ECE 546
Lecture 11
MOS Amplifiers

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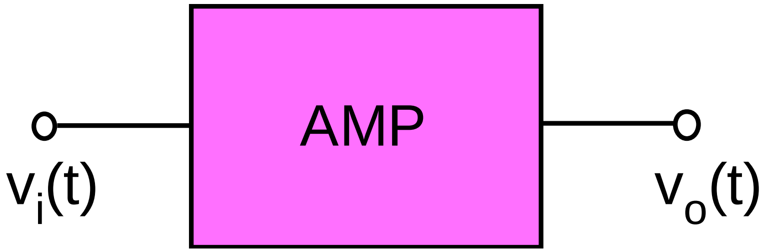
Amplifiers

- **Definitions**
  - Used to increase the amplitude of an input signal to a desired level
  - This is a fundamental signal processing function
  - Must be linear (free of distortion) – Shape of signal preserved



$$v_o(t) = Av_i(t),\ \ where\ A\ is\ the\ voltage\ gain$$

$$Voltage\ Gain: A_v = \frac{v_o}{v_i}$$

$$Power\ \ Gain: A_p = \frac{Load\ \ Power\ (P_L)}{Input\ \ Power\ (P_I)}$$

Amplifiers

$$A_p = \frac{v_o i_o}{v_I i_I}$$

$$Current\ Gain: A_i = \frac{i_o}{i_i}$$

$$Note: A_p = A_v A_i$$

Expressing gain in dB (decibels)

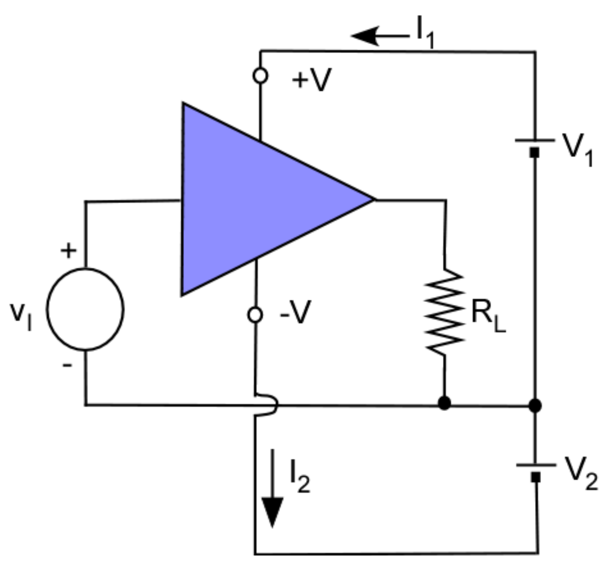
$$Voltage\ gain\ in\ dB = 20\log\left|A_V\right|$$

$$Current\ gain\ in\ dB = 20\log\left|A_I\right|$$

$$Power\ gain\ in\ dB = 10\log\left|A_P\right|$$

Amplifiers

Since output associated with the signal is larger than the input signal, power must come from DC supply



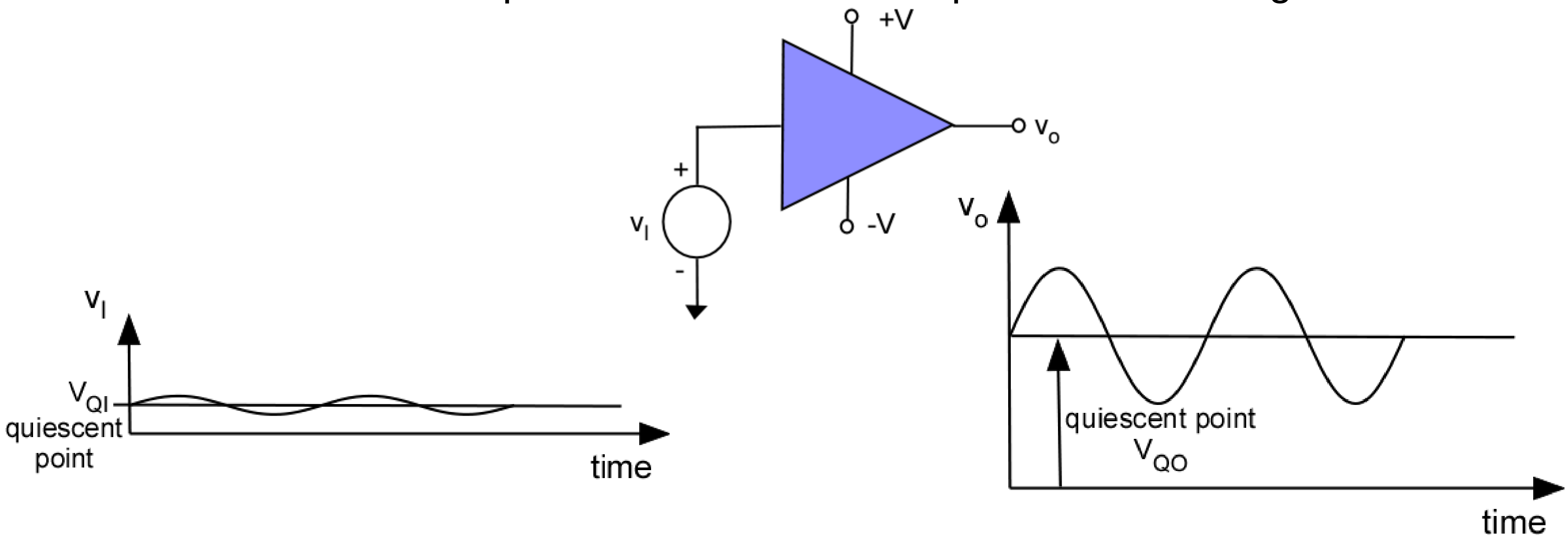
$$P_{DC} = V_1 I_1 + V_2 I_2$$

$$P_{DC} + P_I = P_L + P_{dissipated}$$

$$\eta = \frac{P_L}{P_{DC}} \times 100 = Power\ Efficiency$$

Biasing of Amp

Bias will provide quiescent points for input and output about which variations will take place. Bias maintain amplifier in active region.



