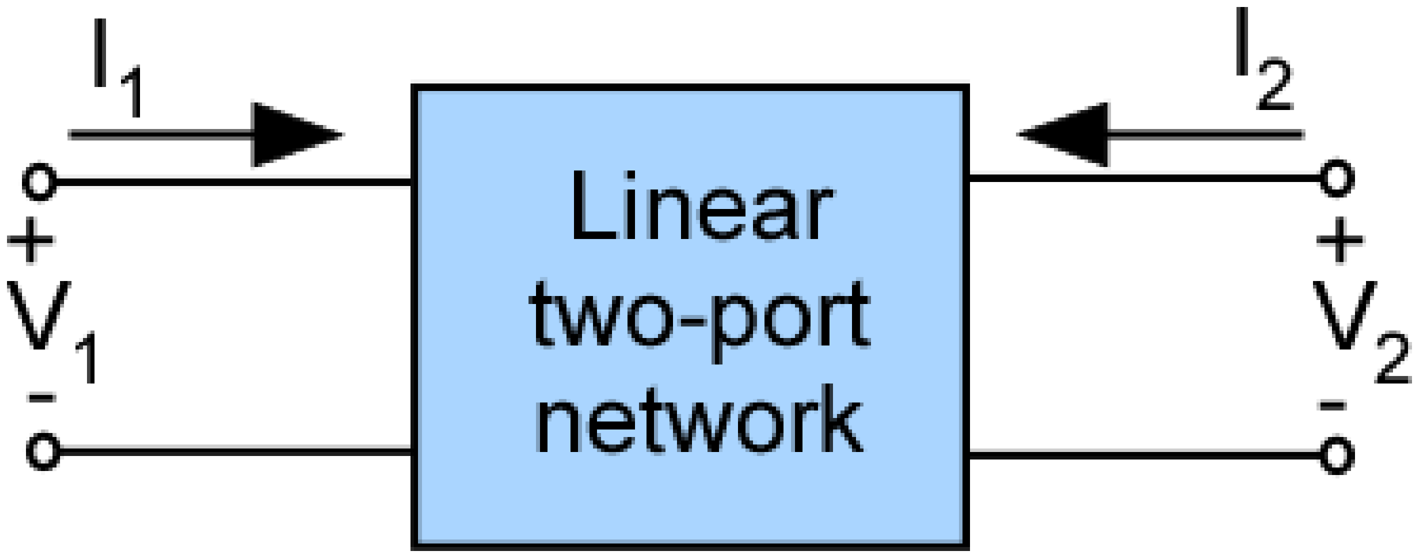
ECE 546
Lecture -13
Scattering Parameters

Spring 2026

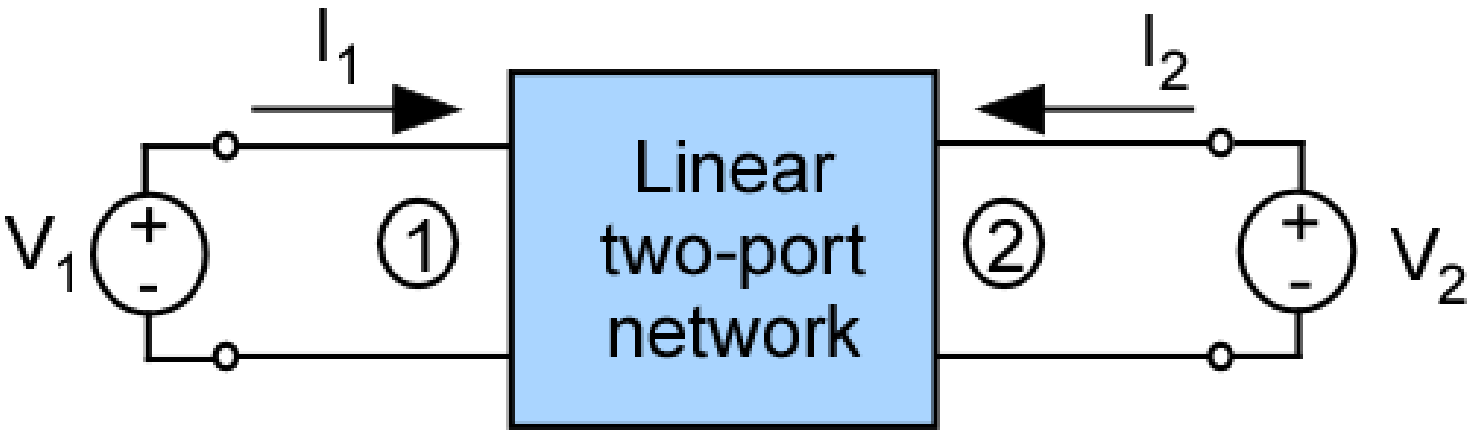
Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jesa@illinois.edu

Transfer Function Representation



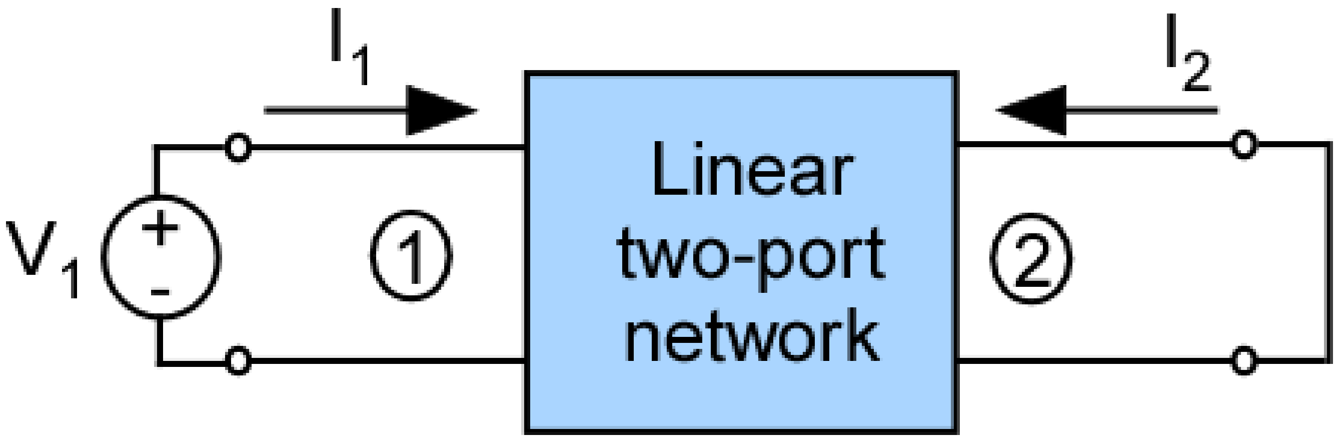
Use a two-terminal representation of system for input and output

