ECE 546
Lecture - 06
Coupled Lines

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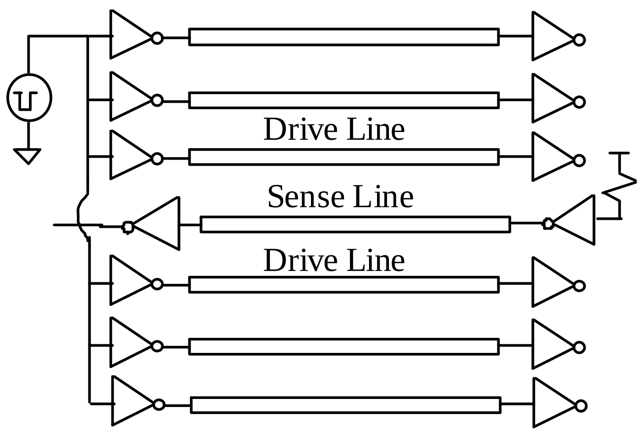
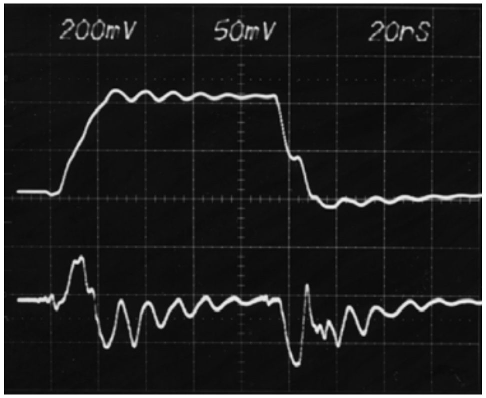
Crosstalk Noise

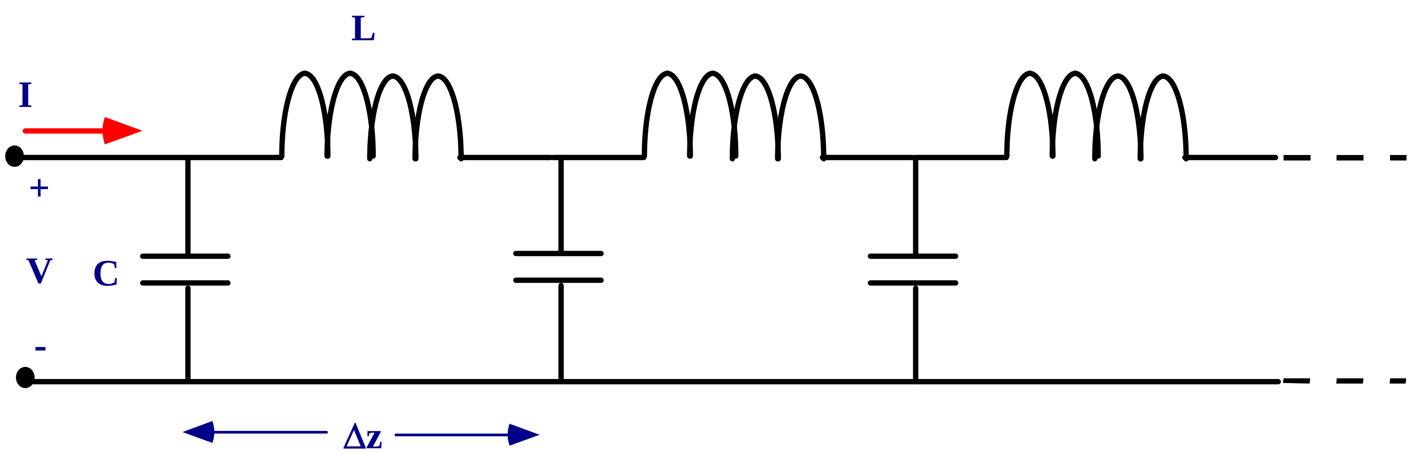
Signal Integrity

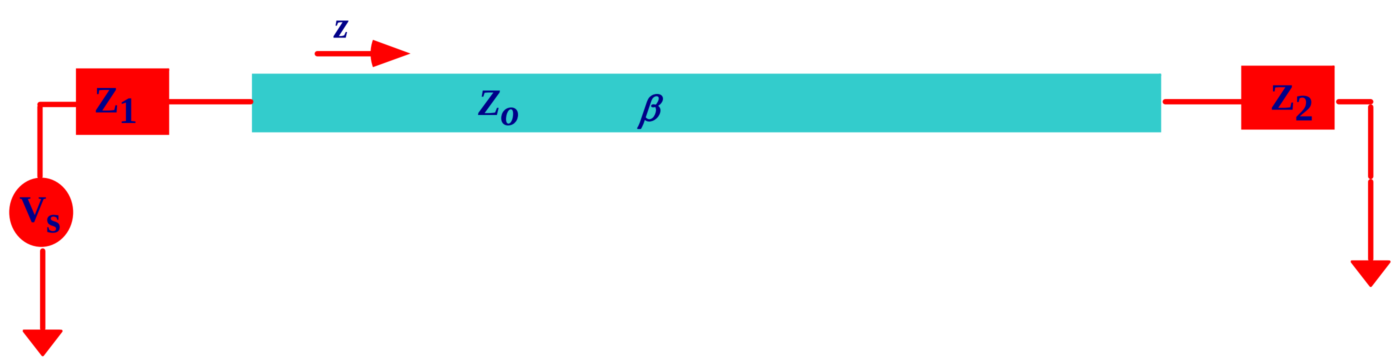
Crosstalk	**Dispersion**	**Attenuation**
Reflection	**Distortion**	**Loss**
Delta I Noise	**Ground Bounce**	**Radiation**



