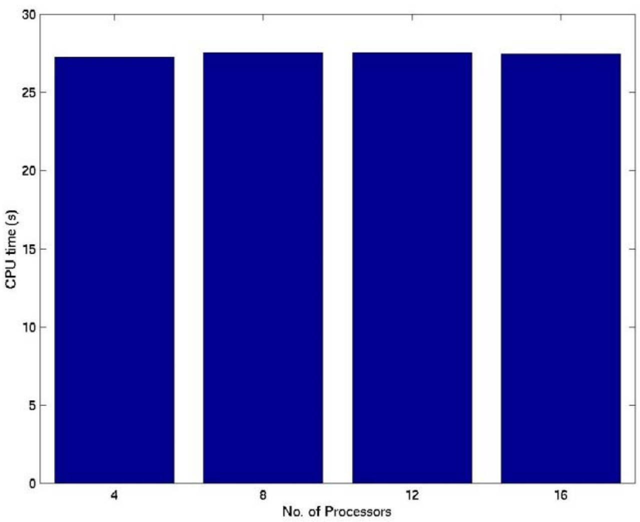
PCB modeling challenges:

1. Size of the problem - determined by the ratio of board size and finest features
2. Simulations time

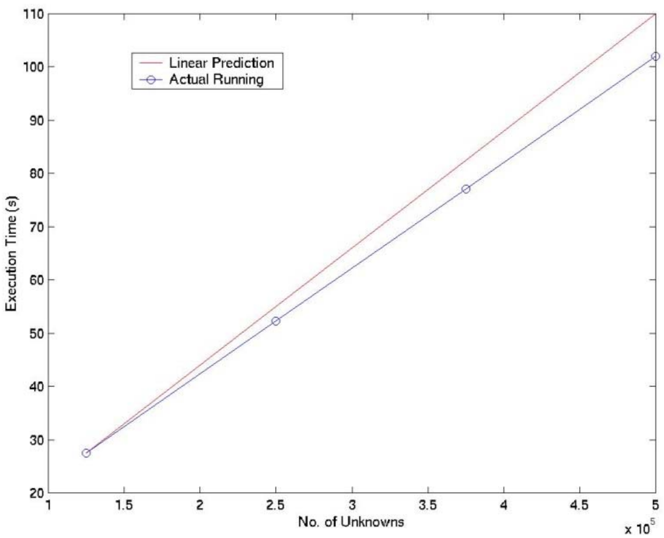
Solutions: ~~Supercomputer~~
Mini-computer cluster **Gflops/dollar**