Y-parameter Representation



$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

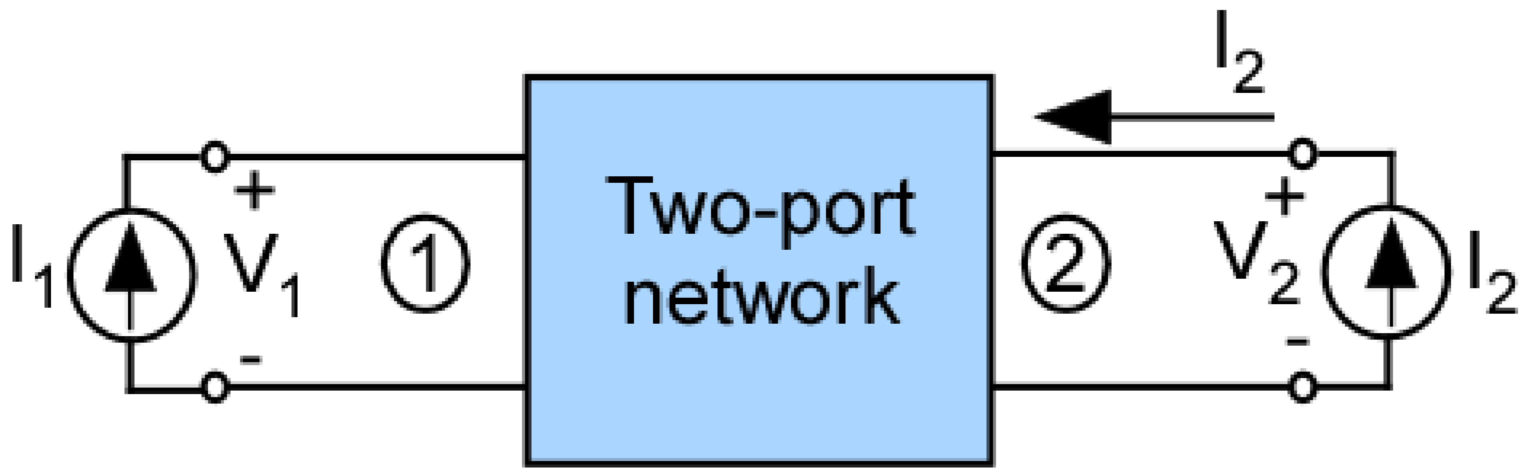
Y Parameter Calculations



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

To make $V_2 = 0$, place a short at port 2

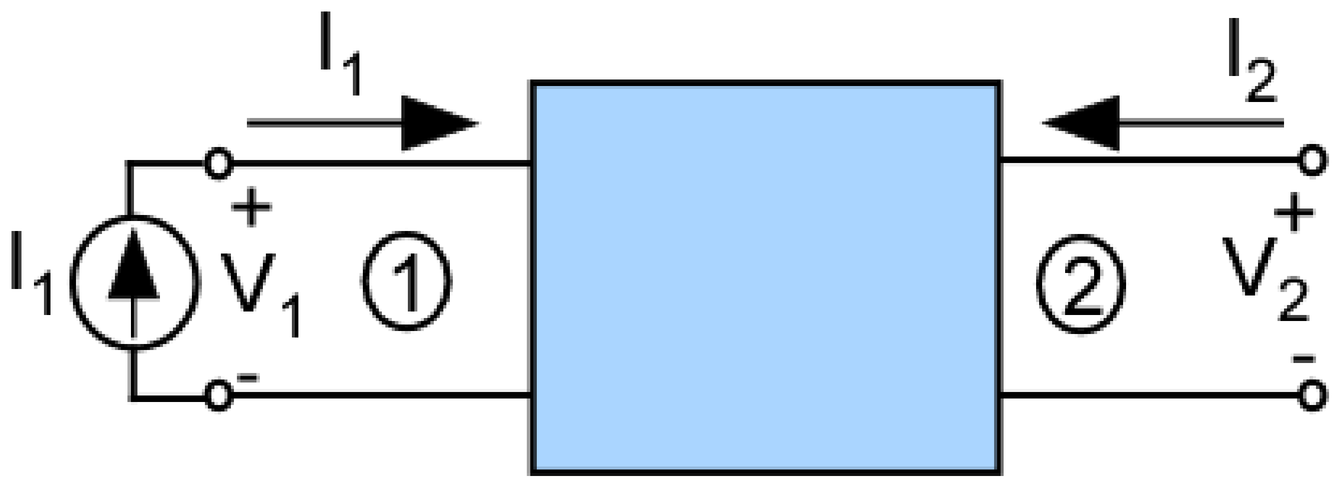
Z Parameters



$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

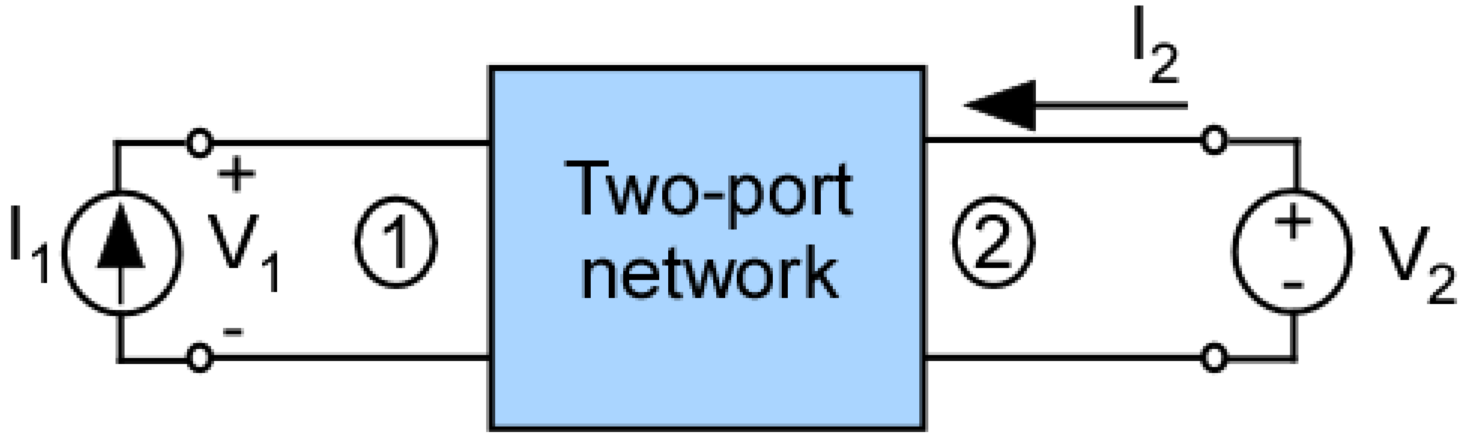
Z-parameter Calculations



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

To make $I_2 = 0$, place an open at port 2

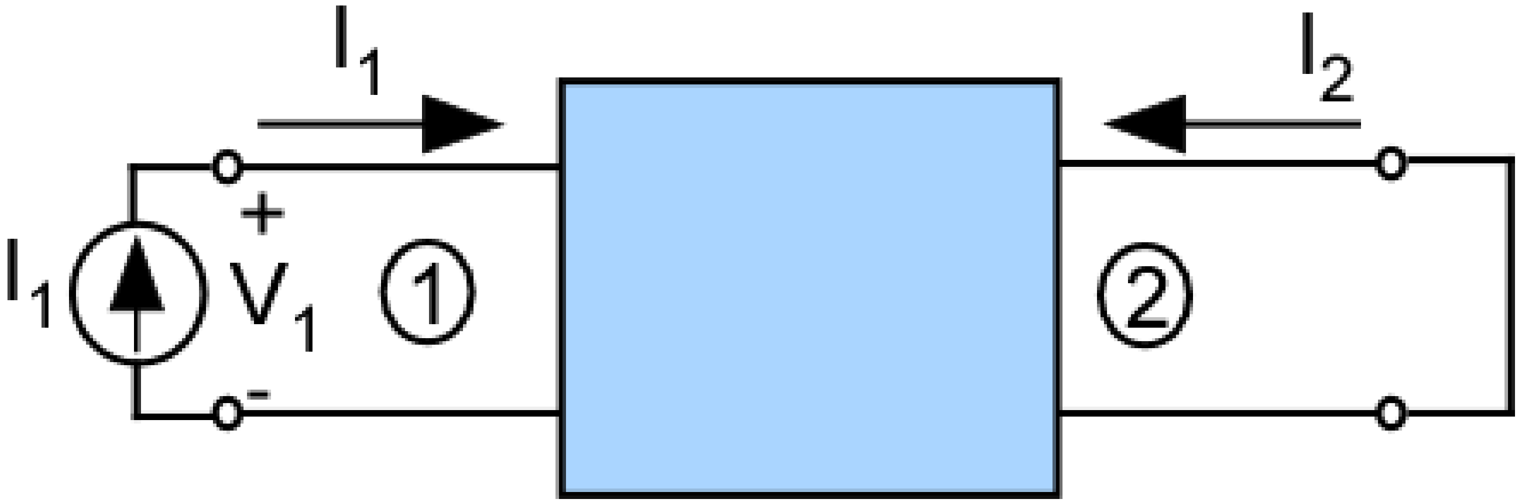
H Parameters



$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

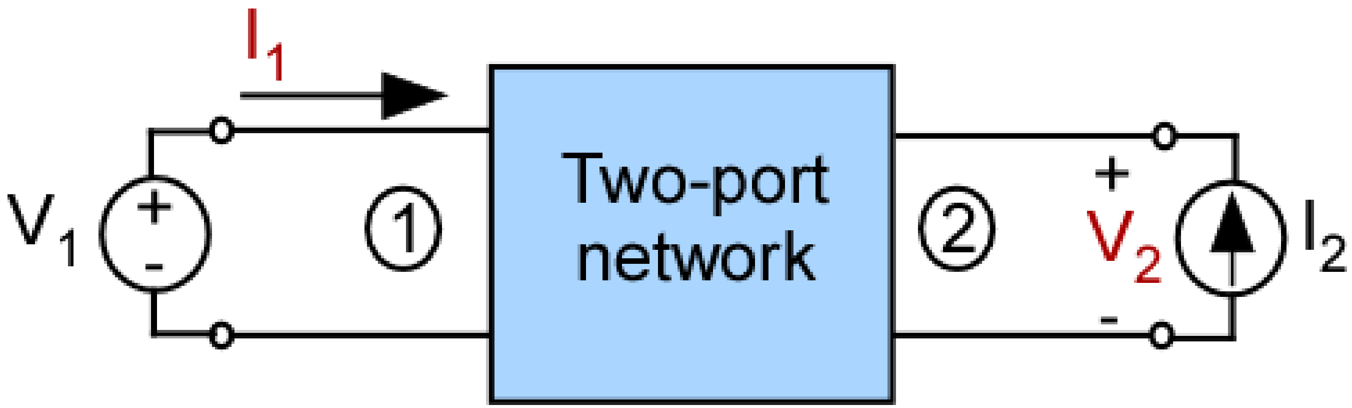
H Parameter Calculations



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

To make $V_2 = 0$, place a short at port 2

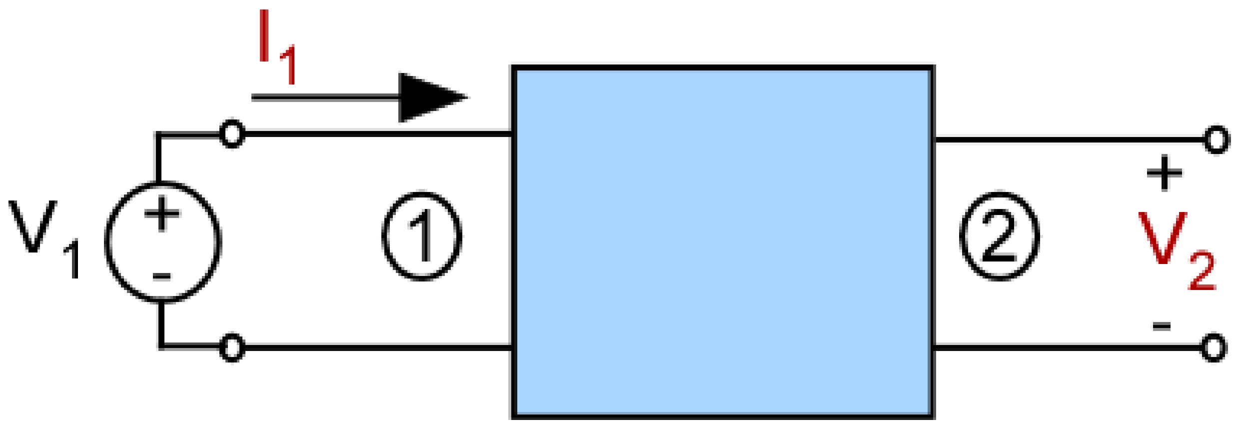
G Parameters



$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

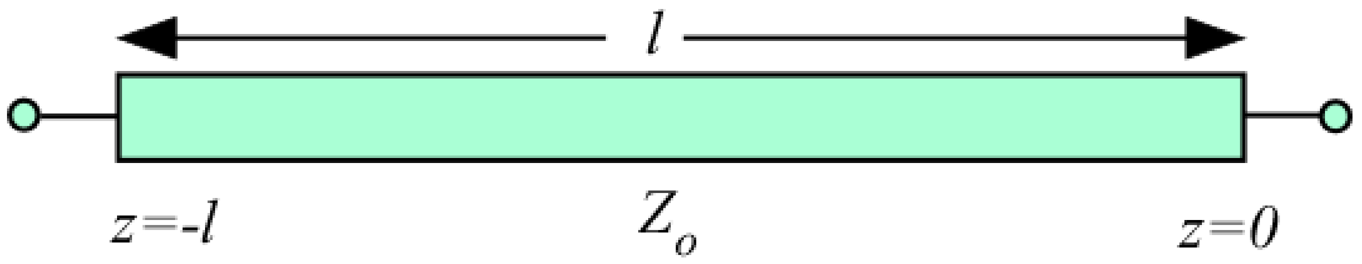
G-Parameter Calculations



$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \qquad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$