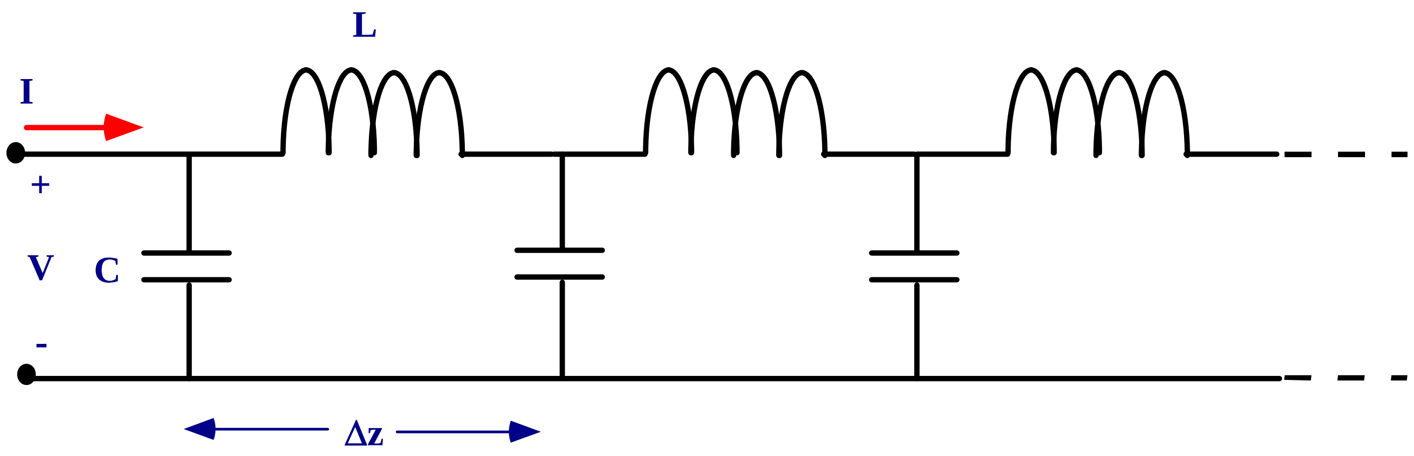


TEM PROPAGATION





Telegrapher's Equations



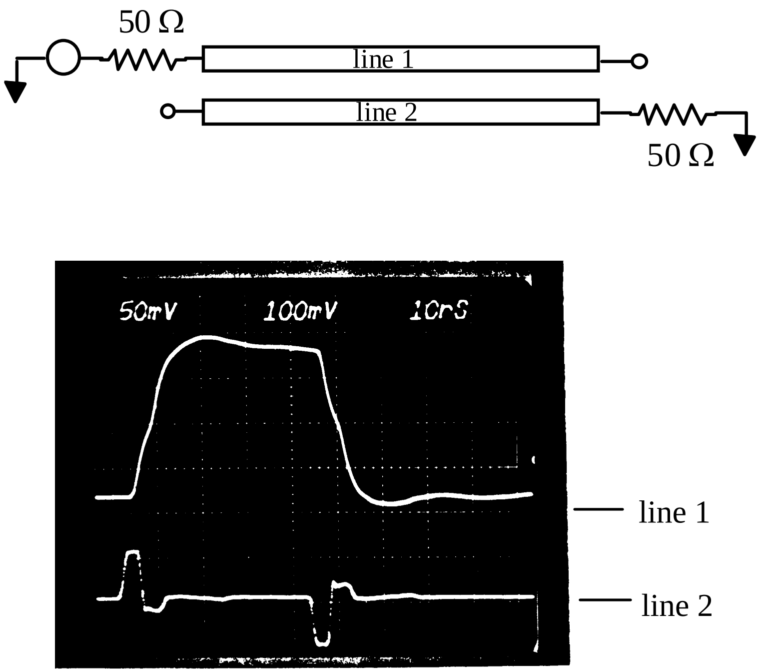
$$-\frac{\partial V}{\partial z} = L\frac{\partial I}{\partial t}$$

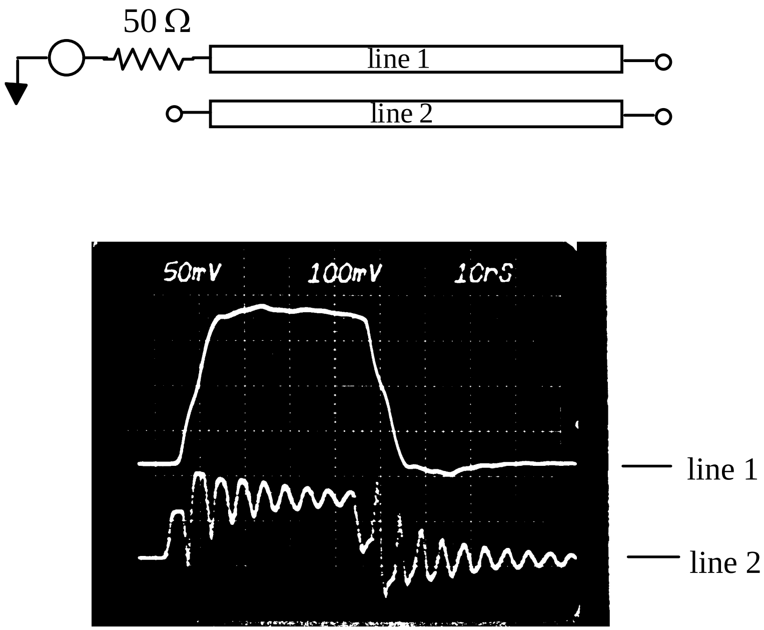
$$-\frac{\partial I}{\partial z} = C\frac{\partial V}{\partial t}$$