$$V_I(t) = V_{QI} + v_I(t)$$

$$V_o(t) = V_{QO} + v_o(t)$$

$$v_o(t) = A_v v_I(t)$$

$$A_v = \left.\frac{dv_o}{dv_I}\right|_{at\ Q}$$

Amplifier characteristics are determined by bias point

Small-Signal Model

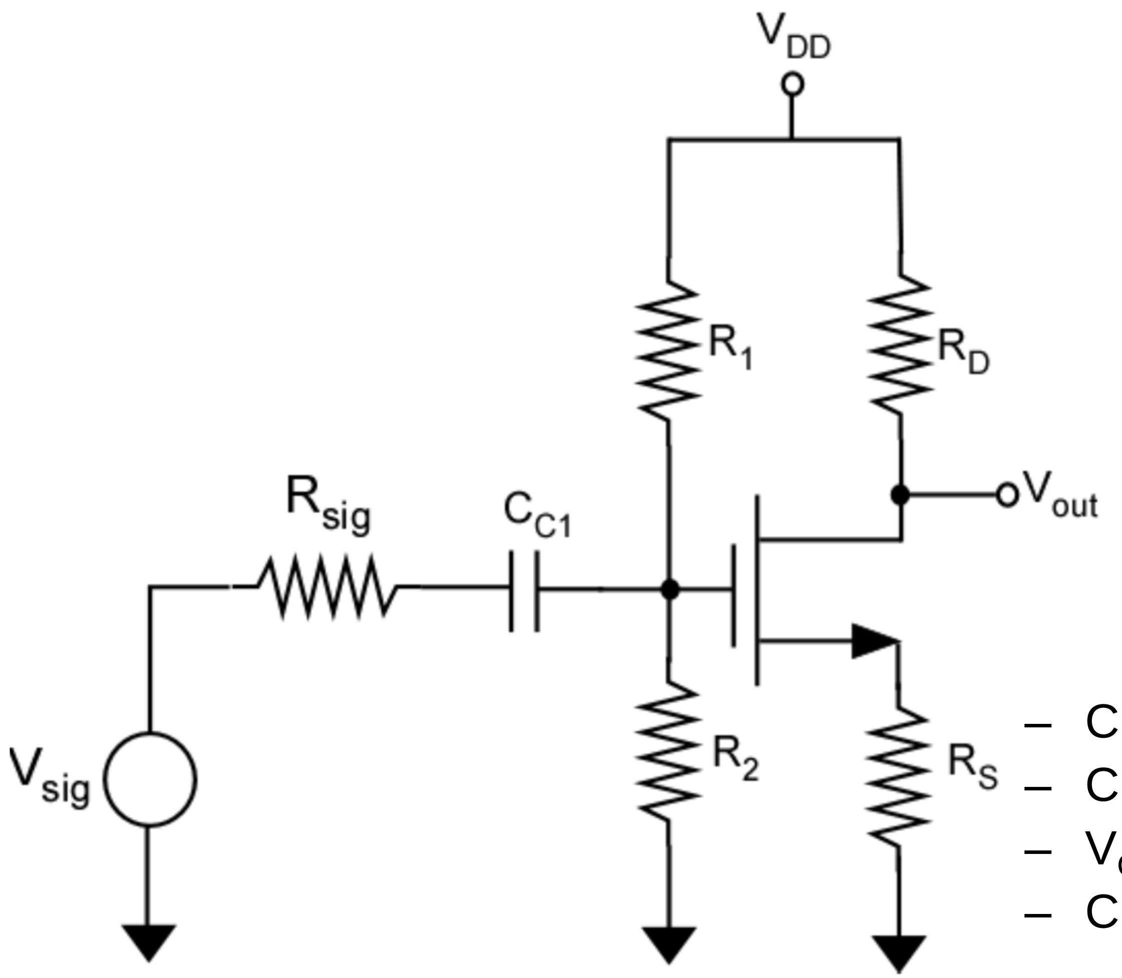
- **What is a small-signal incremental model?**
  - Equivalent circuit that only accounts for signal level fluctuations about the DC bias operating points
  - Fluctuations are assumed to be small enough so as not to drive the devices out of the proper range of operation
  - Assumed to be linear
  - Derives from superposition principle

Biasing of MOS Transistors

- **Bias Characteristics**
  - Operation in saturation region
  - Stable and predictable drain current

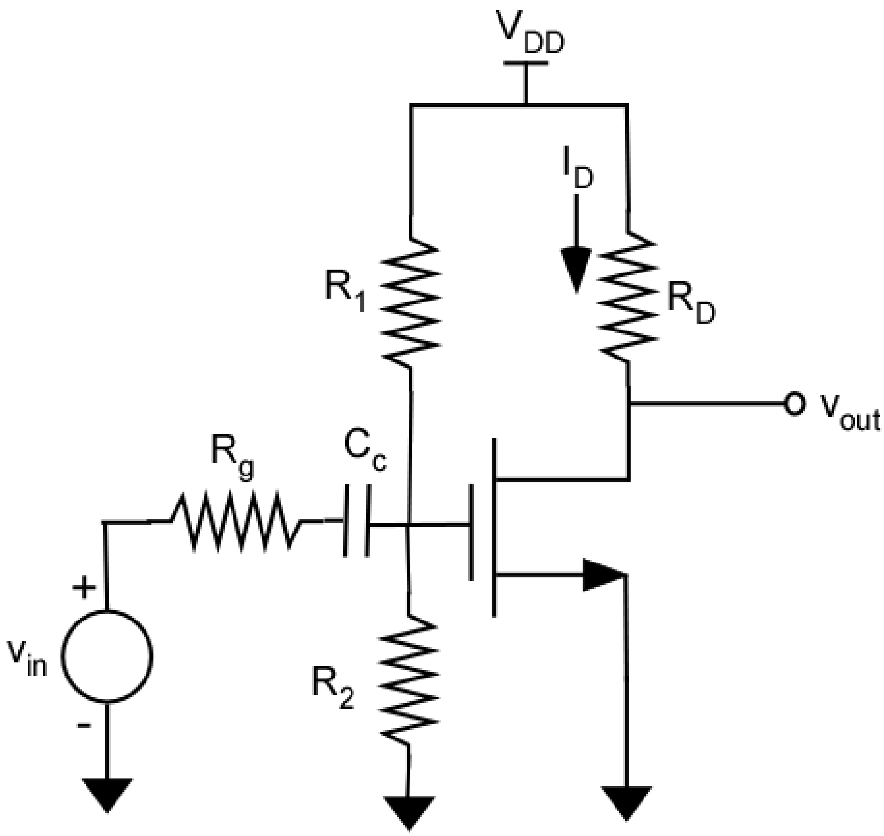
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_T\right)^2$$

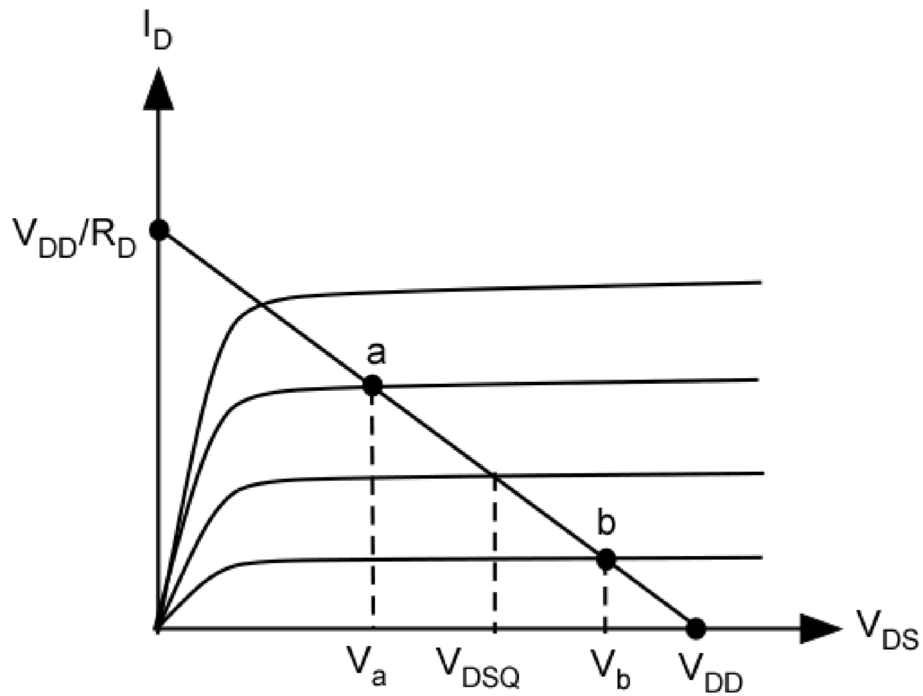
Single-Supply MOS Bias



- Choose R_1 and R_2 to fix V_G
- Choose R_S and R_2 to fix V_S
- V_{GS} determines I_D
- Choose R_D to fix V_D