Parallel-FDTD

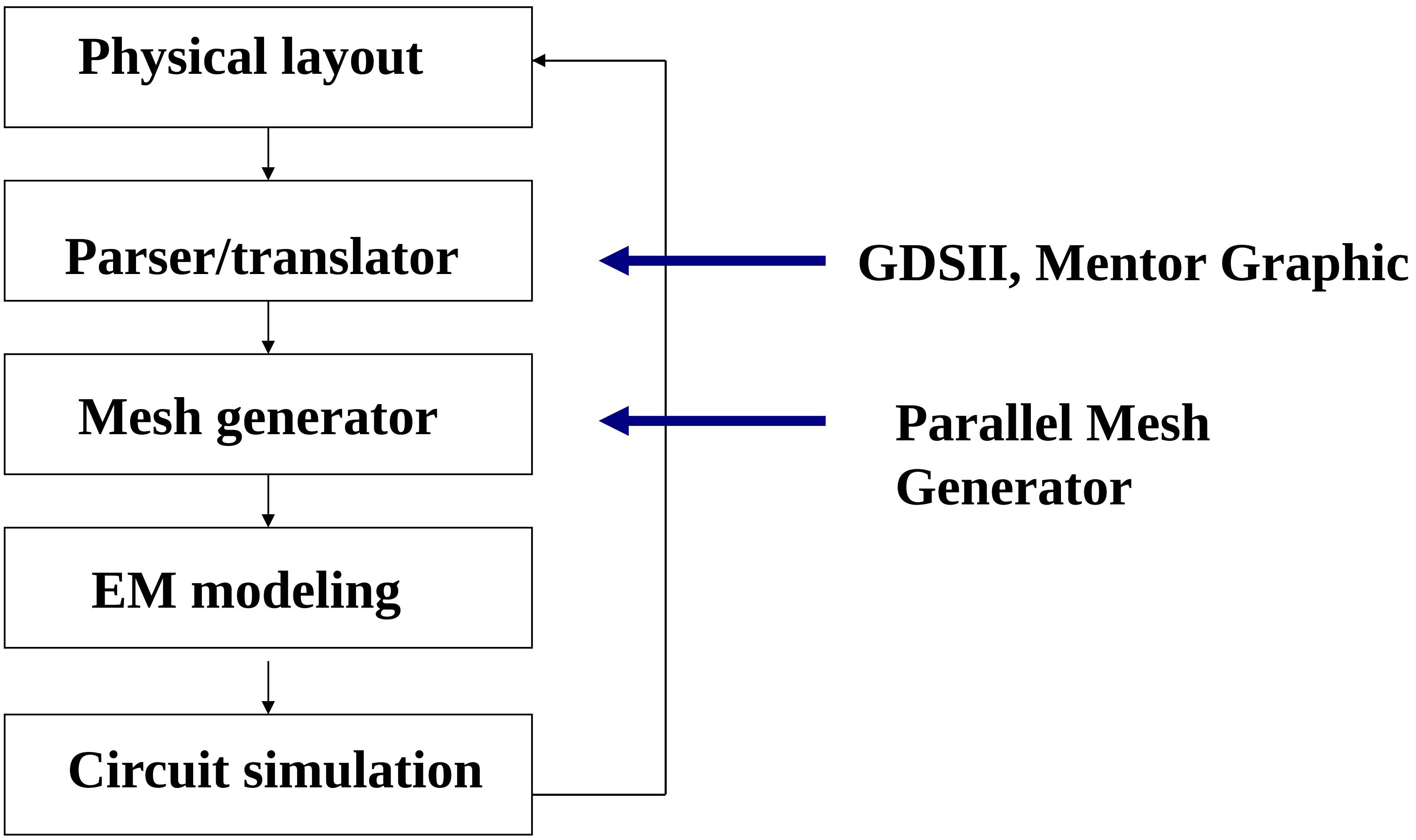
Scalability



Speed vs. problem size



Parallel-FDTD



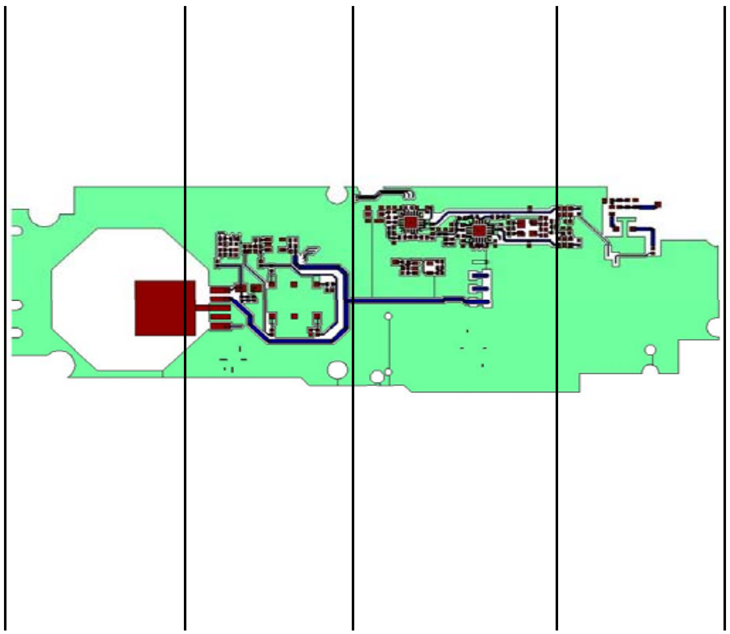
Parallel-FDTD

Max(computation/communication)

Maximize the ratio of volume to surface

↓

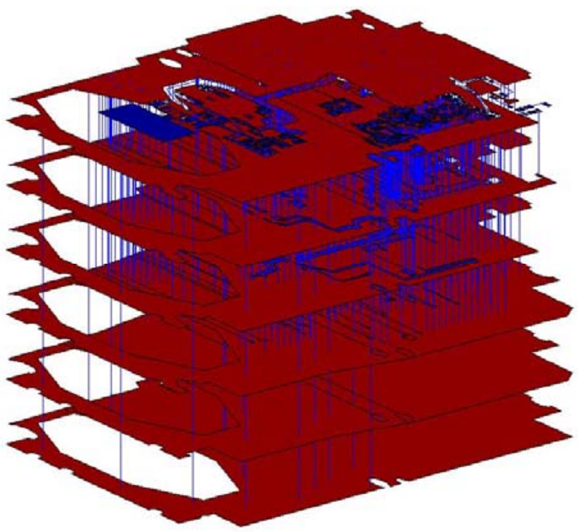
Partition along the largest dimension



Parallel-FDTD

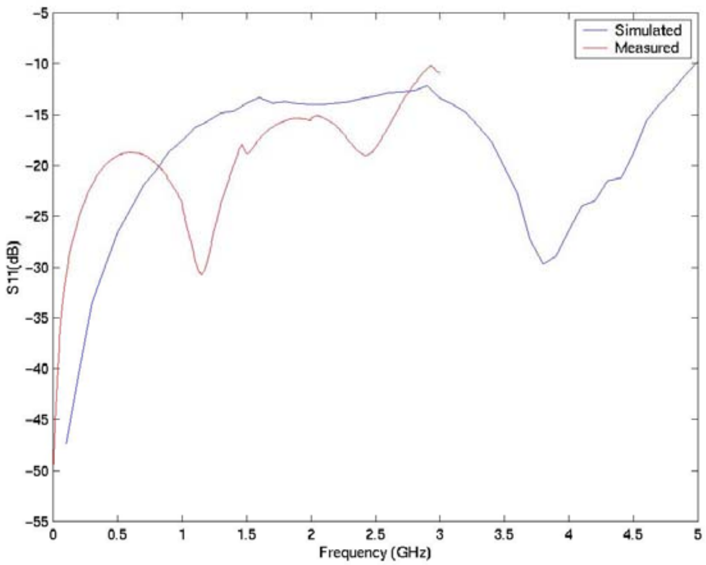
A cellular phone PCB test board

- six layered test board
- over 4000 vias
- dielectric materials FR 4
- total thickness < 20 mils
- over 60 traces
- around 30 PEC fills
- around 300 component pads