***L*: Inductance per unit length.**

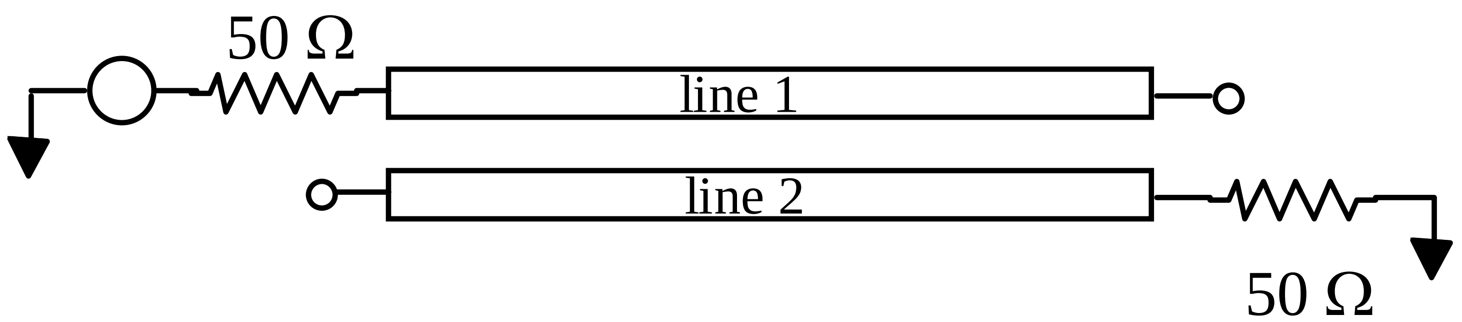
***C*: Capacitance per unit length.**

Crosstalk noise depends on termination

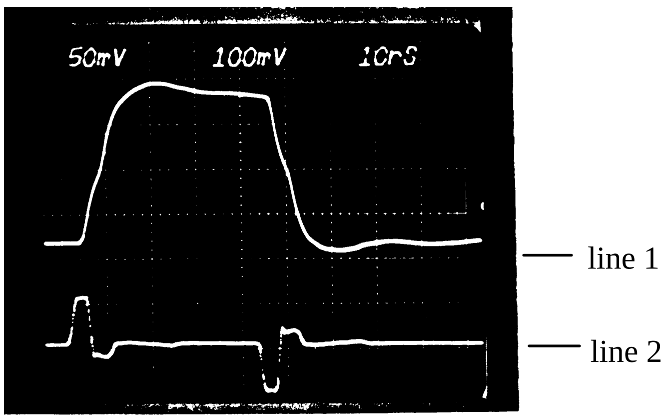




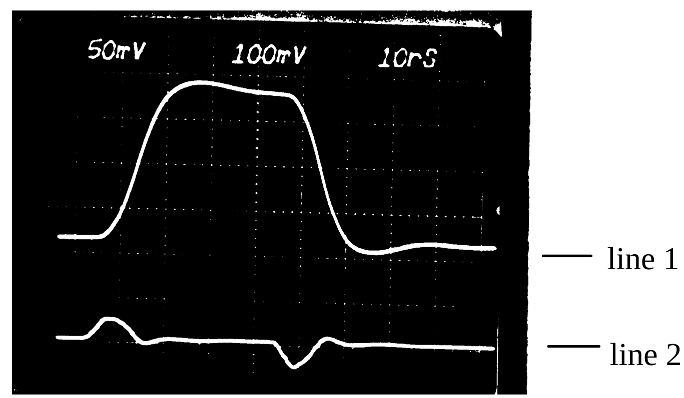
Crosstalk depends on signal rise time



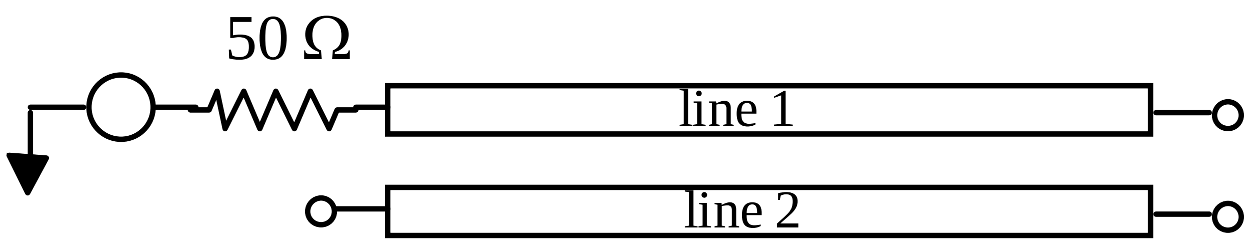
t_r = 1 ns



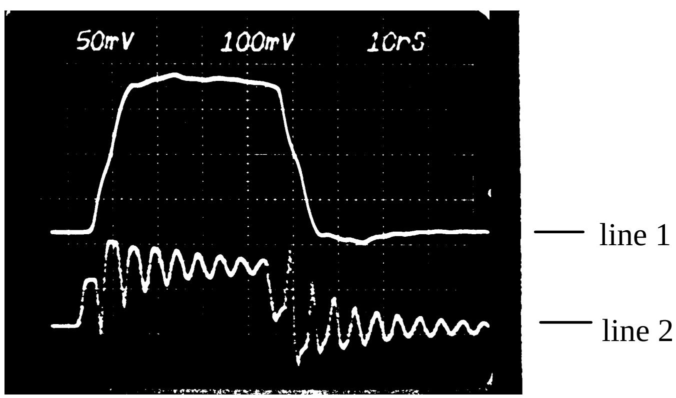
t_r = 7 ns



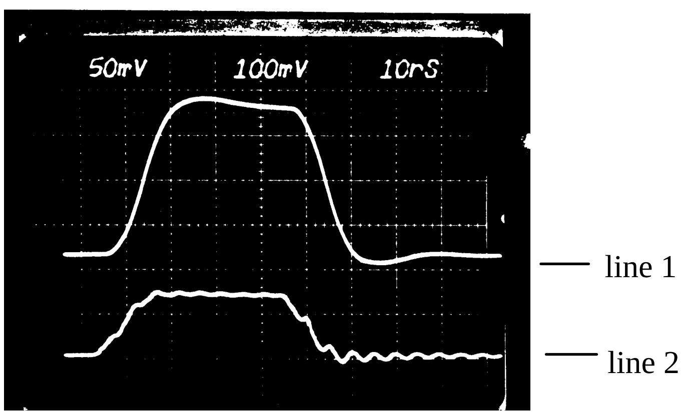
Crosstalk depends on signal rise time



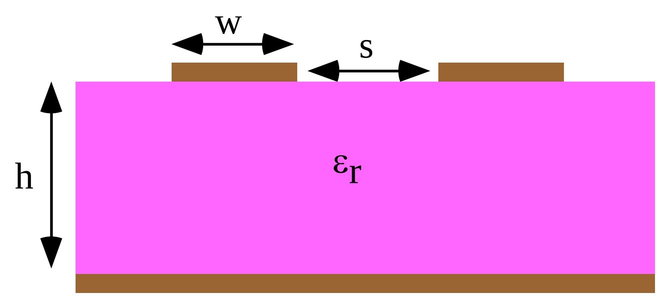
t_r = 1 ns

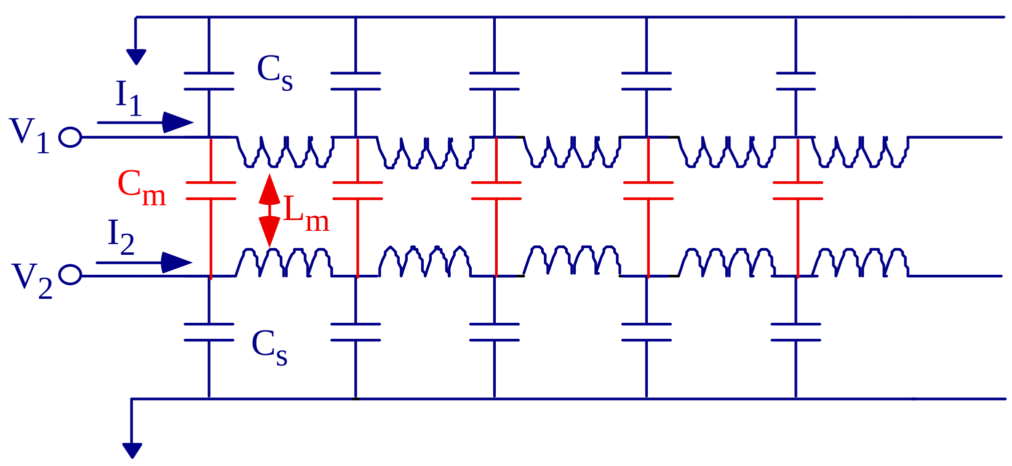


t_r = 7 ns



Coupled Transmission Lines





Telegraphers Equations for Coupled Transmission Lines

Maxwellian Form

$$-\frac{\partial V_1}{\partial z} = L_{11}\frac{\partial I_1}{\partial t} + L_{12}\frac{\partial I_2}{\partial t}$$

$$-\frac{\partial V_2}{\partial z} = L_{21}\frac{\partial I_1}{\partial t} + L_{22}\frac{\partial I_2}{\partial t}$$

$$-\frac{\partial I_1}{\partial z} = C_{11}\frac{\partial V_1}{\partial t} + C_{12}\frac{\partial V_2}{\partial t}$$

$$-\frac{\partial I_2}{\partial z} = C_{21}\frac{\partial V_1}{\partial t} + C_{22}\frac{\partial V_2}{\partial t}$$

Telegraphers Equations for Coupled Transmission Lines

Physical form

$$-\frac{\partial V_1}{\partial z} = L_s \frac{\partial I_1}{\partial t} + L_m \frac{\partial I_2}{\partial t}$$

$$-\frac{\partial V_2}{\partial z} = L_m \frac{\partial I_1}{\partial t} + L_s \frac{\partial I_2}{\partial t}$$

$$-\frac{\partial I_1}{\partial z} = C_s \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_1}{\partial t} - C_m \frac{\partial V_2}{\partial t}$$

$$-\frac{\partial I_2}{\partial z} = -C_m \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_2}{\partial t} + C_s \frac{\partial V_2}{\partial t}$$

Relations Between Physical and Maxwellian Parameters (symmetric lines)

$$L_{11} = L_{22} = L_s$$

$$L_{12} = L_{21} = L_m$$

$$C_{11} = C_{22} = C_s + C_m$$

$$C_{12} = C_{21} = -C_m$$

Even Mode

$$-\frac{\partial V_e}{\partial z} = \left(L_{11} + L_{12}\right)\frac{\partial I_e}{\partial t}$$

$$-\frac{\partial I_e}{\partial z} = \left(C_{11} + C_{12}\right)\frac{\partial I_e}{\partial t}$$

Add voltage and current equations

V_e : Even mode voltage $\quad V_e = \frac{1}{2}\left(V_1 + V_2\right)$

I_e : Even mode current $\quad I_e = \frac{1}{2}\left(I_1 + I_2\right)$

$$Z_e = \sqrt{\frac{L_{11} + L_{12}}{C_{11} + C_{12}}} = \sqrt{\frac{L_s + L_m}{C_s}}$$

Impedance

$$v_e = \frac{1}{\sqrt{(L_{11} + L_{12})(C_{11} + C_{12})}} = \frac{1}{\sqrt{(L_s + L_m)C_s}}$$

velocity

Odd Mode

$$-\frac{\partial V_d}{\partial z} = \left(L_{11} - L_{12}\right)\frac{\partial I_d}{\partial t}$$

$$-\frac{\partial I_d}{\partial z} = \left(C_{11} - C_{12}\right)\frac{\partial I_d}{\partial t}$$