To make $I_2 = 0$, place an open at port 2

Y-Parameters of TL



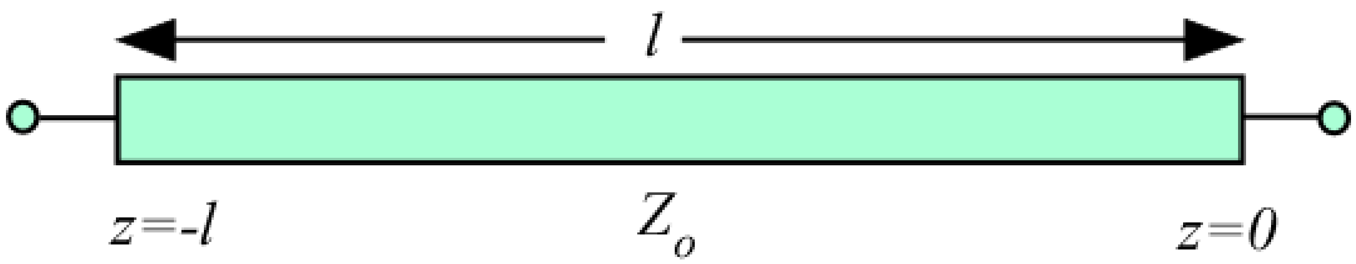
Find the Y-parameters of a lossless transmission line with propagation constant β and characteristic impedance Z_o (admittance Y_o)

$$V(z) = V_+ e^{-j\beta z} + V_- e^{+j\beta z}$$

$$I(z) = Y_o \left(V_+ e^{-j\beta z} - V_- e^{+j\beta z} \right)$$

Let port 1 be at $z=-l$ and port 2 at $z=0$

Y-Parameters of TL



at port 1

$$V_1 = V_+ e^{+j\beta l} + V_- e^{-j\beta l}$$

$$I_1 = Y_o \left(V_+ e^{+j\beta l} - V_- e^{-j\beta l} \right)$$

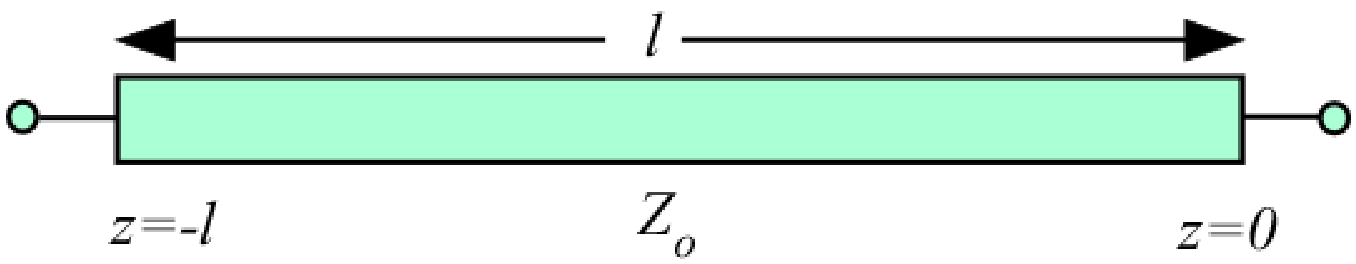
at port 2 ($z = 0$)

$$V_2 = V_+ + V_-$$

$$I_2 = -Y_o \left(V_+ - V_- \right)$$

$$V_+ = \frac{V_2 - Z_o I_2}{2} \quad \text{and} \quad V_- = \frac{V_2 + Z_o I_2}{2}$$

Y-Parameters of TL



So that

$$V_1 = \left(\frac{V_2 - Z_o I_2}{2}\right) e^{+j\beta l} + \left(\frac{V_2 + Z_o I_2}{2}\right) e^{-j\beta l}$$

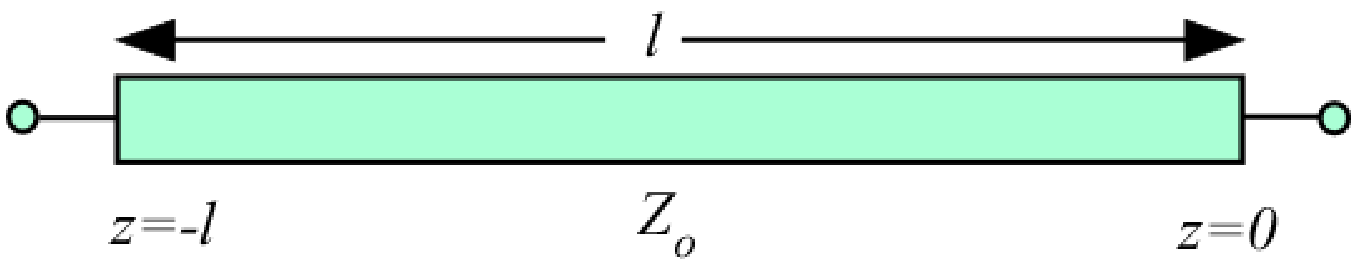
$$I_1 = Y_o \left(\frac{V_2 - Z_o I_2}{2}\right) e^{+j\beta l} - Y_o \left(\frac{V_2 + Z_o I_2}{2}\right) e^{-j\beta l}$$

and

$$V_1 = V_2 \cos \beta l - Z_o I_2 j \sin \beta l$$

$$I_1 = +Y_o V_2 j \sin \beta l - I_2 \cos \beta l$$

Y-Parameters of TL



Using definitions for Y_{11}

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{-I_2 \cos\beta l}{-jZ_o I_2 \sin\beta l} = \frac{-jY_o \cos\beta l}{\sin\beta l}$$

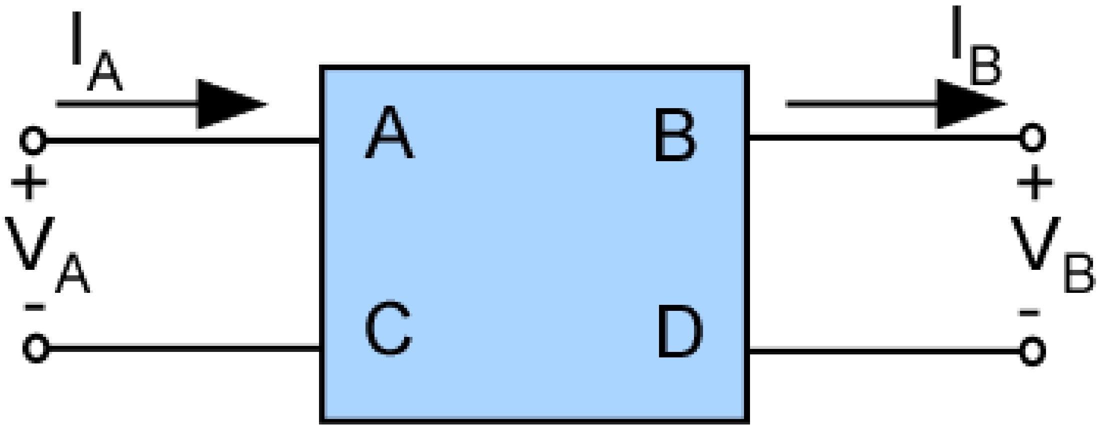
and

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-I_2}{-jZ_o I_2 \sin\beta l} = \frac{+jY_o}{\sin\beta l}$$

$$Y_{22} = Y_{11} \quad \text{by symmetry}$$

$$Y_{12} = Y_{21} \quad \text{by reciprocity}$$

ABCD -Parameters



$$V_A = AV_B + BI_B$$

$$I_A = CV_B + DI_B$$

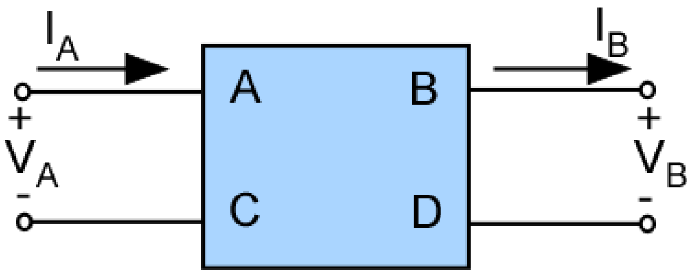
ABCD -Parameters

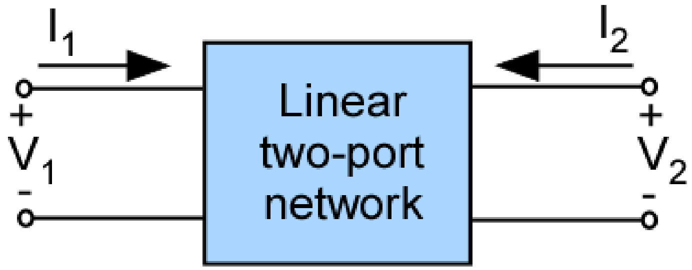
$$V_A = V_1$$

$$V_B = V_2$$

$$I_A = I_1$$

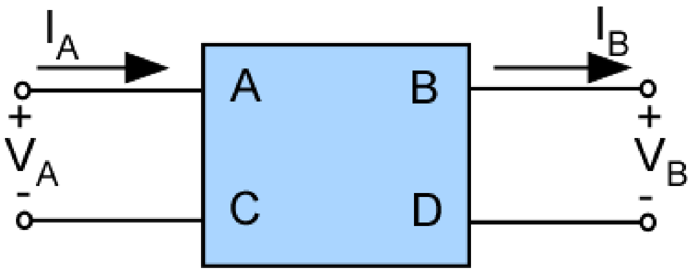
$$I_B = -I_2$$

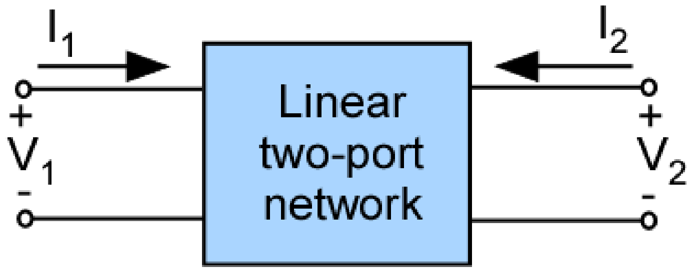




Relationship with Z parameters is obtained by first expressing ABCD parameters in terms of Z parameters