Common Source MOSFET Amplifier

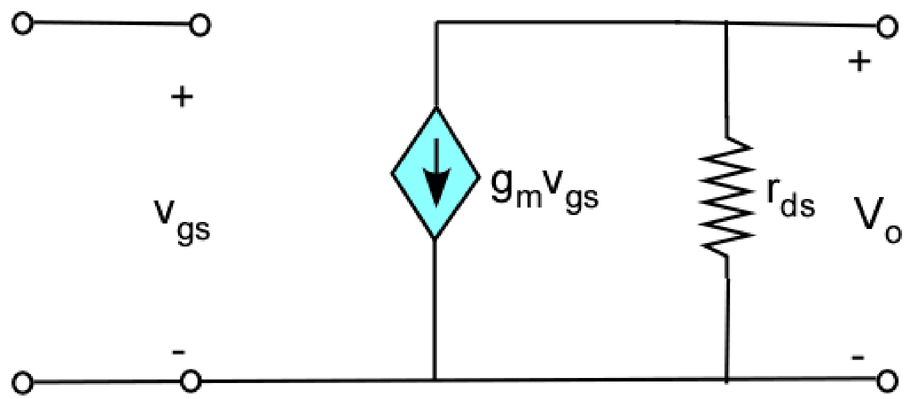




Bias is to keep MOS in saturation region

Common Source MOSFET Amplifier

Small-Signal Equivalent Circuit for MOS (device only)



$$I_D = \frac{1}{2} k_n^{'} \frac{W}{L} \left(V_{GS} - V_T\right)^2$$

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS}=V_{GSQ}} = \frac{2I_D}{V_{eff}}$$

$$where\, V_{GS} - V_T = V_{eff}$$

Which leads to

$$g_m = \sqrt{2k_n^{'}}\,\sqrt{W/L}\,\sqrt{I_D}$$

$$g_m \text{ is proportional to } = \sqrt{W/L}$$

MOSFET Output Impedance

To calculate r_{ds}, account for λ

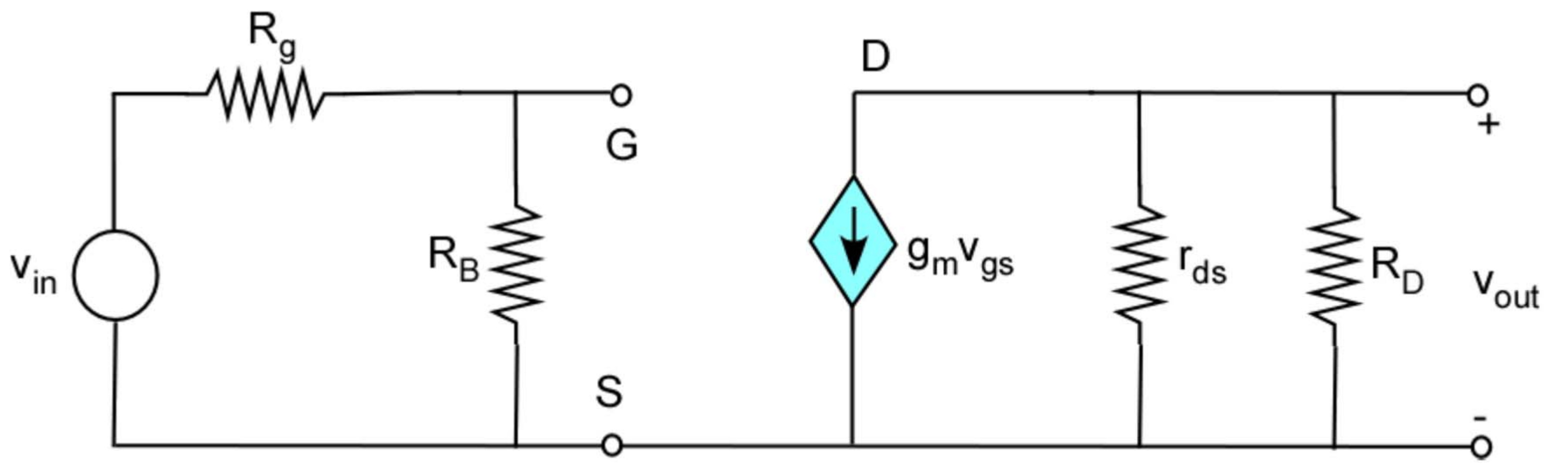
$$r_{ds} = \left.\frac{\partial V_{DS}}{\partial I_D}\right|_{V_{GS}=V_{GSQ}} = \frac{1}{\lambda\mu\frac{W}{2L}C_{ox}\left[V_{GS}-V_T\right]^2} = \frac{1}{\lambda I_{DP}}$$

$$I_{DP} = \frac{1}{2}k_n^{'}\frac{W}{L}\left(V_{GS}-V_T\right)^2$$

r_{ds}, accounts for channel width modulation resistance.

Midband Frequency Gain

Incremental model for complete amplifier



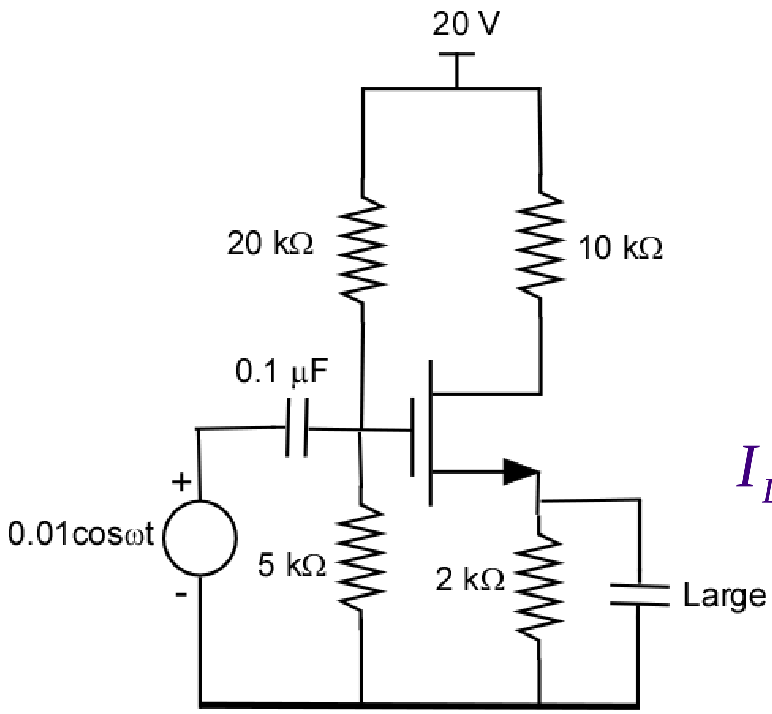
$$A_{MB} = \frac{v_{out}}{v_{in}} = -\frac{R_B}{R_B + R_g} g_m \frac{r_{ds} R_D}{r_{ds} + R_D}$$

Example

For the circuit shown, k=75 μA/V², V_T=1 V, λ=0

(a) Find V_{DQ}, V_{SQ}

(b) Find the midband gain



$$V_{GQ} = V_{DD}\frac{R_2}{R_1+R_2} = \frac{20\times 5}{25} = 4\,V$$

$$V_{GSQ} = V_{GQ} - V_{SQ} = 4 - 2I_{DQ}$$

$$I_{DQ} = K\left[V_{GSQ} - V_T\right]^2 = 0.075\left[4 - 2I_{DQ} - 1\right]^2$$

$$I_{DQ} = 0.075(9 - 12I_{DQ} + 4I_{DQ}^2)$$

$$4I_{DQ}^2 - 12I_{DQ} + 9 = 13.3I_{DQ} \Rightarrow I_{DQ}^2 - 6.33I_{DQ} + 2.25 = 0$$

Example (Cont')

$$I_{DQ} = 3.167 \pm \frac{\sqrt{6.33^2 - 9}}{2} = 0.378\, mA \text{ or } 5.953\, mA$$

reject since voltage drop across R_D will be too large

$$I_{DQ} = 0.378\, mA$$

$$V_{DQ} = V_{DD} - R_D I_{DQ} = 20 - 10 \times 0.378 = 16.22\, V$$

$$V_{SQ} = R_S I_{DQ} = 2 \times 0.378 = 0.756\, V$$

$$\boxed{\begin{aligned} V_{DQ} &= 16.22\, V \\ V_{SQ} &= 0.756\, V \end{aligned}}$$