Subtract voltage and current equations

V_d : Odd mode voltage $\quad \mathrm{V_d} = \frac{1}{2}\left(V_1 - V_2\right)$

I_d : Odd mode current $\quad \mathrm{I_d} = \frac{1}{2}\left(I_1 - I_2\right)$

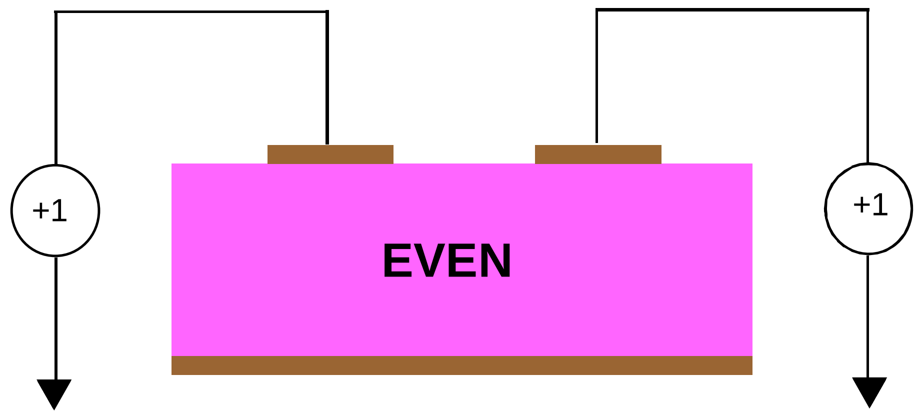
$$\mathrm{Z_d} = \sqrt{\frac{L_{11} - L_{12}}{C_{11} - C_{12}}} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}}$$

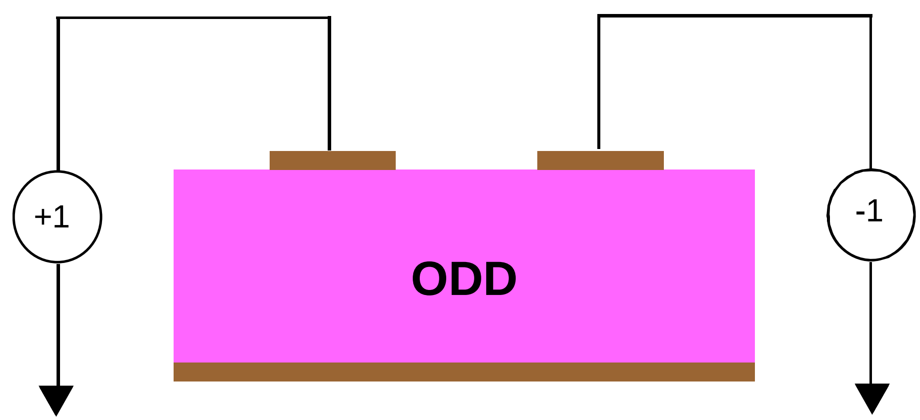
Impedance

$$\mathrm{v_d} = \frac{1}{\sqrt{(L_{11} - L_{12})(C_{11} - C_{12})}} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}$$

velocity

Mode Excitation





PHYSICAL SIGNIFICANCE OF EVEN- AND ODD-MODE IMPEDANCES

* Z_e and Z_d are the wave resistance seen by the even and odd mode travelling signals respectively.

* The impedance of each line is no longer described by a single characteristic impedance; instead, we have

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Definitions

Even-Mode Impedance: Z_e

Impedance seen by wave propagating through the coupled-line system when excitation is symmetric (1, 1).

Odd-Mode Impedance: Z_d

Impedance seen by wave propagating through the coupled-line system when excitation is anti-symmetric (1, -1).

Common-Mode Impedance: $Z_c = 0.5Z_e$

Impedance seen by a pair of line and a common return by a common signal.

Differential Impedance: $Z_{diff} = 2Z_d$

Impedance seen across a pair of lines by differential mode signal.

EVEN AND ODD-MODE IMPEDANCES

Z_{11}, Z_{22} : Self Impedances

Z_{12}, Z_{21} : Mutual Impedances

For symmetrical lines,

$Z_{11} = Z_{22}$ and $Z_{12} = Z_{21}$

Coupled Lines

Line Space

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Modal Space

$$V_e = Z_e I_e$$

$$V_d = Z_d I_d$$

$$\begin{bmatrix} V_e \\ V_d \end{bmatrix} = \begin{bmatrix} Z_e & 0 \\ 0 & Z_d \end{bmatrix} \begin{bmatrix} I_e \\ I_d \end{bmatrix}$$

EXAMPLE
(Microstrip)

$\varepsilon_r = 4.3$
$Z_s = 56.4\ \Omega$

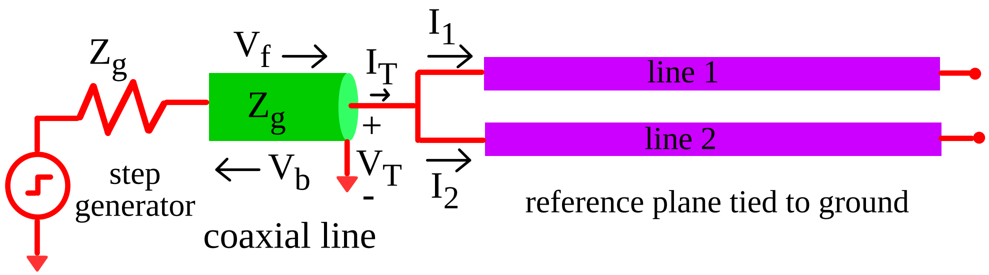
Single Line
Dielectric height = 6 mils
Width = 8 mils

Coupled Lines
Height = 6 mils
Width = 8 mils
Spacing = 12 mils

$\varepsilon_r = 4.3$

$$\mathbf{Z_e = 68.1\ \Omega \quad Z_d = 40.8\ \Omega}$$
$$\mathbf{Z_{11} = 54.4\ \Omega \quad Z_{12} = 13.6\ \Omega}$$

Even Mode



$$I_{tdr} = \left[\frac{a_e(t,0)}{Z_e} + \frac{a_d(t,0)}{Z_d}\right] + \left[\frac{a_e(t,0)}{Z_e} - \frac{a_d(t,0)}{Z_d}\right]$$

$$V_{tdr} = a_e(t,0) - a_d(t,0)$$

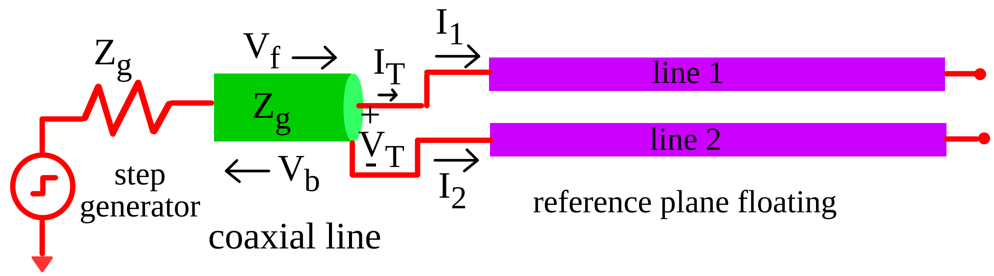
$$\mathrm{a_d}(t,0) = 0$$

$$\frac{V_{tdr}}{I_{tdr}} = \frac{Z_e}{2}$$

$$\mathrm{Z_e} = 2\left(\frac{1+\rho_e}{1-\rho_e}\right)\mathrm{Z_g}$$

$$\mathrm{v_e} = \frac{2l}{\tau_e}$$

Odd Mode



$$V_{tdr} = a_e(t,0)+a_d(t,0)-\left[a_e(t,0)-a_d(t,0)\right] = V_f + V_b$$

$$I_{tdr} = \left[\frac{a_e(t,0)}{Z_e} + \frac{a_d(t,0)}{Z_d}\right] \qquad I_{tdr} = -\left[\frac{a_e(t,0)}{Z_e} - \frac{a_d(t,0)}{Z_d}\right]$$

$$a_e(t,0) = 0, \qquad \frac{V_{tdr}}{I_{tdr}} = 2Z_d$$

$$Z_d = \frac{1}{2}\left(\frac{1+\rho_d}{1-\rho_d}\right)Z_g, \qquad v_d = \frac{2l}{\tau_d}$$