ABCD -Parameters





From

$$V_A = Z_{11} I_A - Z_{12} I_B$$

$$V_B = Z_{21} I_A - Z_{22} I_B$$

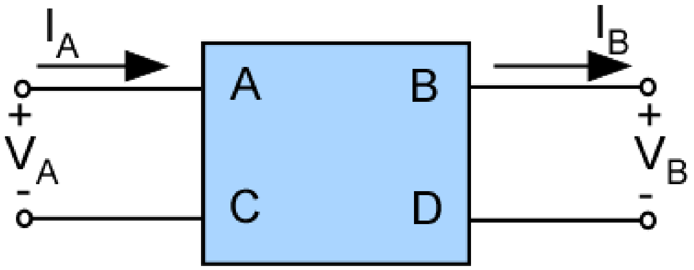
We get

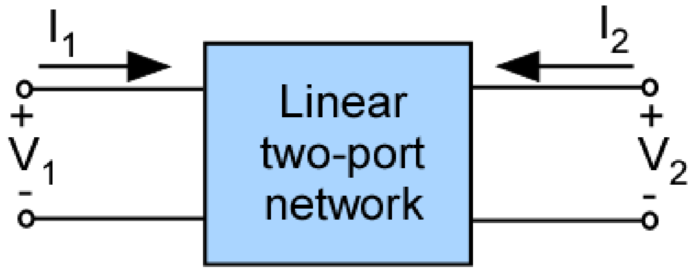
$$A = \frac{Z_{11}}{Z_{21}} \qquad B = \frac{\Delta}{Z_{21}}$$

$$C = \frac{1}{Z_{21}} \qquad D = \frac{Z_{22}}{Z_{21}}$$

$$\Delta = Z_{11} Z_{22} - Z_{12} Z_{21}$$

ABCD -Parameters





$$Z_{11}=\frac{A}{C} \qquad Z_{11}=\frac{(AD-BC)}{C}$$

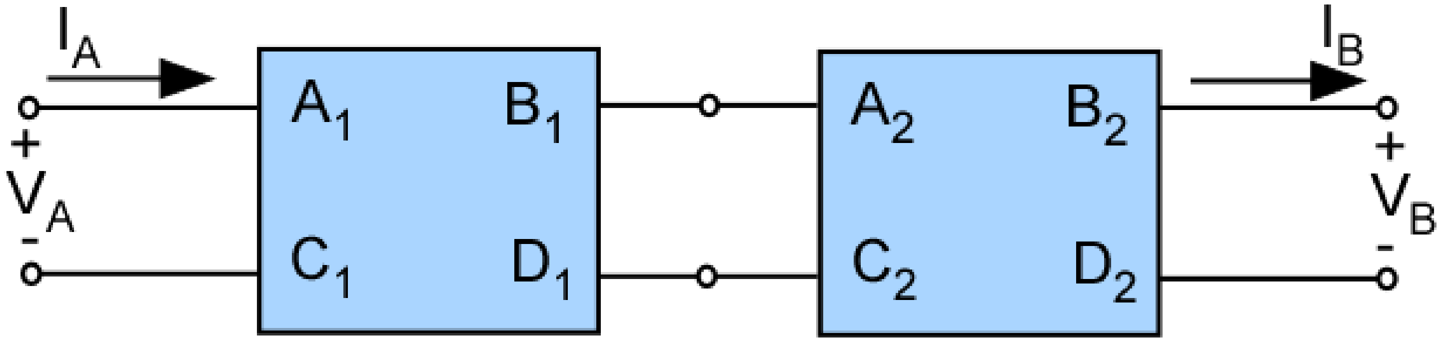
$$Z_{21}=\frac{1}{C} \qquad Z_{22}=\frac{1}{C}$$

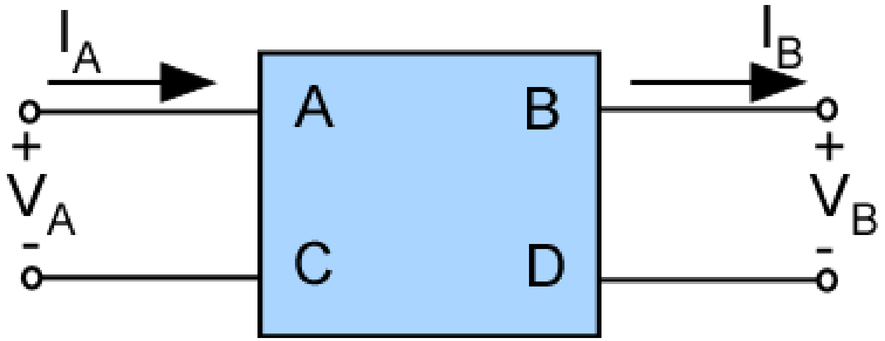
For a reciprocal network, $Z_{21} = Z_{12}$, therefore

$$AD-BC=1$$

← Reciprocity condition for ABCD parameters

ABCD -Parameters

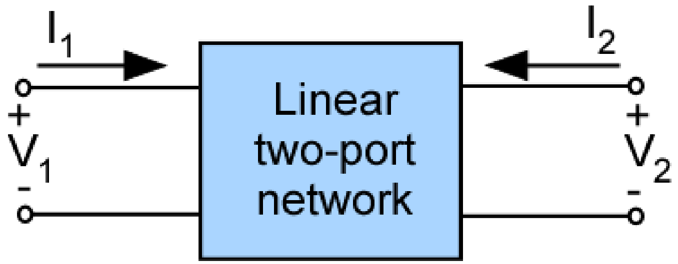




When cascading two-ports, it is best to use ABCD parameters. Put voltage and currents in cascadable form with the input variables in terms of the output variables

$$\mathbf{ABCD} = (\mathbf{ABCD})_1 \cdot (\mathbf{ABCD})_2$$

TWO-PORT NETWORK REPRESENTATION



<u>Z Parameters</u>

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

<u>Y Parameters</u>

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

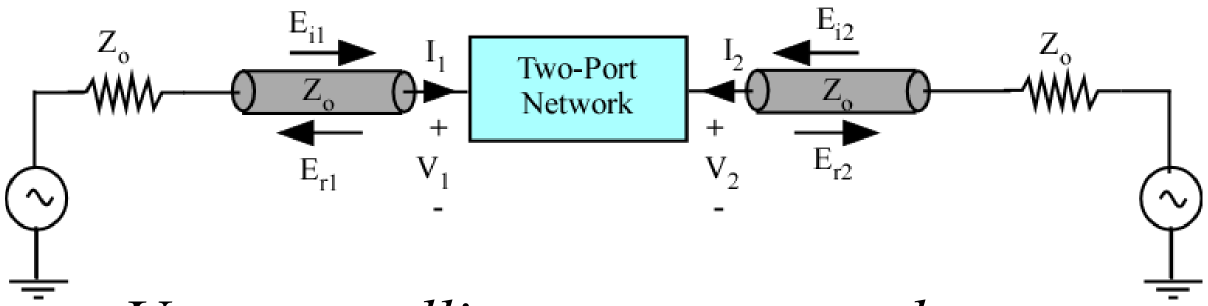
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

- At microwave frequencies, it is more difficult to measure total voltages and currents.

- Short and open circuits are difficult to achieve at high frequencies.

- Most active devices are not short- or open-circuit stable.

Wave Approach



Use a travelling wave approach

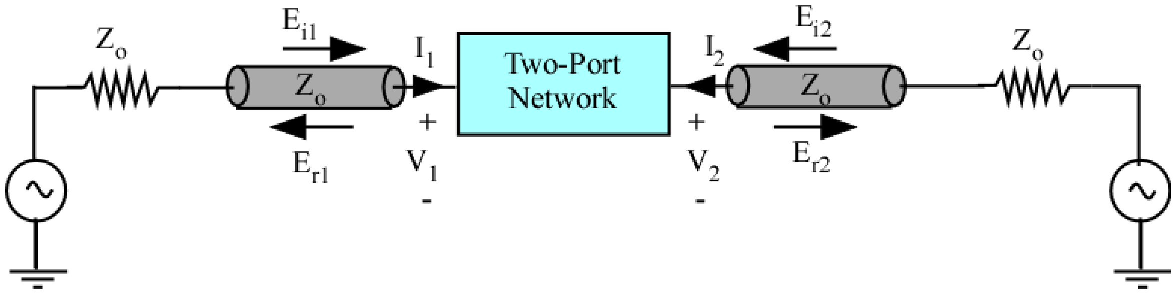
$$V_1 = E_{i1} + E_{r1} \qquad V_2 = E_{i2} + E_{r2}$$

$$I_1 = \frac{E_{i1} - E_{r1}}{Z_o} \qquad I_2 = \frac{E_{i2} - E_{r2}}{Z_o}$$

- **Total voltage and current are made up of sums of forward and backward traveling waves.**

- **Traveling waves can be determined from standing-wave ratio.**

Wave Approach



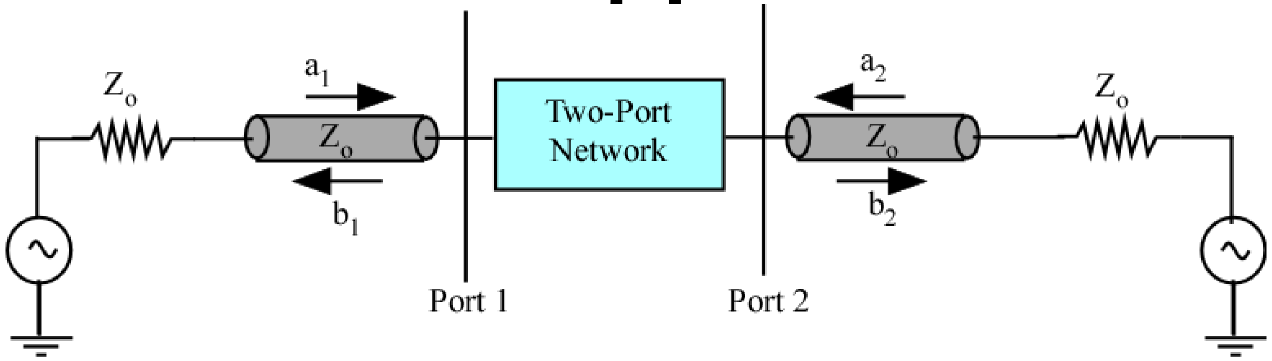
$$a_1 = \frac{E_{i1}}{\sqrt{Z_o}} \qquad a_2 = \frac{E_{i2}}{\sqrt{Z_o}}$$

$$b_1 = \frac{E_{r1}}{\sqrt{Z_o}} \qquad b_2 = \frac{E_{r2}}{\sqrt{Z_o}}$$

Z_o is the reference impedance of the system

$$\mathbf{b_1 = S_{11}\, a_1 + S_{12}\, a_2}$$
$$\mathbf{b_2 = S_{21}\, a_1 + S_{22}\, a_2}$$

Wave Approach



$$S_{11} = \frac{b_1}{a_1}\Big|_{a2=0} \qquad S_{21} = \frac{b_2}{a_1}\Big|_{a2=0}$$

$$S_{12} = \frac{b_1}{a_2}\Big|_{a1=0} \qquad S_{22} = \frac{b_2}{a_2}\Big|_{a1=0}$$

To make $a_i = 0$

1) Provide no excitation at port i

2) Match port i to the characteristic impedance of the reference lines.