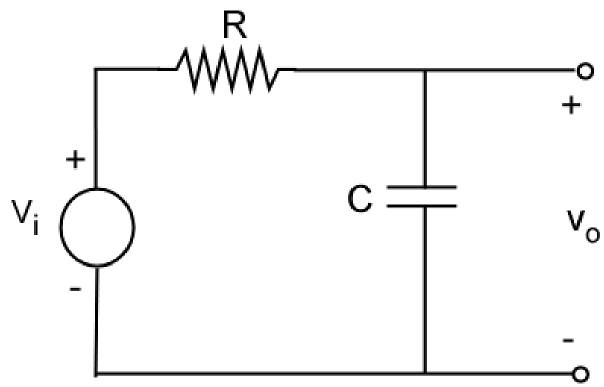
Example (Cont')

$$g_m = \sqrt{2k_n' \frac{W}{L} I_{DQ}} = \sqrt{4 \times 0.075 \times 0.378} = 0.337$$

$$A_{MB} = -g_m R_D = -0.337 \times 10 = -3.37$$

$$\boxed{A_{MB} = -3.37}$$

Low-Pass Circuit



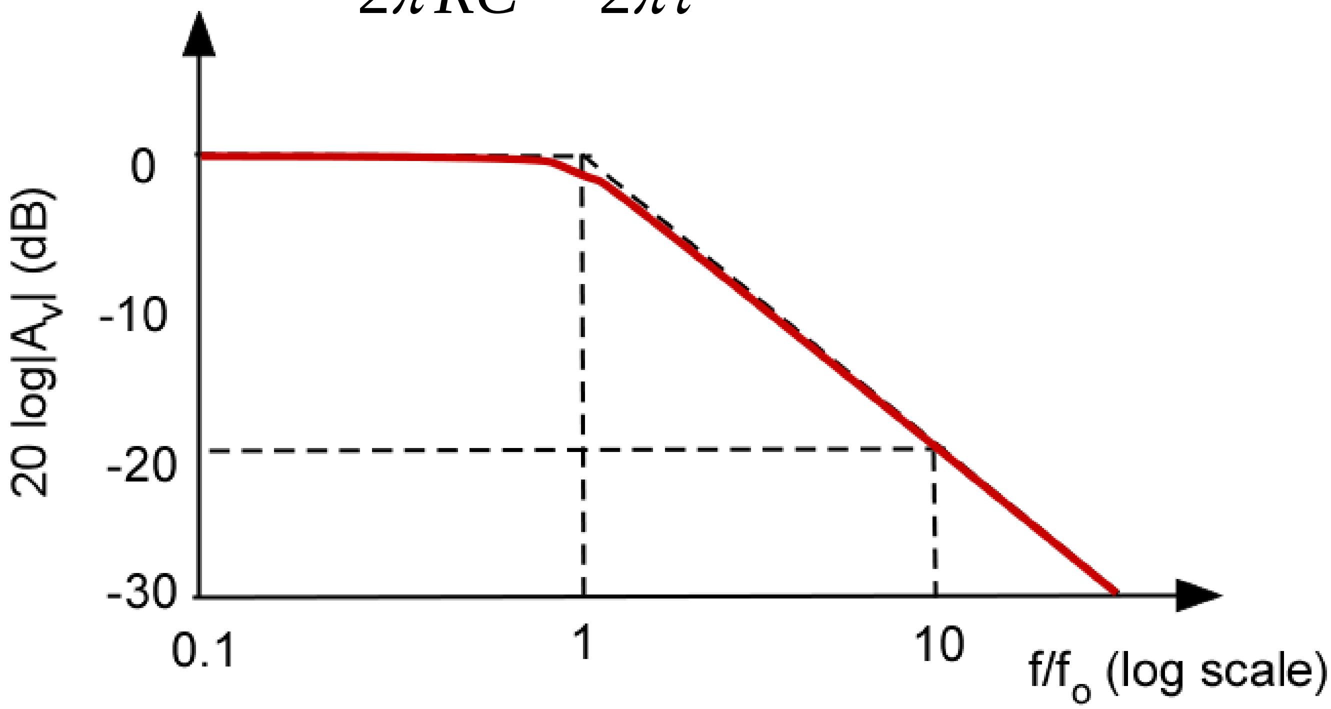
In frequency domain: $$V_o = \frac{V_i}{R + \frac{1}{j\omega C}} \cdot \frac{1}{j\omega C}$$

$$V_o = \frac{V_i}{1 + j\omega RC} \Rightarrow A_v = \frac{V_o}{V_i} = \frac{1}{1 + j\omega RC}$$

$$A_v = \frac{1}{1 + j\omega RC} = \frac{1}{1 + jf / f_2}$$

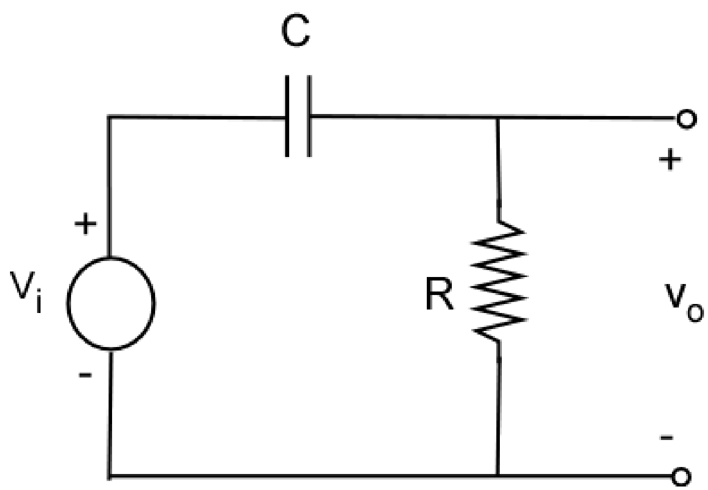
Low-Pass Circuit

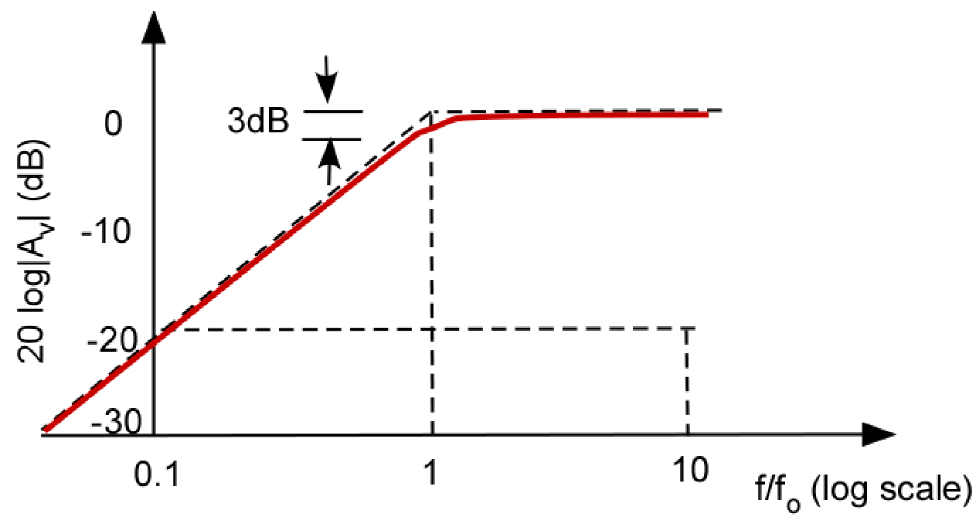
$$f_2 = \frac{1}{2\pi RC} = \frac{1}{2\pi\tau}$$



$$\tau = 2\pi RC = \text{time constant}$$

High-Pass Circuit





$$V_o = \frac{V_i R}{R + \dfrac{1}{j\omega C}} = \frac{V_i}{1 + \dfrac{1}{j\omega RC}}$$