EXTRACT INDUCTANCE AND CAPACITANCE COEFFICIENTS

$$L_s = \frac{1}{2}\left[\frac{Z_e}{v_e} + \frac{Z_d}{v_d}\right]$$

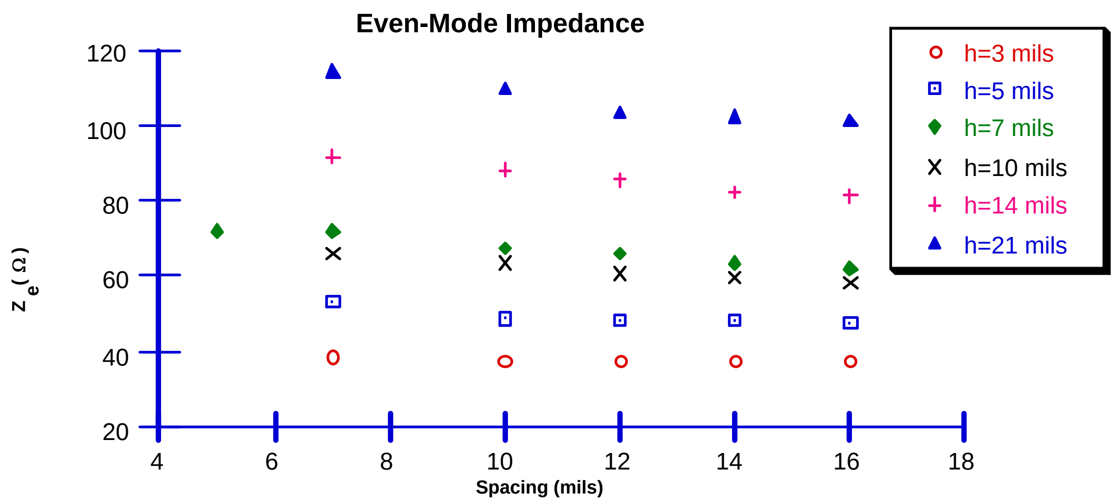
$$C_s = \frac{1}{Z_e v_e}$$

$$L_m = \frac{1}{2}\left[\frac{Z_e}{v_e} - \frac{Z_d}{v_d}\right]$$

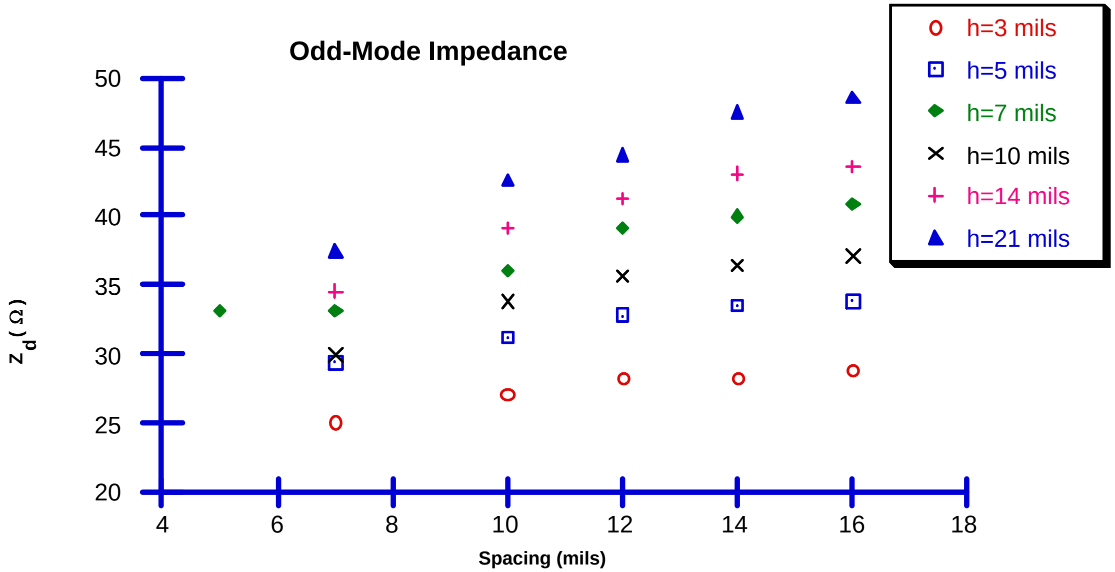
$$C_m = \frac{1}{2}\left[\frac{1}{Z_e v_e} - \frac{1}{Z_d v_d}\right]$$