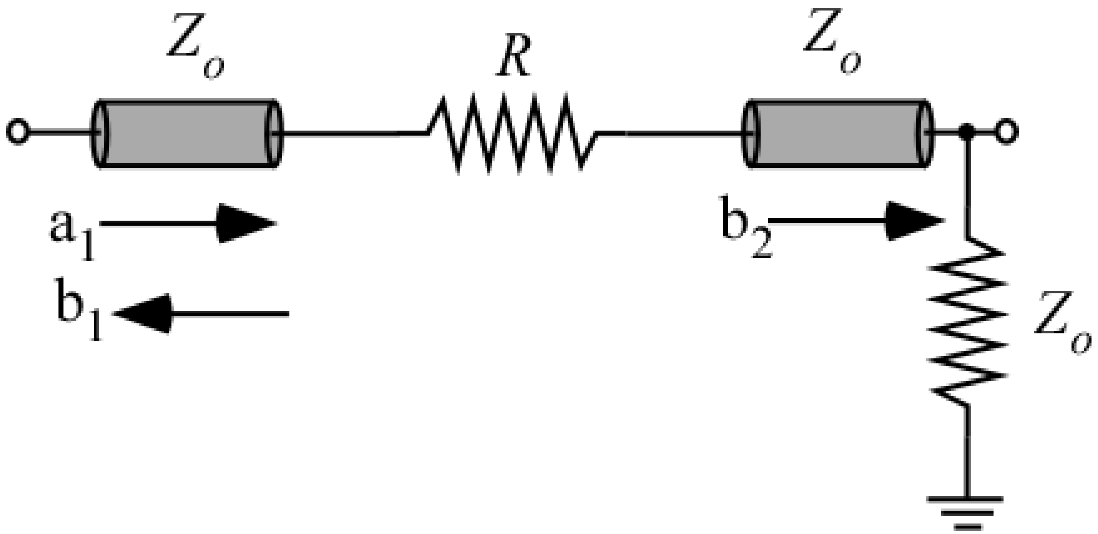
CAUTION : a_i and b_i are the traveling waves in the reference lines.

S-Parameters of Resistor

Determine S-Parameter of 2-port resistance

- Insert R between two reference TL
- Provide excitation at port 1 for S_{11} and S_{21}
- Provide excitation at port 2 for S_{12} and S_{22}
- Can use symmetry and reciprocity

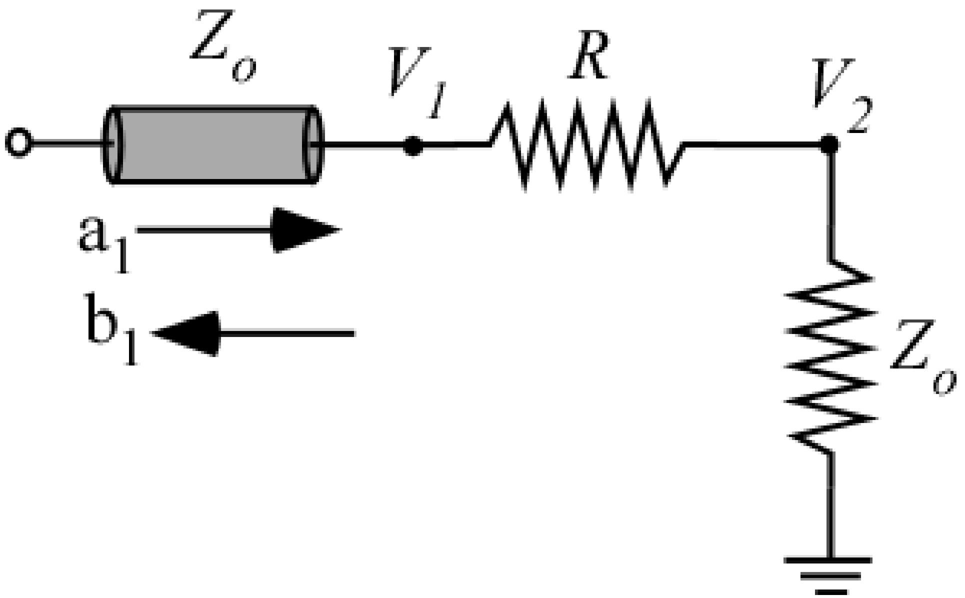
S-Parameters of Resistor



$$S_{11} = \frac{b_1}{a_1} = \Gamma = \frac{(R+Z_o)-Z_o}{(R+Z_o)+Z_o} = \frac{R}{R+2Z_o}$$

$$S_{11} = \frac{R}{R+2Z_o}$$ **and by symmetry,** $$S_{22} = \frac{R}{R+2Z_o}$$

Calculating S_{21} of Resistor

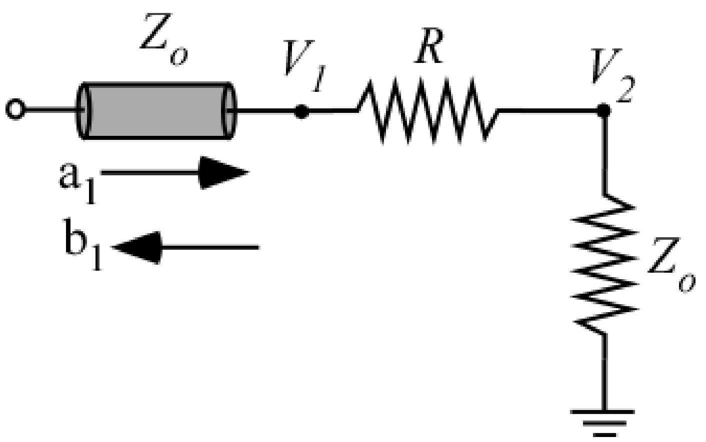


Since a_2=0, the total voltage in port 2 is: $V_2 = b_2\sqrt{Z_o}$

$$V_2 = \frac{V_1 Z_o}{R+Z_o} = \frac{\sqrt{Z_o}\left(a_1+b_1\right)Z_o}{R+Z_o} = \frac{\sqrt{Z_o}\left(a_1+S_{11}a_1\right)Z_o}{R+Z_o}$$

S-Parameters of Resistor

$$V_2 = \frac{Z_o\sqrt{Z_o}\left(1+S_{11}\right)a_1}{R+Z_o} = \frac{2Z_o a_1\sqrt{Z_o}}{R+2Z_o}$$



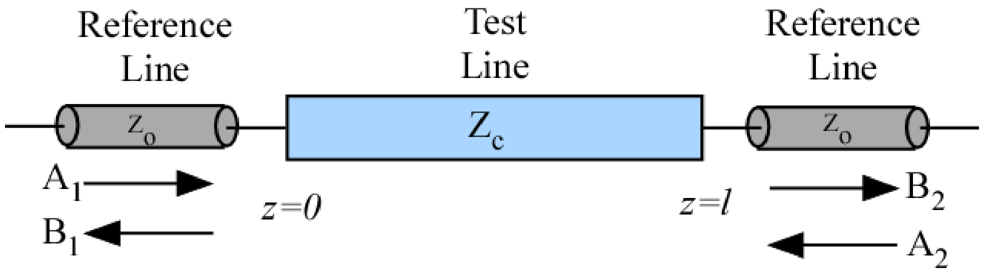
$$S_{21} = \frac{b_2}{a_1} = \frac{V_2}{\sqrt{Z_o}}\frac{1}{a_1} = \frac{2Z_o}{R+2Z_o}$$

$$S_{21} = \frac{2Z_o}{R+2Z_o}$$ **and by reciprocity,** $$S_{12} = \frac{2Z_o}{R+2Z_o}$$

S parameters of resistor R

$$S = \begin{bmatrix} \frac{R}{R+2Z_o} & \frac{2Z_o}{R+2Z_o} \\ \frac{2Z_o}{R+2Z_o} & \frac{R}{R+2Z_o} \end{bmatrix}$$