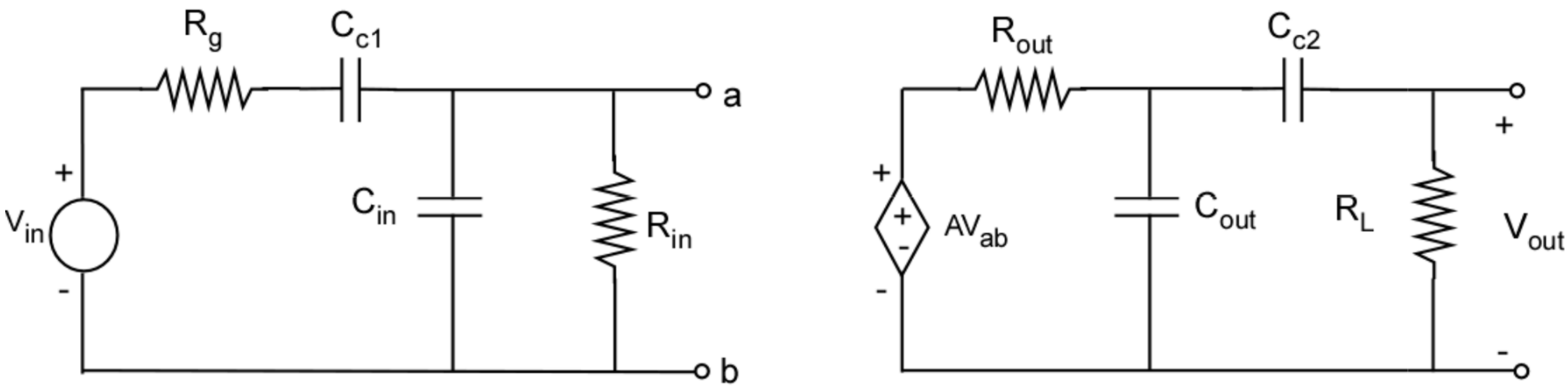
$$A_v = \frac{V_o}{V_i} = \frac{1}{1 - j\dfrac{1}{2\pi fRC}} = \frac{1}{1 - jf_2/f}$$

$$f_2 = \frac{1}{2\pi RC}$$

Model for general Amplifying Element

C_{c1} and C_{c2} are coupling capacitors (large) ➔ μF

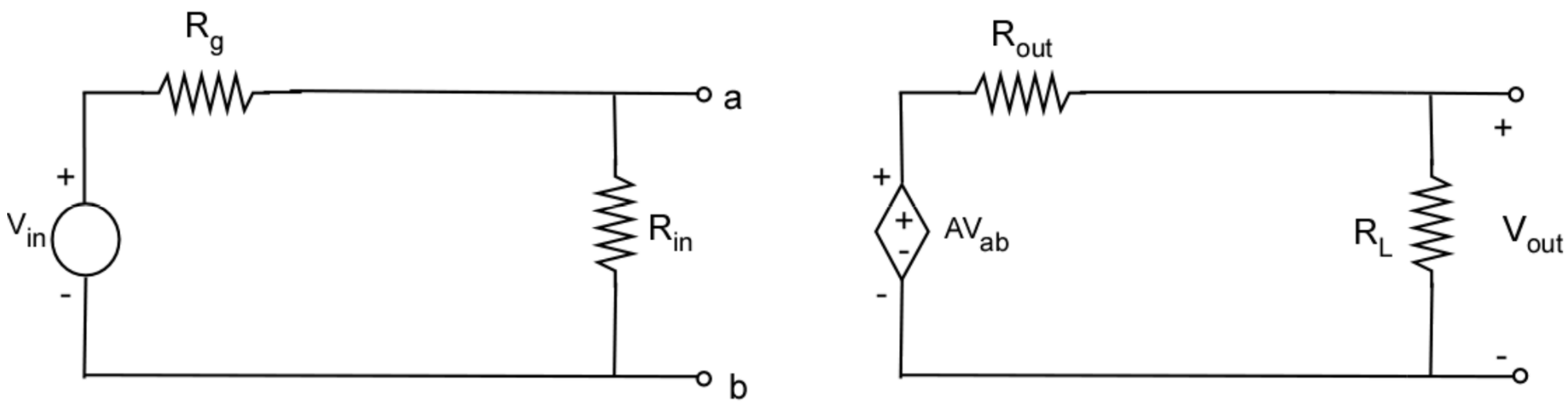
C_{in} and C_{out} are parasitic capacitors (small) ➔ pF



Midband Frequencies

- Coupling capacitors are short circuits

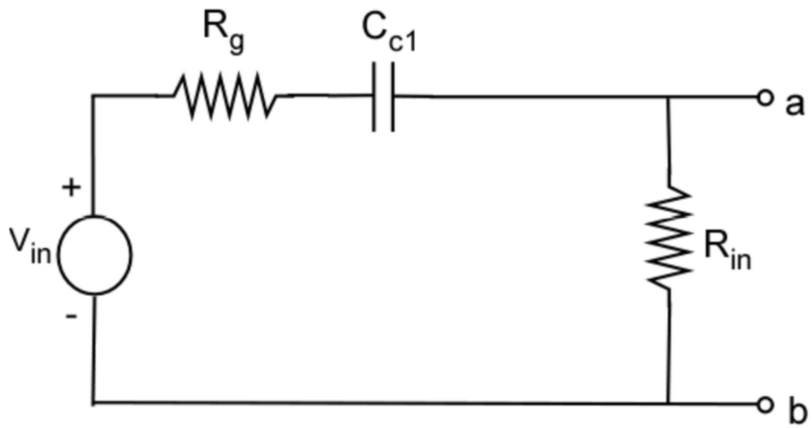
- Parasitic capacitors are open circuits

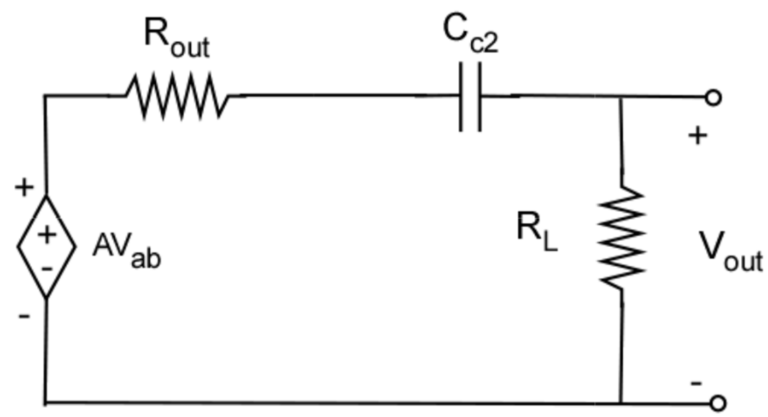


$$A_{MB} = \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_g + R_{in}} A \frac{R_L}{R_{out} + R_L}$$