$$Z_d < Z_s < Z_e$$

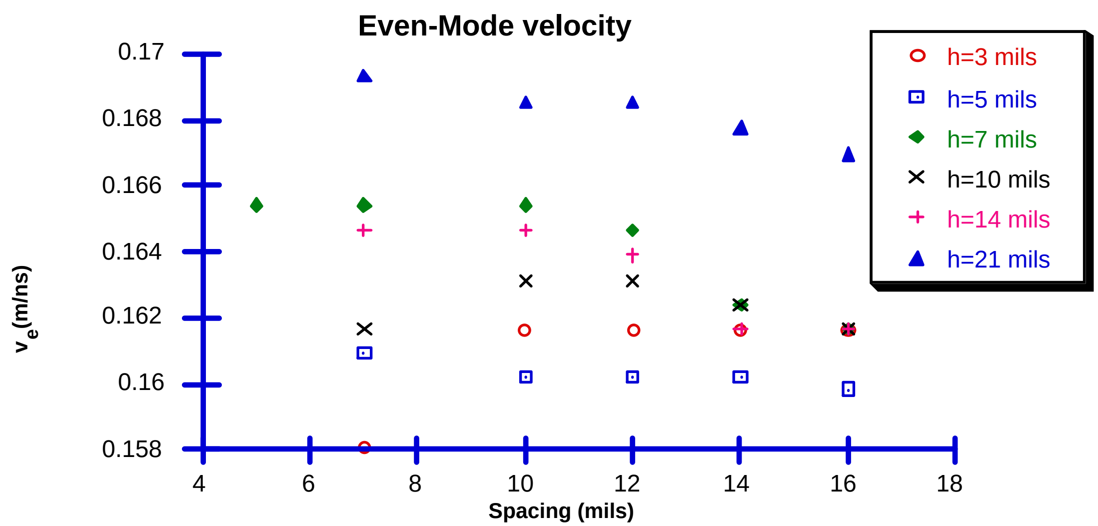
Measured even-mode impedance



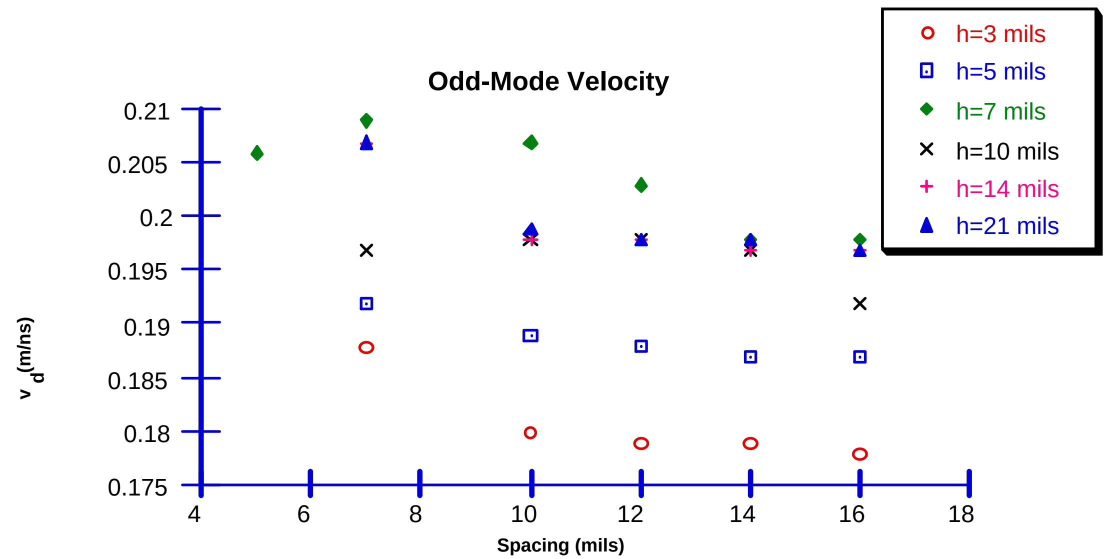
Measured odd-mode impedance



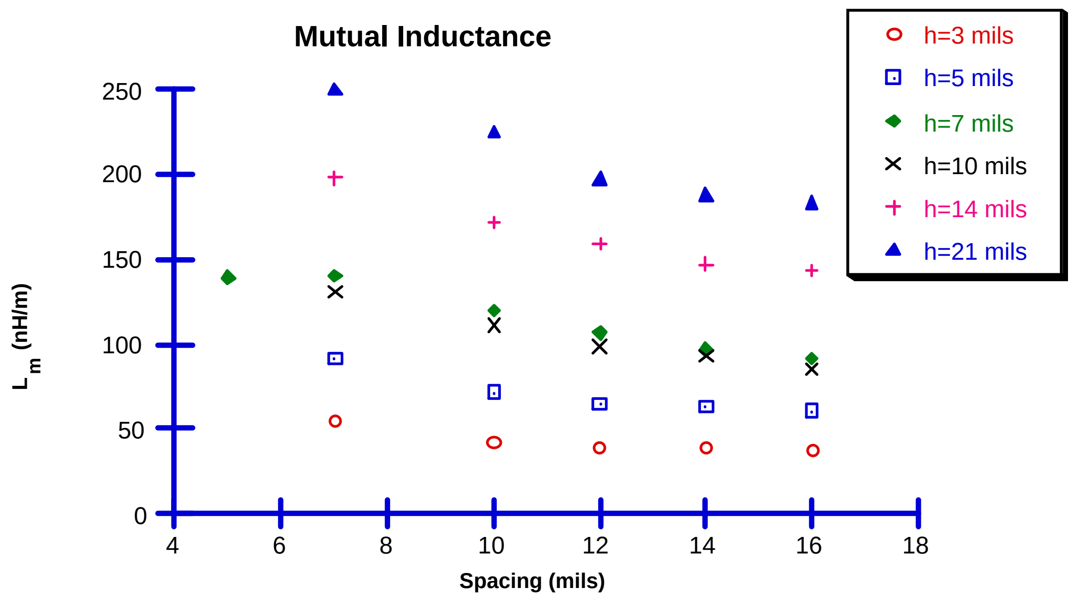
Measured even-mode velocity



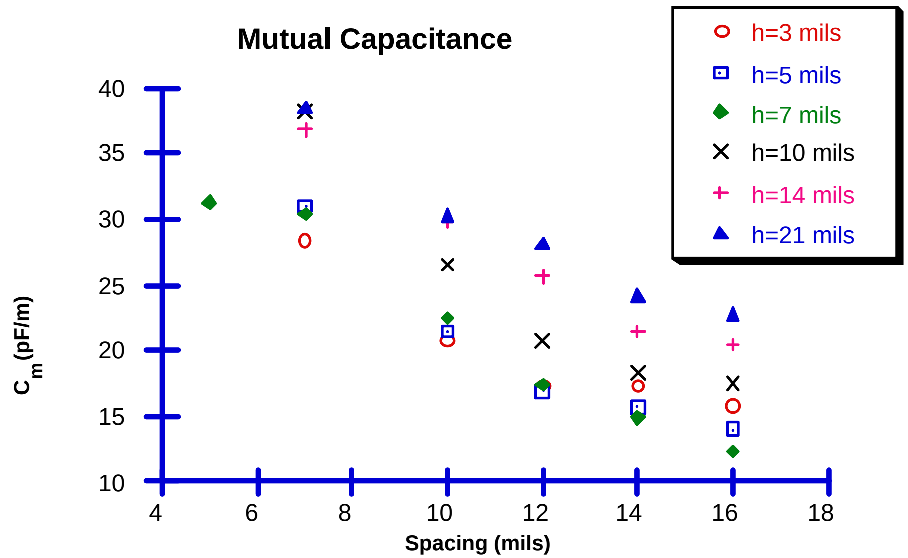
Measured odd-mode velocity



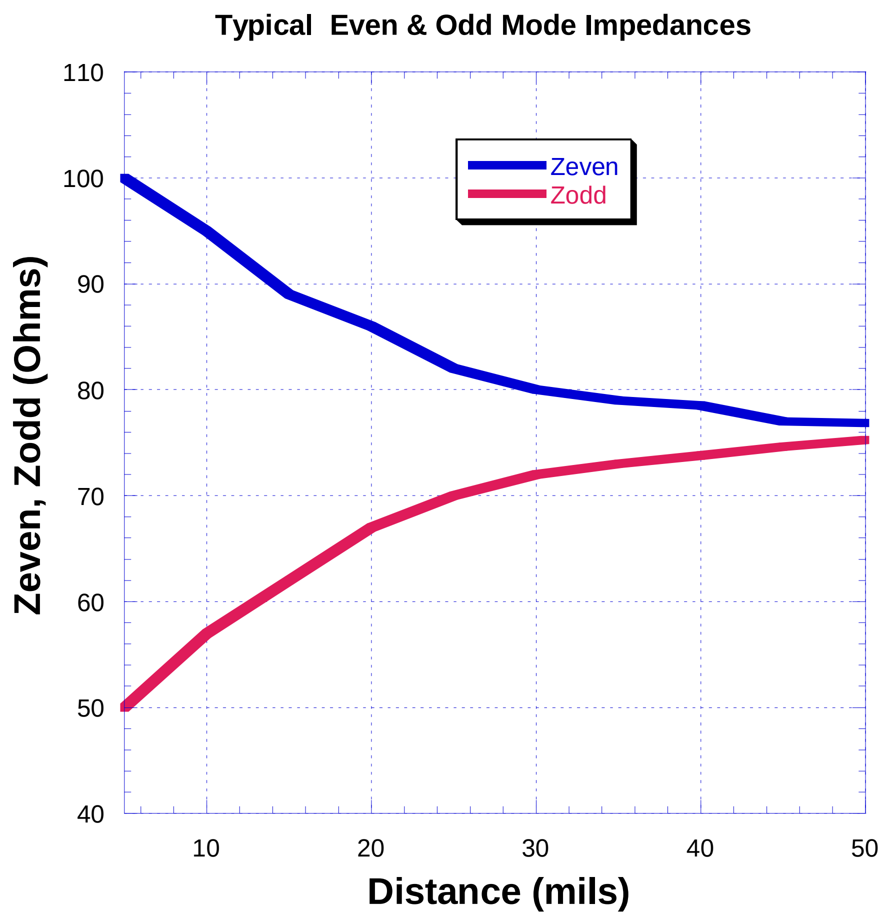
Measured mutual inductance



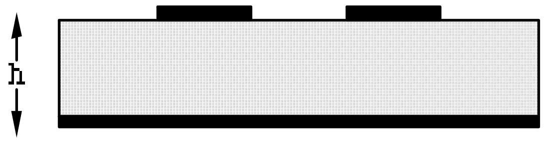
Measured mutual capacitance

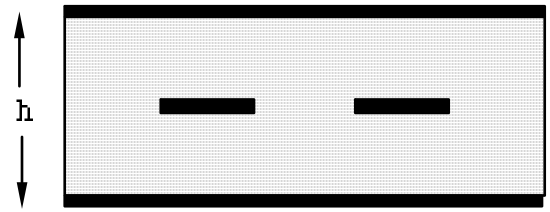


Even & Odd Mode Impedances



Modal Velocities in Stripline and Microstrip

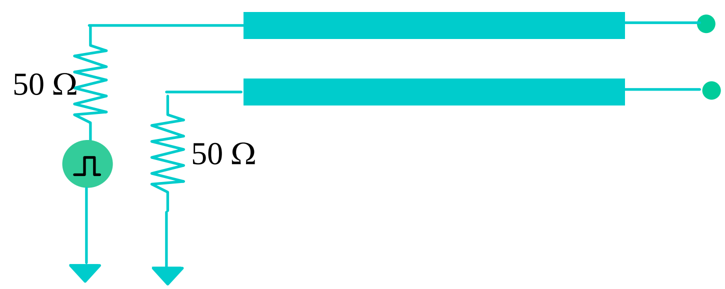




Microstrip : Inhomogeneous structure, odd and even-mode velocities must have different values.