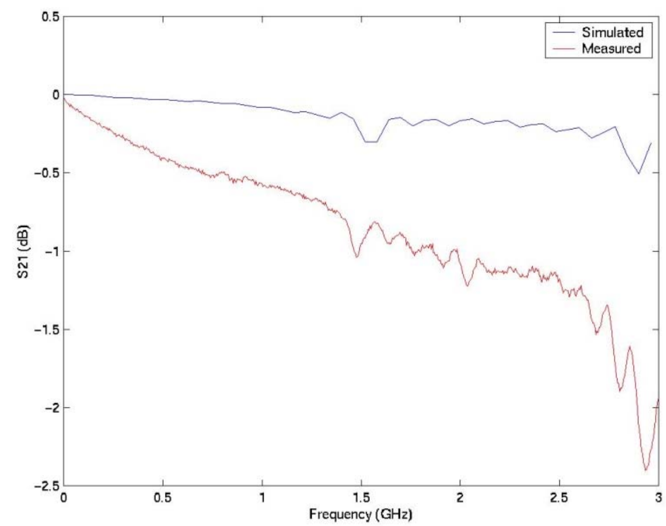
Parallel-FDTD

Comparison between measurement and simulation



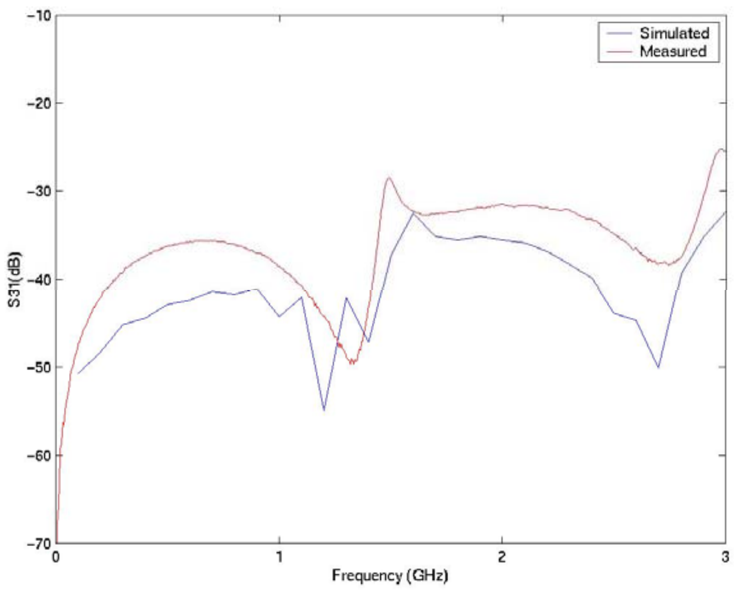
S11

S21

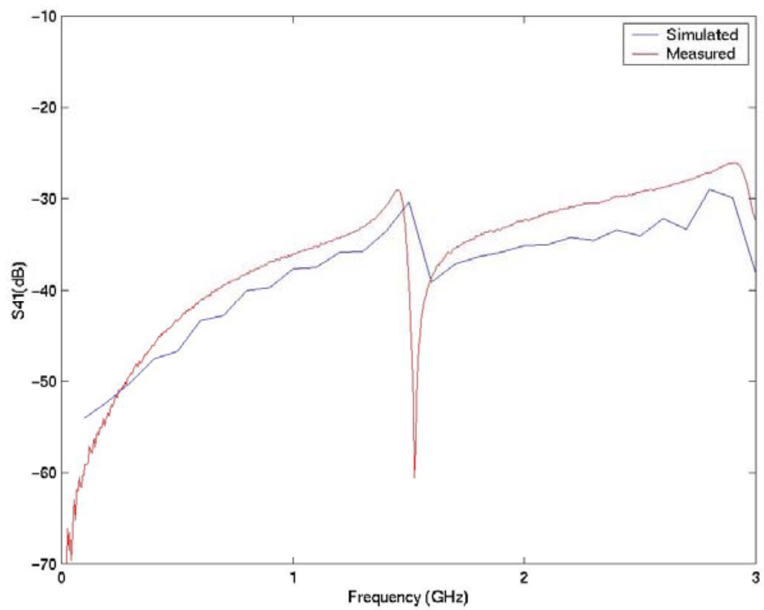


Parallel-FDTD

S31



S41



- **Reference**
  - F. Liu, J. Schutt-Ainé and J. Chen, "Full Wave Analysis and Modeling of Multiconductor Transmission Lines via 2-D-FDTD and Signal-Processing Techniques," IEEE Trans. Microwave Theory Tech. vol. 50, pp. 570-577, February 2002.