Low Frequency Model

- Coupling capacitors are present
- Parasitic capacitors are open circuits

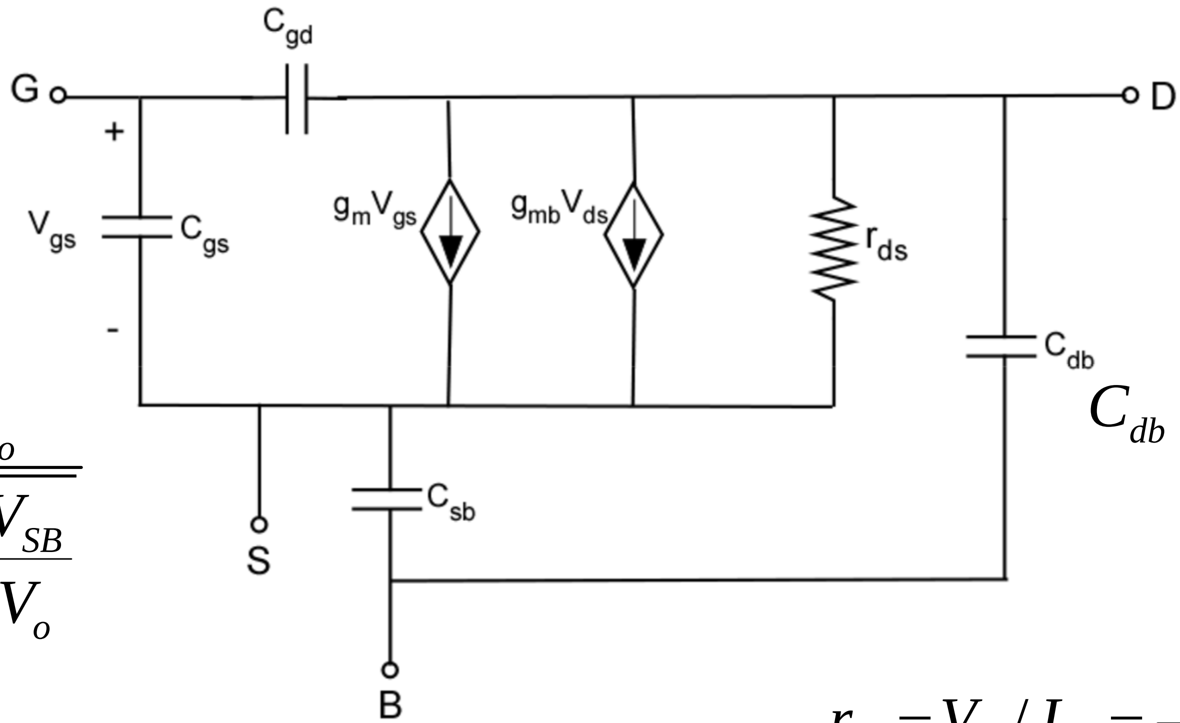




$$v_{ab} = \frac{v_{in} R_{in}}{R_g + R_{in} + \dfrac{1}{j\omega C_{c1}}} = \frac{v_{in}\, j\omega C_{c1} R_{in}}{1 + j\omega C_{c1}(R_g + R_{in})}$$

$$v_{ab} = v_{in} \frac{R_{in}}{R_g + R_{in}} \cdot \frac{j\omega C_{c1}(R_g + R_{in})}{\left[1 + j\omega C_{c1}(R_g + R_{in})\right]}$$

MOSFET High-Frequency Model



$$C_{sb} = \frac{C_{sbo}}{\sqrt{1+\frac{V_{SB}}{V_o}}}$$

$$C_{db} = \frac{C_{dbo}}{\sqrt{1+\frac{V_{DB}}{V_o}}}$$

$$g_m = \mu_n C_{ox}\frac{W}{L}V_{eff} = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \frac{2I_D}{V_{eff}}$$

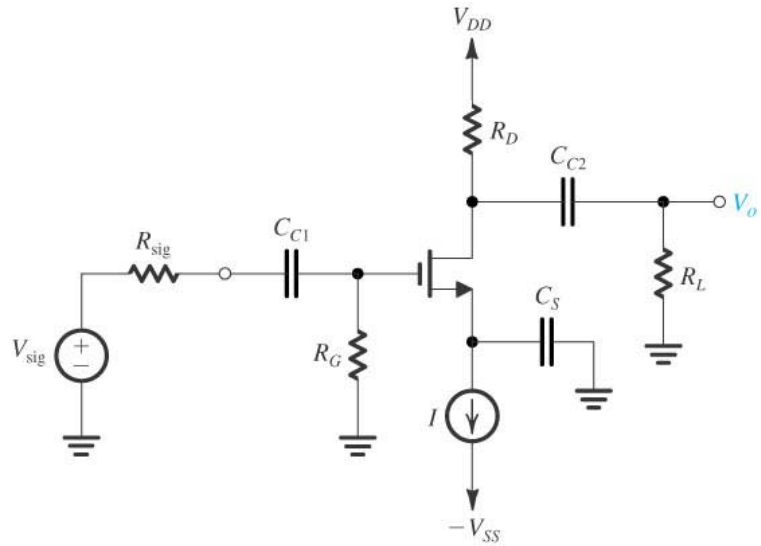
$$g_{mb} = \chi g_m = \frac{\gamma}{2\sqrt{2\phi_F + V_{sb}}}g_m$$

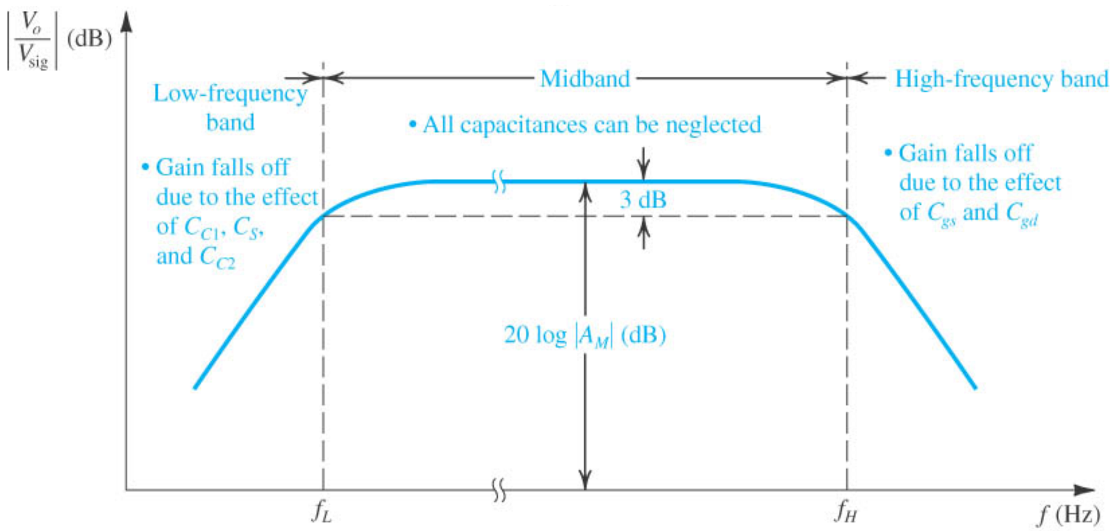
$$r_{ds} = V_A / I_D = \frac{1}{\lambda I_D}$$

$$C_{gs} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$$

$$C_{gd} = WL_{ov}C_{ox}$$

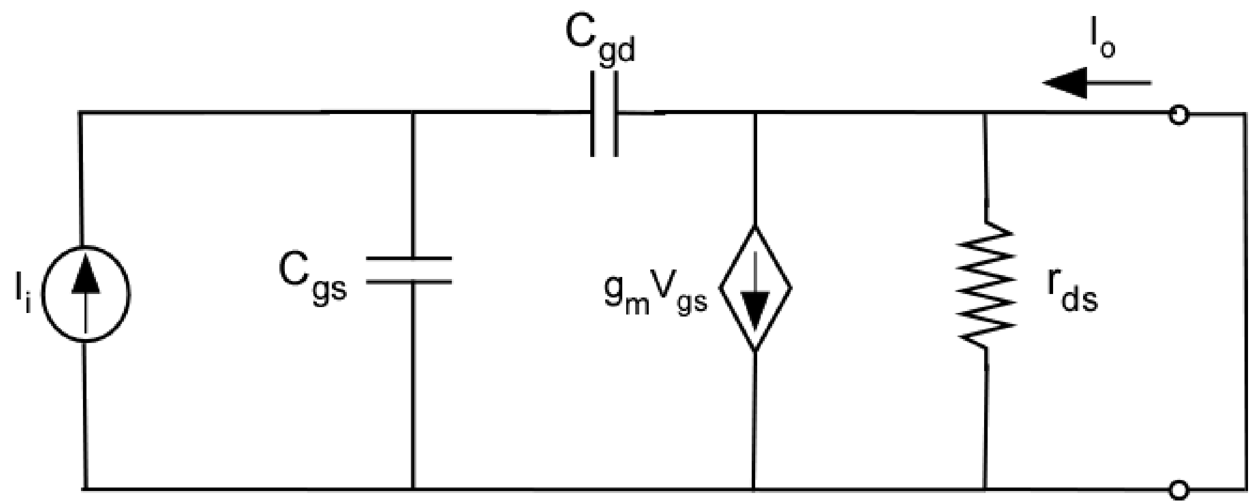
CS - Three Frequency Bands





Unity-Gain Frequency f_T

f_T is defined as the frequency at which the short-circuit current gain of the common source configuration becomes unity



Define:

$$s = j\omega$$

(neglect $sC_{gd}V_{gs}$ since C_{gd} is small)

$$I_o = g_m V_{gs} - sC_{gd} V_{gs}$$

$$\frac{I_o}{I_i} = \frac{g_m}{s\left(C_{gs} + C_{gd}\right)}$$

$$I_o \simeq g_m V_{gs} \qquad V_{gs} = \frac{I_i}{s\left(C_{gs} + C_{gd}\right)}$$