S-Parameters of TL



$$S_{11} = S_{22} = \frac{(1 - X^2)\Gamma}{1 - X^2\Gamma^2}$$

$$S_{12} = S_{21} = \frac{(1 - \Gamma^2)X}{1 - X^2\Gamma^2}$$

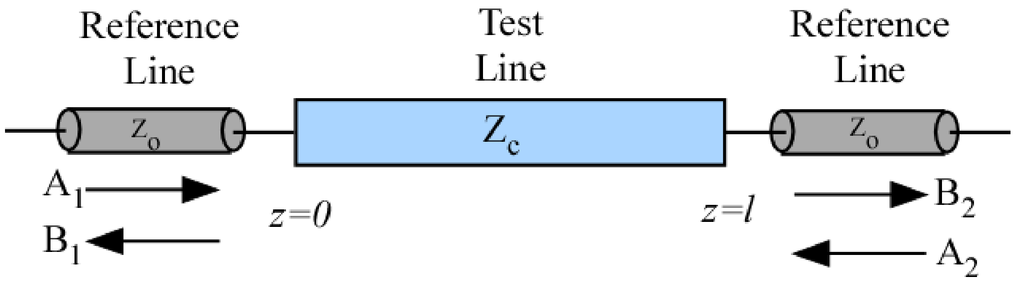
$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\Gamma = \frac{Z_c - Z_{ref}}{Z_c + Z_{ref}}$$

$$Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$X = e^{-\gamma l}$$

S-Parameters of Lossless TL



$$\beta = \omega\sqrt{LC}$$

$$S_{11} = S_{22} = \frac{(1-X^2)\Gamma}{1-X^2\Gamma^2}$$

$$\Gamma = \frac{Z_c - Z_{ref}}{Z_c + Z_{ref}}$$

$$Z_c = \sqrt{\frac{L}{C}}$$

$$S_{12} = S_{21} = \frac{(1-\Gamma^2)X}{1-X^2\Gamma^2}$$

$$X = e^{-j\beta l}$$

If $Z_c = Z_{ref}$

$$S_{11} = S_{22} = 0$$

$$S_{12} = S_{21} = e^{-j\beta l}$$

N-Port S Parameters

$$\begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdot & \cdot \\ S_{21} & S_{22} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ a_n \end{bmatrix}$$

$$\mathbf{b} = \mathbf{Sa}$$

If $b_i = 0$, then no reflected wave on port *i* ➔ port is matched

$$a_i = \frac{V_i^+}{\sqrt{Z_{oi}}}$$

$$b_i = \frac{V_i^-}{\sqrt{Z_{oi}}}$$

V_i^+ : incident voltage wave in port *i*

V_i^- : reflected voltage wave in port *i*

Z_{oi} : impedance in port *i*

N-Port S Parameters

$$\mathbf{v} = \sqrt{Z_o}(\mathbf{a}+\mathbf{b}) \quad (1) \qquad \mathbf{i} = \frac{1}{\sqrt{Z_o}}(\mathbf{a}-\mathbf{b}) \quad (2) \qquad \mathbf{v} = \mathbf{Z}\mathbf{i} \quad (3)$$

Substitute (1) and (2) into (3)

$$\sqrt{Z_o}(\mathbf{a}+\mathbf{b}) = \mathbf{Z}\frac{1}{\sqrt{Z_o}}(\mathbf{a}-\mathbf{b})$$

Defining S such that b = Sa and substituting for b

$$Z_o(\mathbf{U}+\mathbf{S})\mathbf{a} = Z_o(\mathbf{U}-\mathbf{S})\mathbf{a}$$

U : unit matrix

S→Z

$$\mathbf{Z} = Z_o(\mathbf{U}+\mathbf{S})(\mathbf{U}-\mathbf{S})^{-1}$$

Z→S

$$\mathbf{S} = (\mathbf{Z}+Z_o\mathbf{U})^{-1}(\mathbf{Z}-Z_o\mathbf{U})$$

N-Port S Parameters

If the port reference impedances are different, we define k as

$$\mathbf{k} = \begin{bmatrix} \sqrt{Z_{o1}} & & & \\ & \sqrt{Z_{o2}} & & \\ & & \cdot & \\ & & & \sqrt{Z_{on}} \end{bmatrix}$$

$\mathbf{v} = \mathbf{k}(\mathbf{a}+\mathbf{b})$ and $\mathbf{i} = \mathbf{k}^{-1}(\mathbf{a}-\mathbf{b})$ and $\mathbf{k}(\mathbf{a}+\mathbf{b}) = \mathbf{Z}\mathbf{k}^{-1}(\mathbf{a}-\mathbf{b})$

Z→S

$$\mathbf{S} = \left(\mathbf{Z}\mathbf{k}^{-1}+\mathbf{k}\right)\left(\mathbf{Z}\mathbf{k}^{-1}-\mathbf{k}\right)$$

S→Z

$$\mathbf{Z} = \mathbf{k}\left(\mathbf{U}+\mathbf{S}\right)\left(\mathbf{U}-\mathbf{S}\right)^{-1}\mathbf{k}$$

Normalization

Assume original S parameters as S_1 with system k_1. Then the representation S_2 on system k_2 is given by

Transformation Equation

$$S_2 = \left[k_1(U+S_1)(U-S_1)^{-1}k_1k_2 + k_2\right]^{-1}\left[k_1(U+S_1)(U-S_1)^{-1}k_1k_2 - k_2\right]$$

If Z is symmetric, S is also symmetric

Dissipated Power

$$P_d = \frac{1}{2}\mathbf{a}^T\left(\mathbf{U} - \mathbf{S}^T\mathbf{S}^*\right)\mathbf{a}^*$$

The dissipation matrix D is given by:

$$\mathbf{D} = \mathbf{U} - \mathbf{S}^T\mathbf{S}^*$$

Passivity insures that the system will always be stable provided that it is connected to another passive network

For passivity

- (1) the determinant of D must be ≥ 0
- (2) the determinant of the principal minors must be ≥ 0

Dissipated Power

When the dissipation matrix is 0, we have a lossless network

$$\mathbf{S}^T\mathbf{S}^* = \mathbf{U}$$

The S matrix is unitary.

For a lossless two-port:

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

$$|S_{22}|^2 + |S_{12}|^2 = 1$$

If in addition the network is reciprocal, then

$$S_{12} = S_{21} \quad \text{and} \quad |S_{11}| = |S_{22}| = \sqrt{1 - |S_{12}|^2}$$

Scattering Transfer Parameters

In T-Parameters, traveling waves at the input are related to those at the output

$$b_1 = S_{11}a_1 + S_{12}a_2 \qquad b_1 = T_{11}a_2 + T_{12}b_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \qquad a_1 = T_{21}a_2 + T_{22}b_2$$

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} T_{12}T_{22}^{-1} & T_{11} - T_{12}T_{21}T_{22}^{-1} \\ T_{22}^{-1} & -T_{21}T_{22}^{-1} \end{pmatrix}$$

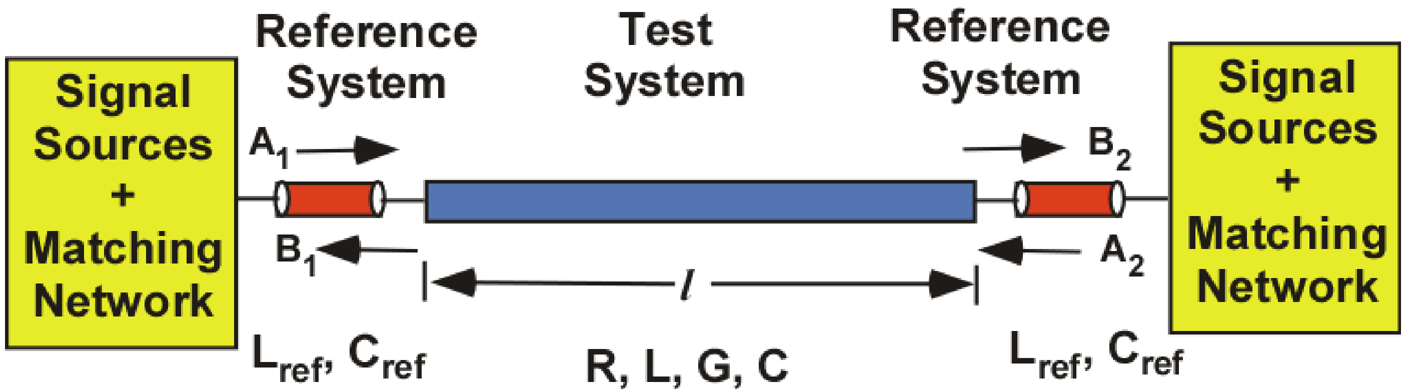
$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} S_{12} - S_{11}S_{22}S_{21}^{-1} & S_{11}S_{21}^{-1} \\ -S_{22}S_{21}^{-1} & S_{21}^{-1} \end{pmatrix}$$

T parameters can be cascaded $\mathbf{T} = \mathbf{T_A} \cdot \mathbf{T_B}$

Applications of S Parameters

- **Measurement**
  - High-frequency (microwaves, mm-wave)
  - Results can be converted to other parameters
- **Modeling**
  - Cascading of linear components
  - De-embedding using scattering transfer parameters
- **Simulation**
  - Transmission lines
  - Blackbox multiport