Stripline : Homogeneous configuration, odd and even-mode velocities have approximately the same values.

Microstrip vs Stripline



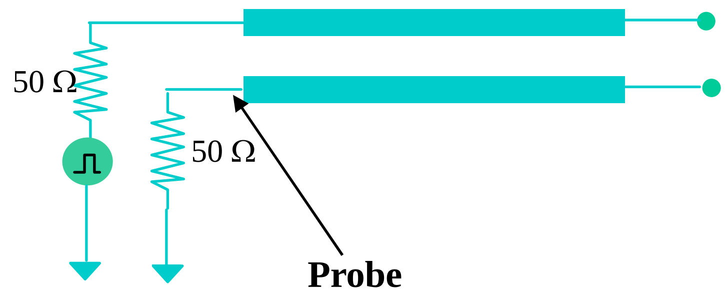
Microstrip (h =8 mils)

- $w = 8$ mils
- $\varepsilon_r = 4.32$
- $L_s = 377$ nH/m
- $C_s = 82$ pF/m
- $L_m = 131$ nH/m
- $C_m = 23$ pF/m
- $v_e = 0.155$ m/ns
- $v_d = 0.178$ m/ns

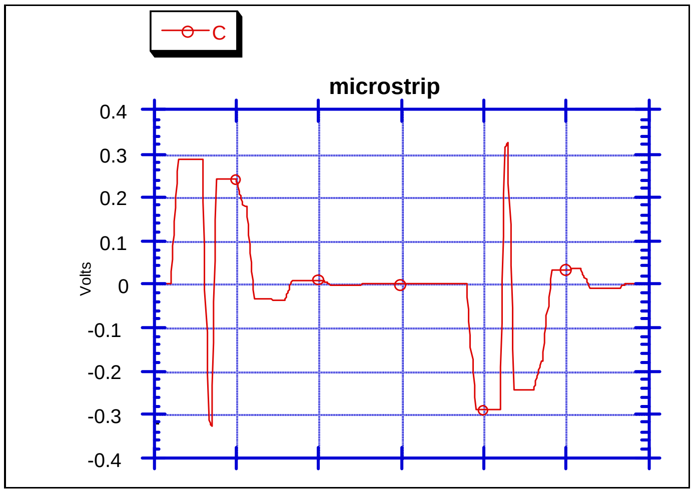
Stripline (h =16 mils)

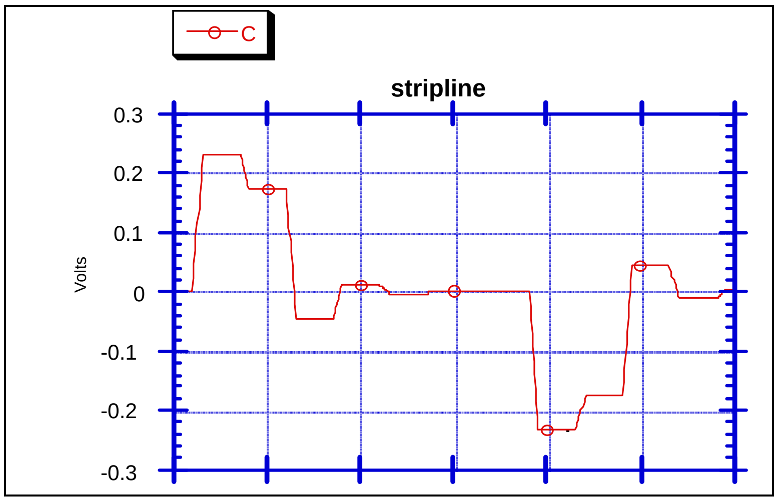
- $w = 8$ mils
- $\varepsilon_r = 4.32$
- $L_s = 466$ nH/m
- $C_s = 86$ pF/m
- $L_m = 109$ nH/m
- $C_m = 26$ pF/m
- $v_e = 0.142$ m/ns
- $v_d = 0.142$ m/ns

Microstrip vs Stripline

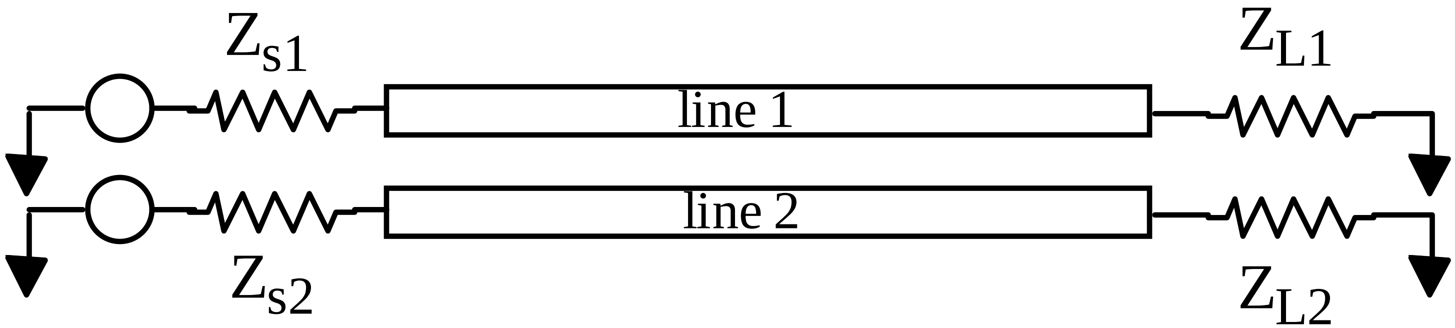


Sense line response at near end





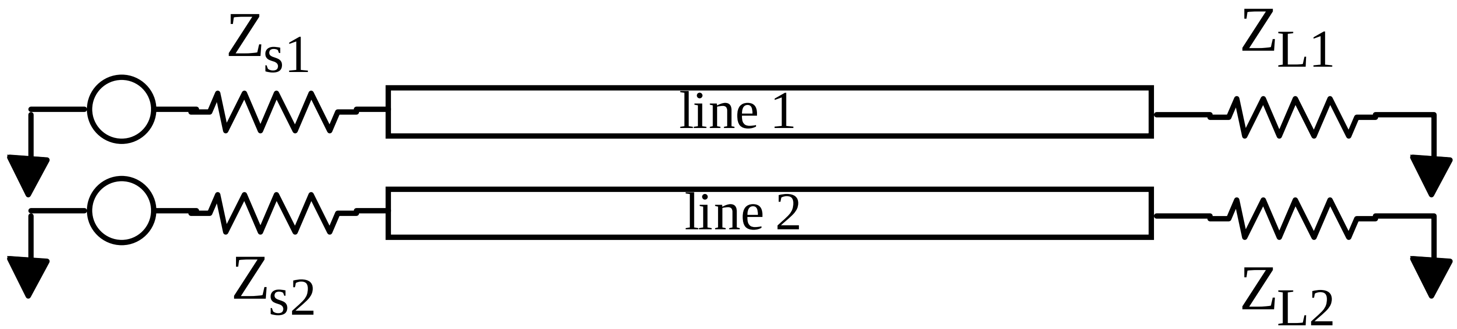
General Solution for Voltages



$$V_1(z) = \underbrace{A_e e^{-\frac{j\omega z}{v_e}} + B_e e^{+\frac{j\omega z}{v_e}}}_{even} + \underbrace{A_d e^{-\frac{j\omega z}{v_d}} + B_d e^{+\frac{j\omega z}{v_d}}}_{odd}$$

$$V_2(z) = \underbrace{A_e e^{-\frac{j\omega z}{v_e}} + B_e e^{+\frac{j\omega z}{v_e}}}_{even} \underbrace{- A_d e^{-\frac{j\omega z}{v_d}} - B_d e^{+\frac{j\omega z}{v_d}}}_{odd}$$