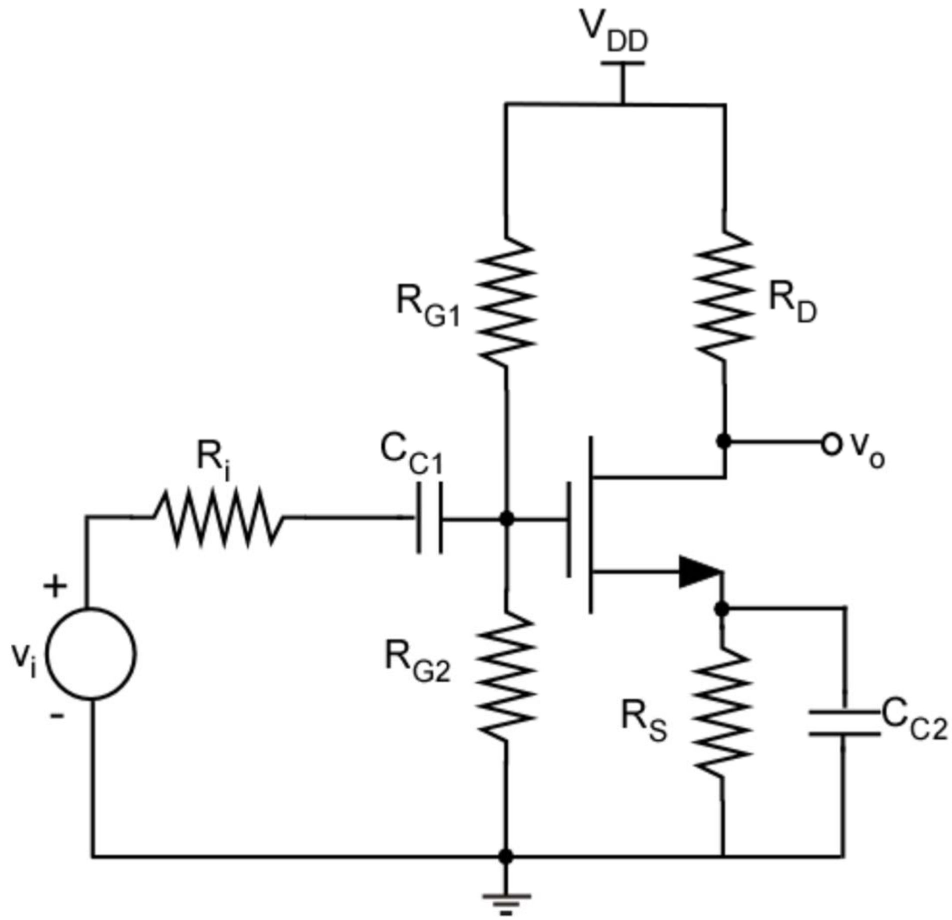
Calculating f_T

For $s=j\omega$, magnitude of current gain becomes unity at

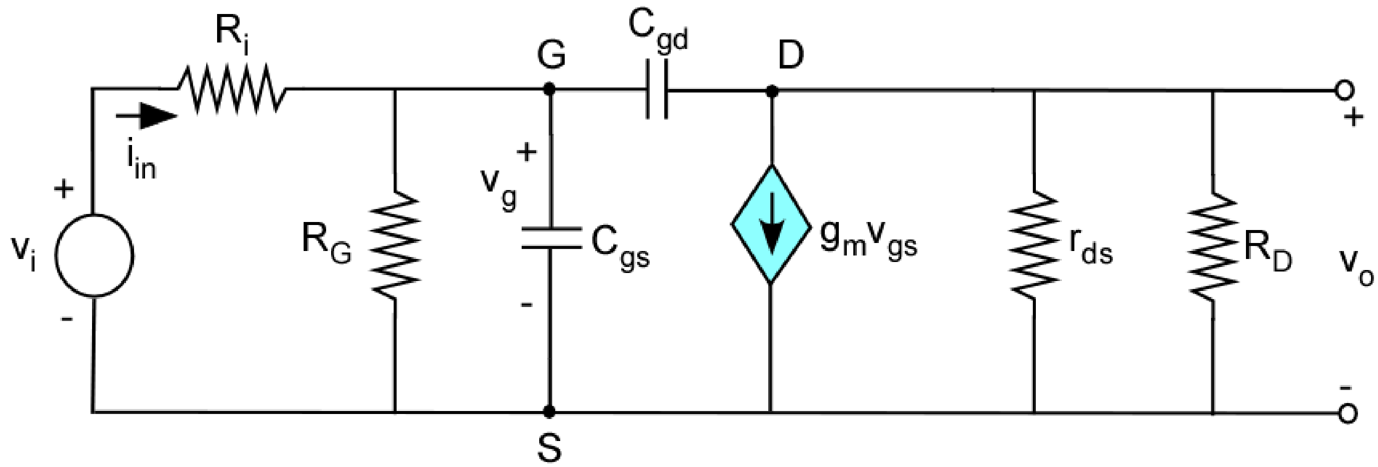
$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}} \Rightarrow f_T = \frac{g_m}{2\pi\left(C_{gs} + C_{gd}\right)}$$

f_T ~ 100 MHz for 5-μm CMOS, f_T ~ several GHz for 0.13μm CMOS

CS - High-Frequency Response



CS – Miller Effect – Exact Analysis



$$G_i = \frac{1}{R_i} \qquad G_D = \frac{1}{R_D} \qquad G_g = \frac{1}{R_g} \qquad g_{ds} = \frac{1}{r_{ds}}$$

$$R_D^{'} = R_D \,||\, r_{ds} = \frac{1}{G_D + g_{ds}}$$

$$\frac{v_o}{v_i} = -\frac{G_i R_D^{'}\left(g_m - sC_{gd}\right)}{G_i + G_g + s\left[C_{gs} + C_{gd}\right] + sC_{gd}R_D^{'}\left[G_i + G_g\right] + sC_{gd}g_m R_D^{'} + s^2 C_{gd}C_{gs}R_D^{'}}$$

CS – Miller Effect – Exact Analysis

We neglect the terms in s^2 since

$$\left|s^2 C_{gd} C_{gs} R_D'\right| << \left|s C_{gd} g_m R_D'\right| \quad or \quad \left|s C_{gs}\right| << \left|g_m\right|$$

$$\frac{v_o}{v_i} = -\frac{G_i R_D'\left(g_m - sC_{gd}\right)}{G_i + G_g + s\left[C_{gs} + \underbrace{C_{gd}\left(1 + g_m R_D'\right)}_{\text{Miller}} + C_{gd} R_D'\left(G_i + G_g\right)\right]}$$

If we multiply through by $R_i = \dfrac{1}{G_i}$

CS – Miller Effect – Exact Analysis

$$\frac{v_o}{v_i} = -\frac{R_D^{'}\left(g_m - sC_{gd}\right)}{1 + R_i G_g + s\left\{R_i\left[C_{gs} + C_{gd}\left(1 + g_m R_D^{'}\right)\right] + C_{gd} R_D^{'}\left(1 + R_s G_g\right)\right\}}$$

From which we extract the 3-dB frequency point

$$f_H = \frac{1 + R_i G_g}{2\pi\left\{R_i\left[C_{gs} + C_{gd}\left(1 + g_m R_D^{'}\right)\right] + C_{gd} R_D^{'}\left(1 + R_i G_g\right)\right\}}$$

CS – Miller Effect – Exact Analysis

If G_g is negligible

$$f_H = \frac{1}{2\pi\left\{R_i\left[C_{gs}+C_{gd}\left(1+g_m R_D^{'}\right)\right]+C_{gd}R_D^{'}\right\}}$$

If R_i =0

$$f_H = \frac{1}{2\pi C_{gd} R_D^{'}}$$

Transfer Function Representation

In general, the transfer function of an amplifier can be expressed as

$$F_H(s) = a_m \frac{(s - Z_1)(s - Z_2)\ldots(s - Z_m)}{(s - P_1)(s - P_2)\ldots(s - P_m)}$$