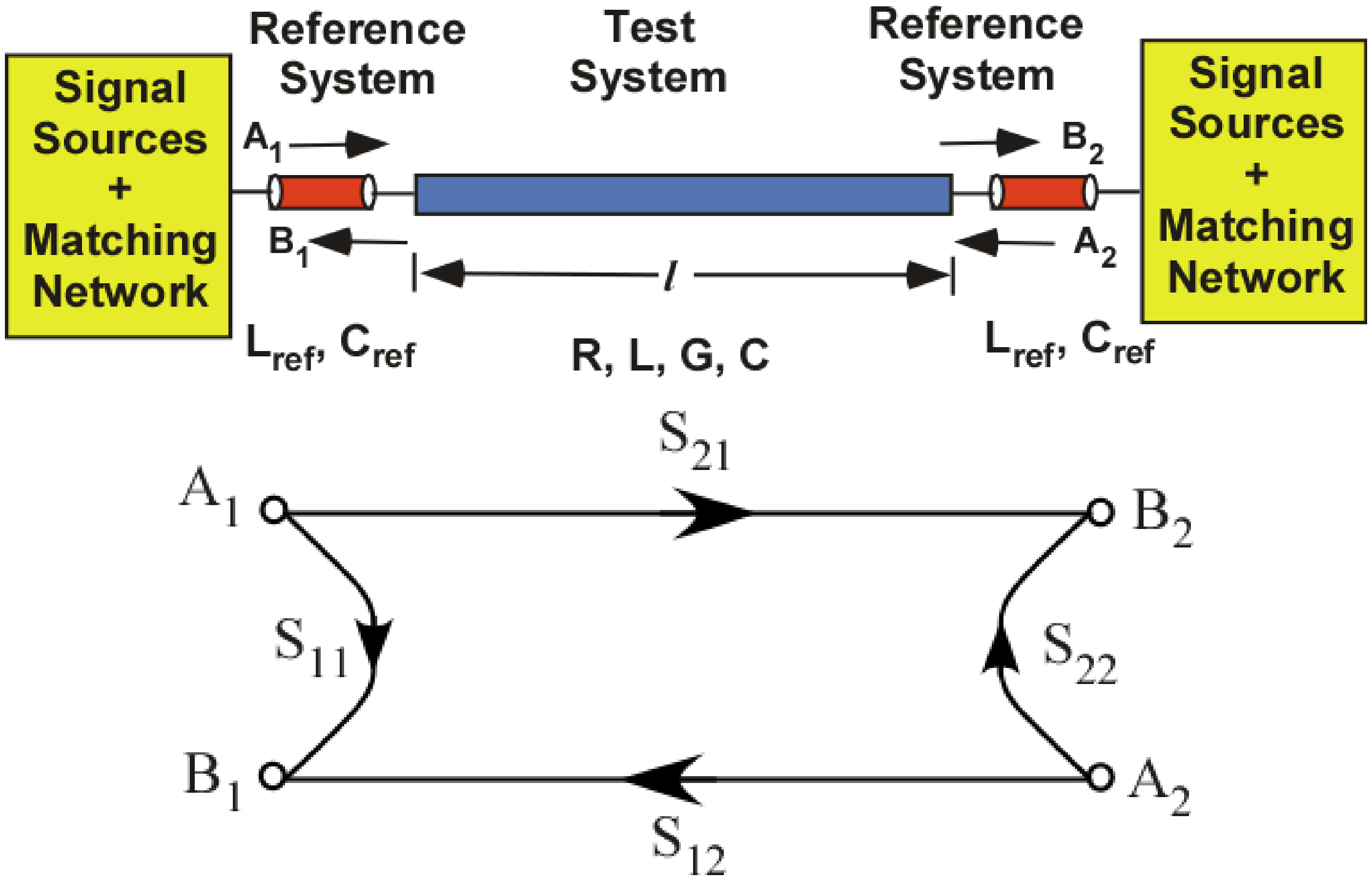
Lossy and Dispersive Line



$$S_{11} = S_{22} = \frac{\left(1-\alpha^2\right)\rho}{1-\rho^2\alpha^2} \qquad S_{21} = S_{12} = \frac{\left(1-\rho^2\right)\alpha}{1-\rho^2\alpha^2}$$

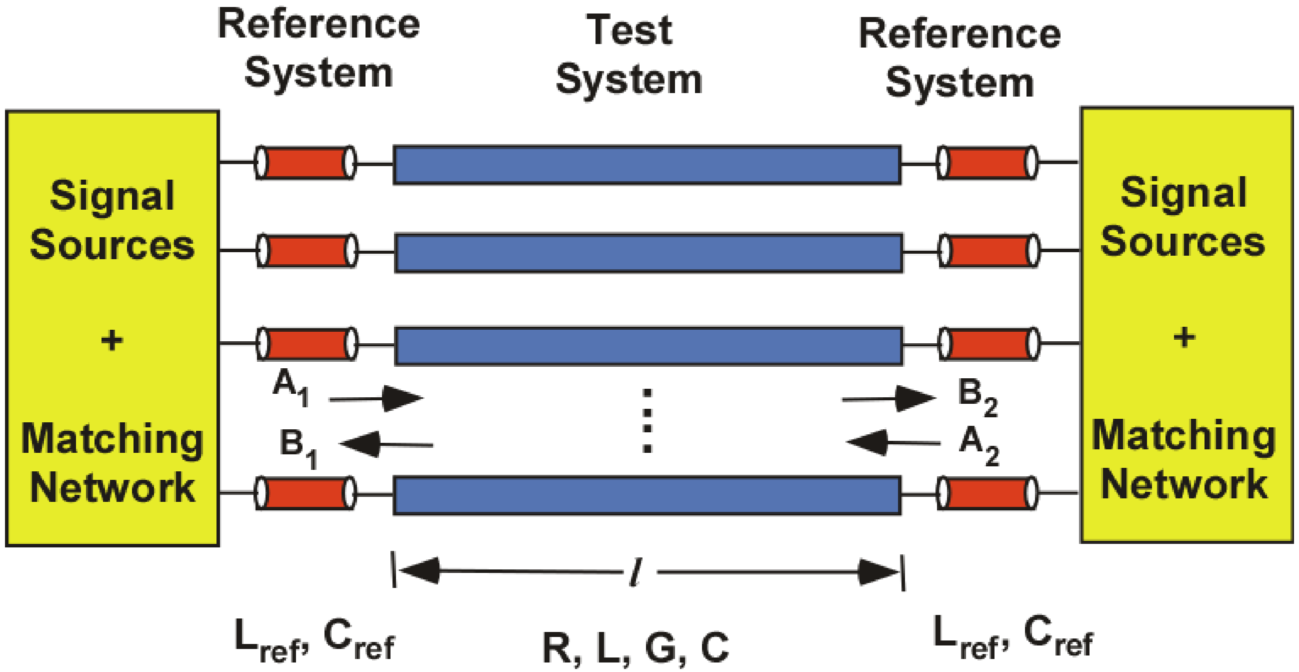
$$\alpha = e^{-\gamma l} \qquad \rho = \frac{Z_c(\omega) - Z_o}{Z_c(\omega) + Z_o}$$

Frequency-Domain Formulation*



* J. E. Schutt-Aine and R. Mittra, "Scattering Parameter Transient analysis of transmission lines loaded with nonlinear terminations," IEEE Trans. Microwave Theory Tech., vol. MTT-36, pp. 529-536, March 1988.

Scattering Parameters for N-Line



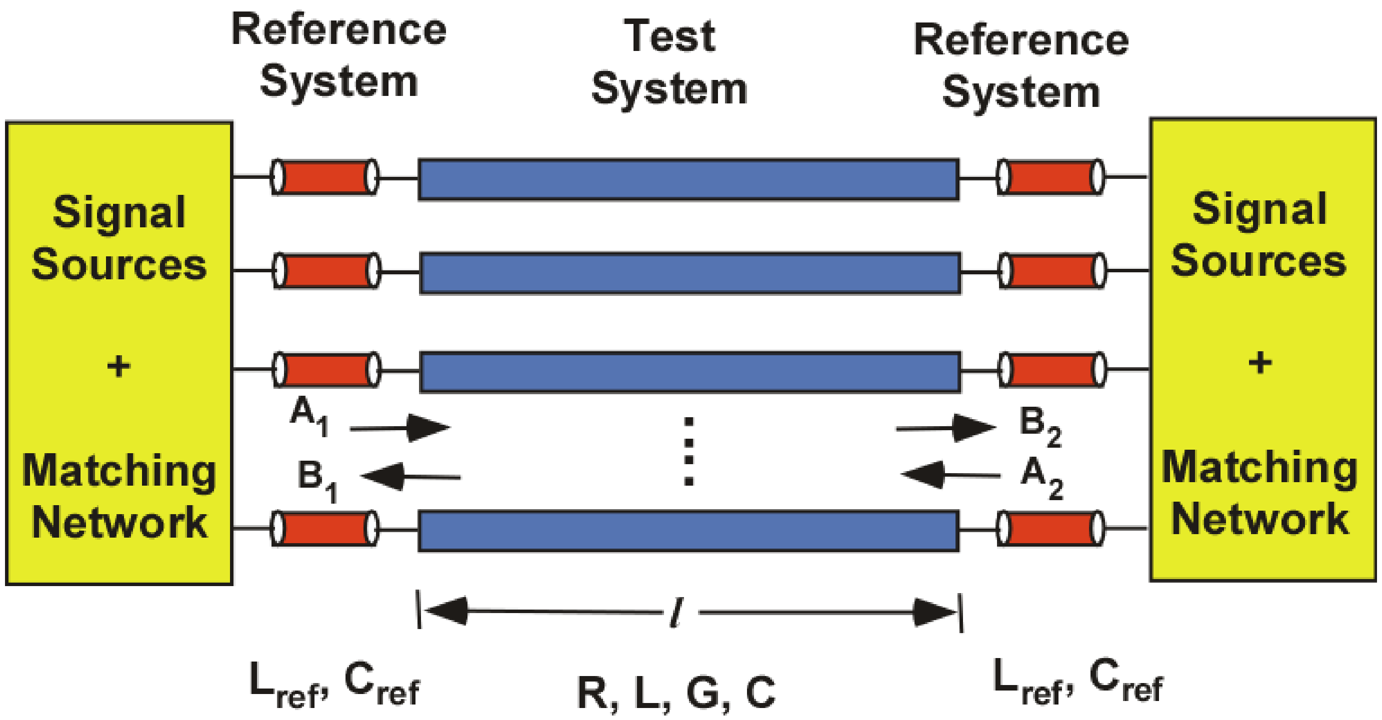
$$S_{21} = S_{12} = 2E_oE^{-1}\left[1-\Gamma\right]\Psi\left[1-\Gamma\Psi\Gamma\Psi\right]^{-1}T$$

$$S_{11} = S_{22} = T^{-1}\left[\Gamma-\Psi\Gamma\Psi\right]\left[1-\Gamma\Psi\Gamma\Psi\right]^{-1}T$$

$$\Gamma = \left[1+EE_o^{-1}Z_oH_oH^{-1}Z_m^{-1}\right]^{-1}\left[1-EE_o^{-1}Z_oH_oH^{-1}Z_m^{-1}\right]$$

$$T = \left[1+EE_o^{-1}Z_oH_oH^{-1}Z_m^{-1}\right]^{-1}EE_o^{-1} \qquad \Psi = W(-l)$$

Lossy Multiconductor S-Parameters*

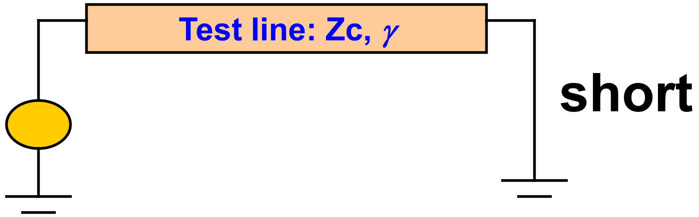


$$B_1 = S_{11}\,A_1 + S_{12}\,A_2 \qquad B_2 = S_{21}\,A_1 + S_{22}\,A_2$$

* J. E. Schutt-Aine and R. Mittra, "Transient analysis of coupled lossy transmission lines with nonlinear terminations," IEEE Trans. Circuit Syst., vol. CAS-36, pp. 959-967, July 1989.

Why Use S Parameters?