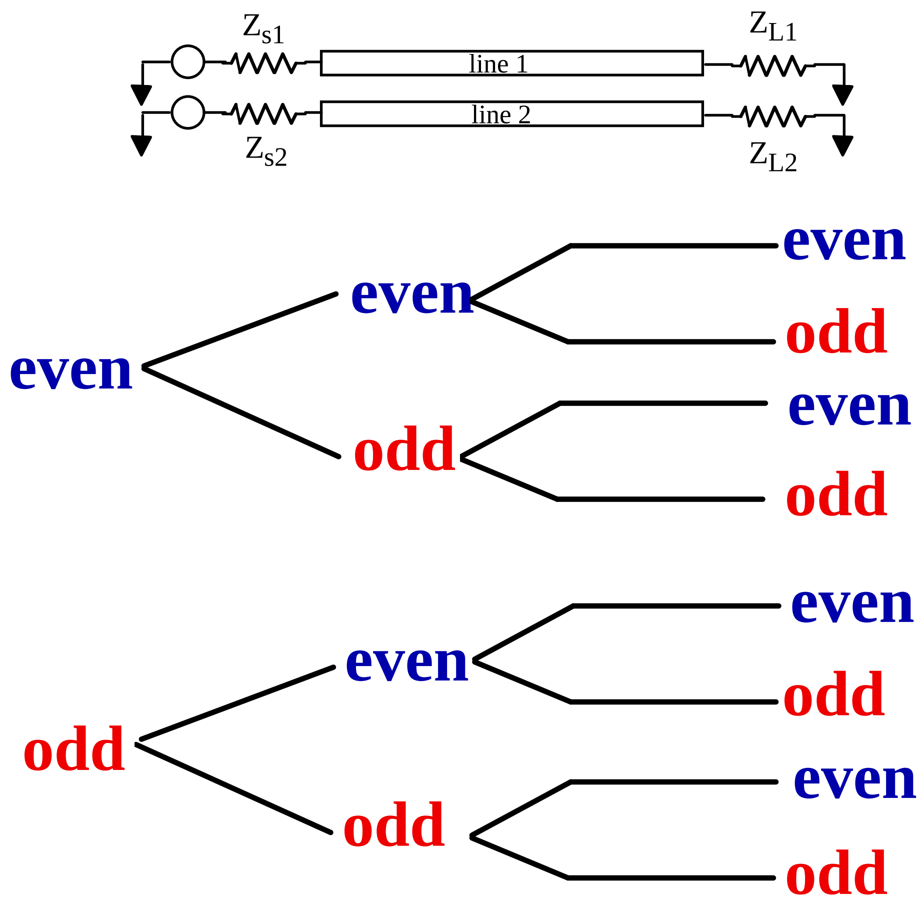
General Solution for Currents



$$I_1(z) = \underbrace{\frac{1}{Z_e}\left[A_e e^{-\frac{j\omega z}{v_e}} - B_e e^{+\frac{j\omega z}{v_e}}\right]}_{even} + \underbrace{\frac{1}{Z_d}\left[A_d e^{-\frac{j\omega z}{v_d}} - B_d e^{+\frac{j\omega z}{v_d}}\right]}_{odd}$$

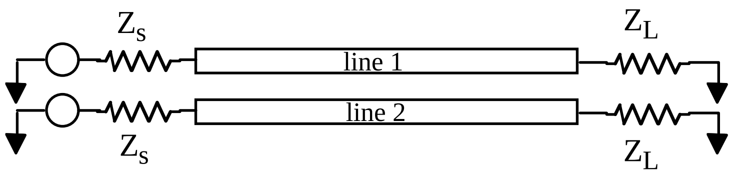
$$I_2(z) = \underbrace{\frac{1}{Z_e}\left[A_e e^{-\frac{j\omega z}{v_e}} - B_e e^{+\frac{j\omega z}{v_e}}\right]}_{even} - \underbrace{\frac{1}{Z_d}\left[A_d e^{-\frac{j\omega z}{v_d}} - B_d e^{+\frac{j\omega z}{v_d}}\right]}_{odd}$$

Coupling of Modes (asymmetric load)



First reflection **Second reflection**

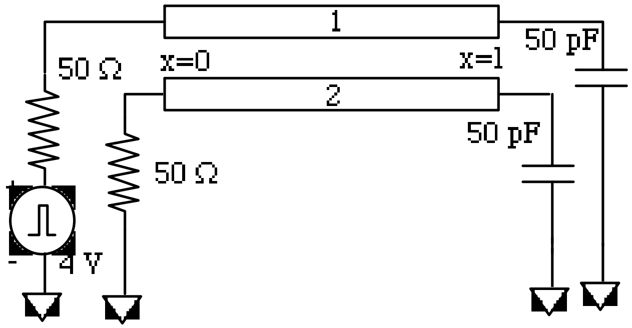
Coupling of Modes (symmetric load)

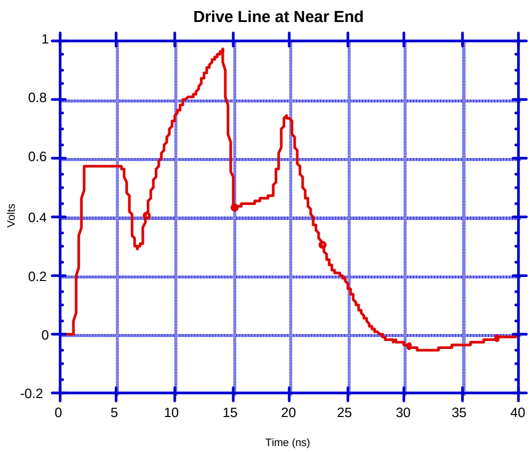


even —— **even** —— **even**

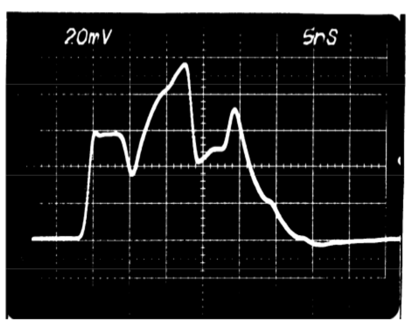
odd —— **odd** —— **odd**

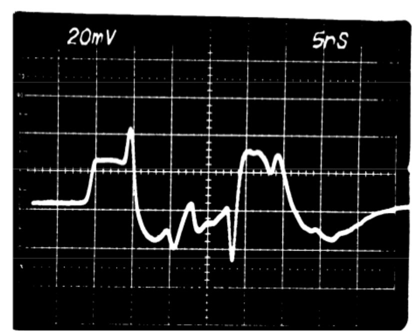
First reflection **Second reflection**


1
2
50 Ω
x=0
x=1
50 pF
50 pF
50 Ω
4 V


Drive Line at Near End
Volts
Time (ns)


Sense Line at Near End
Volts
Time (ns)


20mV
5nS


20mV
5nS