$Z_1, Z_2, \ldots Z_m$ are the **zeros** of the transfer function

$P_1, P_2, \ldots P_m$ are the **poles** of the transfer function

s is a complex number $s = \sigma + j\omega$

3dB Frequency Determination

$$A(s) \equiv A_M F_H(s)$$

- **Designer is interested in midband operation**
- **However needs to know upper 3-dB frequency**
- **In many cases some conditions are met:**
  - **Zeros are infinity or at very high frequencies**
  - **One of the poles (ω_{P1}) is at much lower frequency than other poles (➔dominant pole)**
- **If the conditions are met then $F_H(s)$ can be approximated by:**

$$F_H(s) \equiv \frac{1}{1 + s/\omega_{P1}} \quad \text{and we have} \quad \omega_H \cong \omega_{P1}$$

3dB Frequency Determination

If the lowest frequency pole is at least 4 times away from the nearest pole or zero, it is a **dominant pole**

If there is no dominant pole, the 3-dB frequency ω_H can be approximated by:

$$\omega_H \cong 1/\sqrt{\left(\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}+\ldots\right)-2\left(\frac{1}{\omega_{Z1}^2}+\frac{1}{\omega_{Z2}^2}+\ldots\right)}$$

Open-Circuit Time Constants

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \ldots + a_n s^n}{1 + b_1 s + b_2 s^2 + \ldots b_n s^n}$$

The coefficients *a* and *b* are related to the frequencies of the zeros and poles respectively.

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \ldots + \frac{1}{\omega_{pn}}$$

b_1 can be obtained by summing the individual time constants of the circuit using the *open-circuit time constant method*

Open-Circuit Time Constant Method

- The time constant of each capacitor in the circuit is evaluated. It is the product of the capacitance and the resistance seen across its terminals with:
  - All other internal capacitors open circuited
  - All independent voltage sources short circuited
  - All independent current sources opened

- The value of b_1 is computed by summing the individual time constants

$$b_1 = \sum_{i=1}^{n} C_i R_{io}$$

Open-Circuit Time Constant Method

- **An approximation can be made by using the value of b_1 to determine the 3dB upper frequency point ω_H**
- **If the zeros are not dominant and if one of the poles P1 is dominant, then**

$$b_1 = \frac{1}{\omega_{P1}}$$

Assuming that the 3-dB frequency will be approximately equal to ω_{P1}

$$\omega_H = \frac{1}{b_1} = \frac{1}{\sum_i C_i R_{io}}$$

Bandwidth of Multistage Amplifier

- The poles of a multistage amplifier are difficult to obtain analytically

- An approximate value for the 3dB upper frequency point ω_{3dB} can be obtained by assigning an open circuit time constant τ_{io} to each capacitor C_i

Bandwidth of Multistage Amplifier

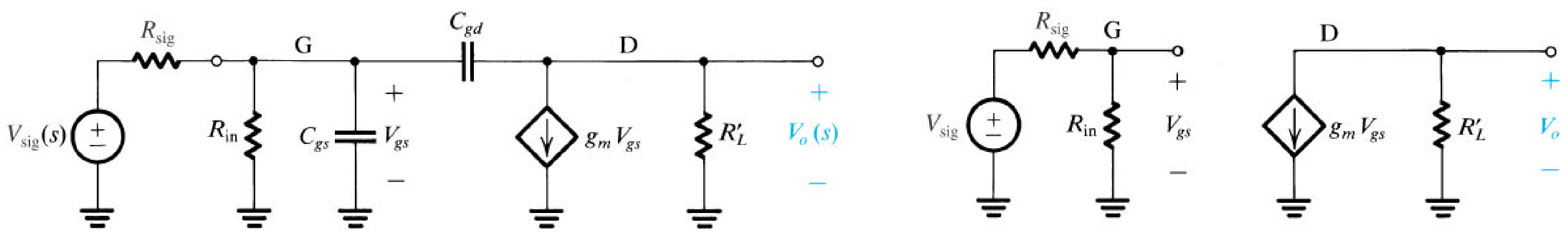
- The time constant τ_{io} is the product of the capacitance and the resistance seen across its terminals with:
  - All other internal capacitors open circuited
  - All independent voltage sources short circuited
  - All independent current sources opened

- The upper 3dB frequency point ω_{3dB} is then found by using :

$$\omega_{3dB} = \frac{1}{\sum \tau_{io}}$$

MOSFET Amp Bandwidth

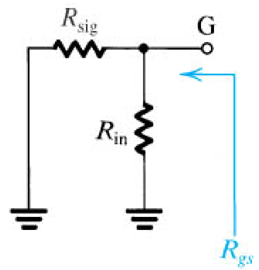
MOSFET amplifier has R_{sig}= 100 kΩ, $C_{gs}=C_{gd}$= 1 pF, g_m = 4 mA/V and R_L' =3.33 kΩ. Find midband voltage gain and 3-dB frequency.



$$A_M = \frac{V_o}{V_{sig}} = -\frac{R_{in}}{R_{in} + R_{sig}}\left(g_m R_L^{'}\right) = -\frac{420}{420+100} \times 4 \times 3.33 = -10.8$$

MOSFET Amp Analysis

To determine the 3-dB frequency, we first evaluate the time constant associated with C_{gs}. First, we determine the resistance R_{gs} seen by C_{gs}. The capacitance C_{gd} is removed and V_{sig} is short-circuited



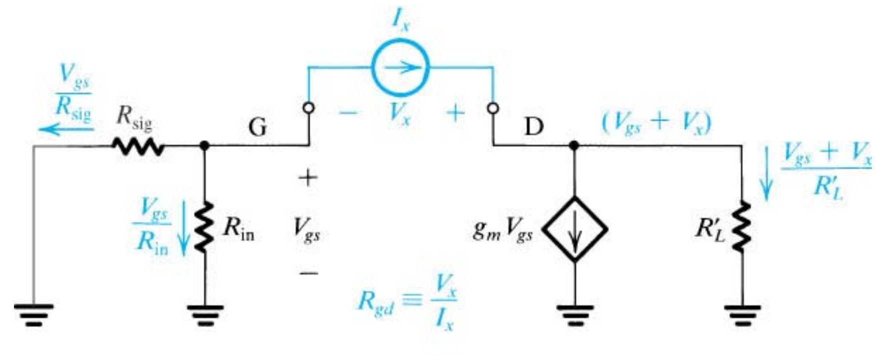
$$R_{gs} = R_{in} \,||\, R_{sig} = 420\ k\Omega \,||\, 100\ k\Omega = 80.8\ k\Omega$$

The time constant associated with C_{gs} is

$$\tau_{gs} = C_{gs} R_{gs} = 1\times 10^{-12} \times 80.8 \times 10^{3} = 80.8\ ns$$

MOSFET Amp Analysis

The resistance R_{gd} seen by C_{gd} is found by setting C_{gs} = 0 and short-circuiting V_{sig}



$$I_x = -\frac{V_{gs}}{R_{in}} - \frac{V_{gs}}{R_{sig}}$$

$$V_{gs} = -I_x R^{'}$$

$$R^{'} = R_{in} \,||\, R_{sig}$$

$$I_x = g_m V_{gs} + \frac{V_{gs} + V_x}{R_L^{'}}$$

$$R_{gd} = \frac{V_x}{I_x} = R^{'} + R_L^{'} + g_m R_L^{'} R^{'}$$

MOSFET Amp Analysis

The open-circuit time constant of C_{gd} is

$$\tau_{gd} = C_{gd}R_{gd} = 1\times10^{-12}\times1.16\times10^{6} = 1160\,ns$$

The upper 3-dB frequency ω_H can now be determined from

$$\omega_H = \frac{1}{\tau_{gs}+\tau_{gd}} = \frac{1}{(80.8+1160)\times10^{-9}} = 806\,krad/s$$

$$f_H = \frac{\omega_H}{2\pi} = 128.3\,kHz$$