Y-Parameter

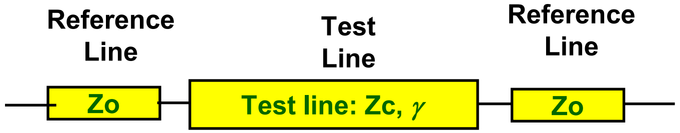


$$Y_{11} = \frac{1 + e^{-2\gamma l}}{Z_c (1 - e^{-2\gamma l})}$$

Z_c : microstrip characteristic impedance
γ : complex propagation constant
l : length of microstrip

Y_{11} can be unstable

S-Parameter



$$S_{11} = \frac{(1 - e^{-2\gamma l})\Gamma}{1 - \Gamma^2 e^{-2\gamma l}}$$

$$\Gamma = \frac{Z_c - Z_o}{Z_c + Z_o}$$

S_{11} is always stable

Choice of Reference

$$\Gamma = \frac{Z_c - Z_{ref}}{Z_c + Z_{ref}} \qquad Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Z_{ref} is arbitrary

What is the best choice for Z_{ref} ?

At high frequencies $Z_c \rightarrow \sqrt{\frac{L}{C}}$

Thus, if we choose $Z_{ref} = \sqrt{\frac{L}{C}}$

$$S_{12} \rightarrow e^{-j\omega\sqrt{LC}d} = X_o \qquad S_{11} \rightarrow 0$$

Choice of Reference

S-Parameter measurements (or simulations) are made using a 50-ohm system. For a 4-port, the reference impedance is given by:

$$Z_o = \begin{matrix} 50.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 50.0 \end{matrix}$$

Z: Impedance matrix (of blackbox)
S: S-parameter matrix
Z_o: Reference impedance
I: Unit matrix

$$S = \left[ZZ_o^{-1} + I\right]^{-1}\left[ZZ_o^{-1} - I\right]$$

$$Z = \left[I + S\right]\left[I - S\right]^{-1} Z_o$$

Reference Transformation

Method: Change reference impedance from uncoupled to coupled system to get new S-parameter representation

$$Z_o = \begin{matrix} 50.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 50.0 \end{matrix}$$

Uncoupled system

$$Z_o = \begin{matrix} 328.0 & 69.6 & 328.9 & 69.6 \\ 69.6 & 328.8 & 69.6 & 328.9 \\ 328.9 & 69.6 & 328.8 & 69.6 \\ 69.6 & 328.9 & 69.6 & 328.8 \end{matrix}$$

Coupled system

as an example…

Choice of Reference

using

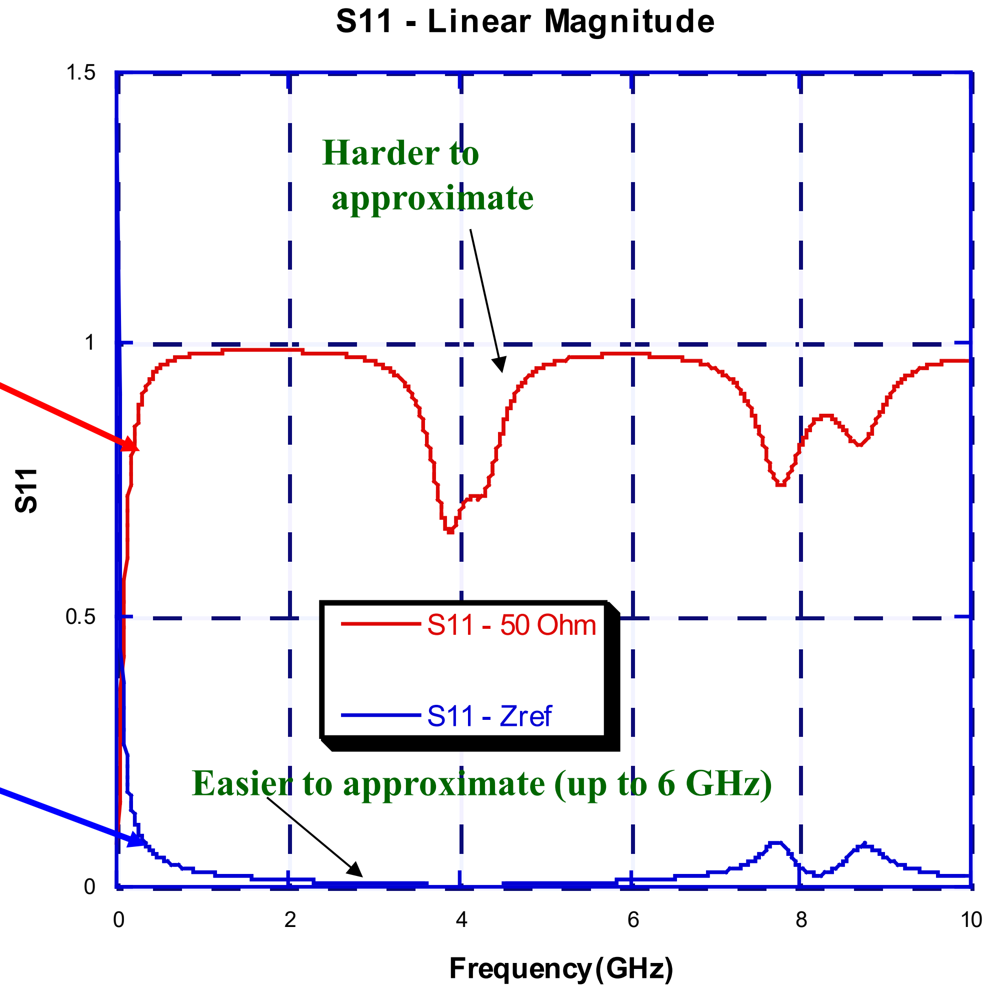
$$Z_o = \begin{matrix} 50.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 50.0 \end{matrix}$$

as reference…

using

$$Z_o = \begin{matrix} 328.0 & 69.6 & 328.9 & 69.6 \\ 69.6 & 328.8 & 69.6 & 328.9 \\ 328.9 & 69.6 & 328.8 & 69.6 \\ 69.6 & 328.9 & 69.6 & 328.8 \end{matrix}$$

as reference…



Choice of Reference

using

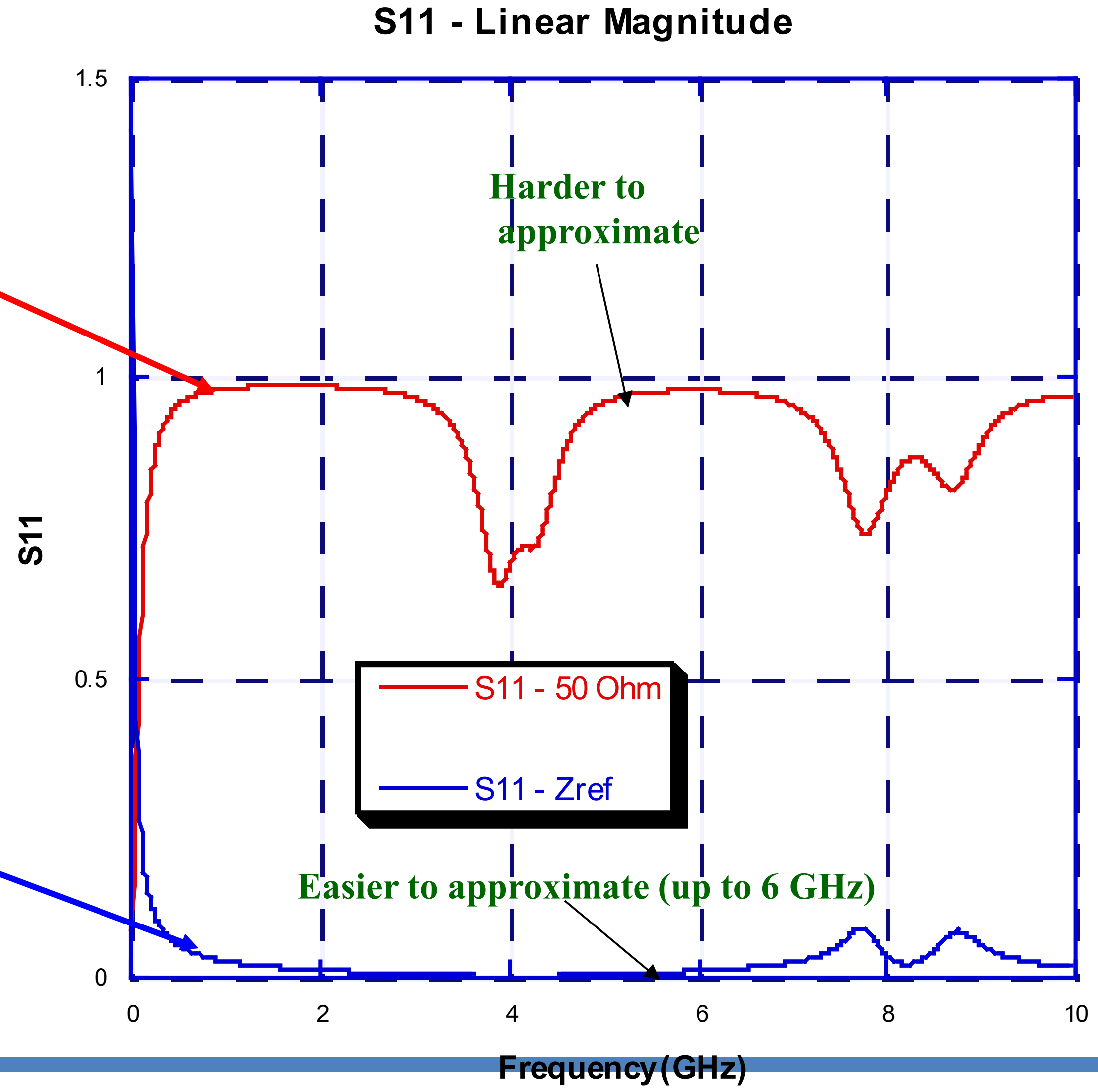
$$Z_o = \begin{matrix} 50.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 50.0 \end{matrix}$$

as reference…

using

$$Z_o = \begin{matrix} 328.0 & 69.6 & 328.9 & 69.6 \\ 69.6 & 328.8 & 69.6 & 328.9 \\ 328.9 & 69.6 & 328.8 & 69.6 \\ 69.6 & 328.9 & 69.6 & 328.8 \end{matrix}$$

as reference…



Choice of Reference

using

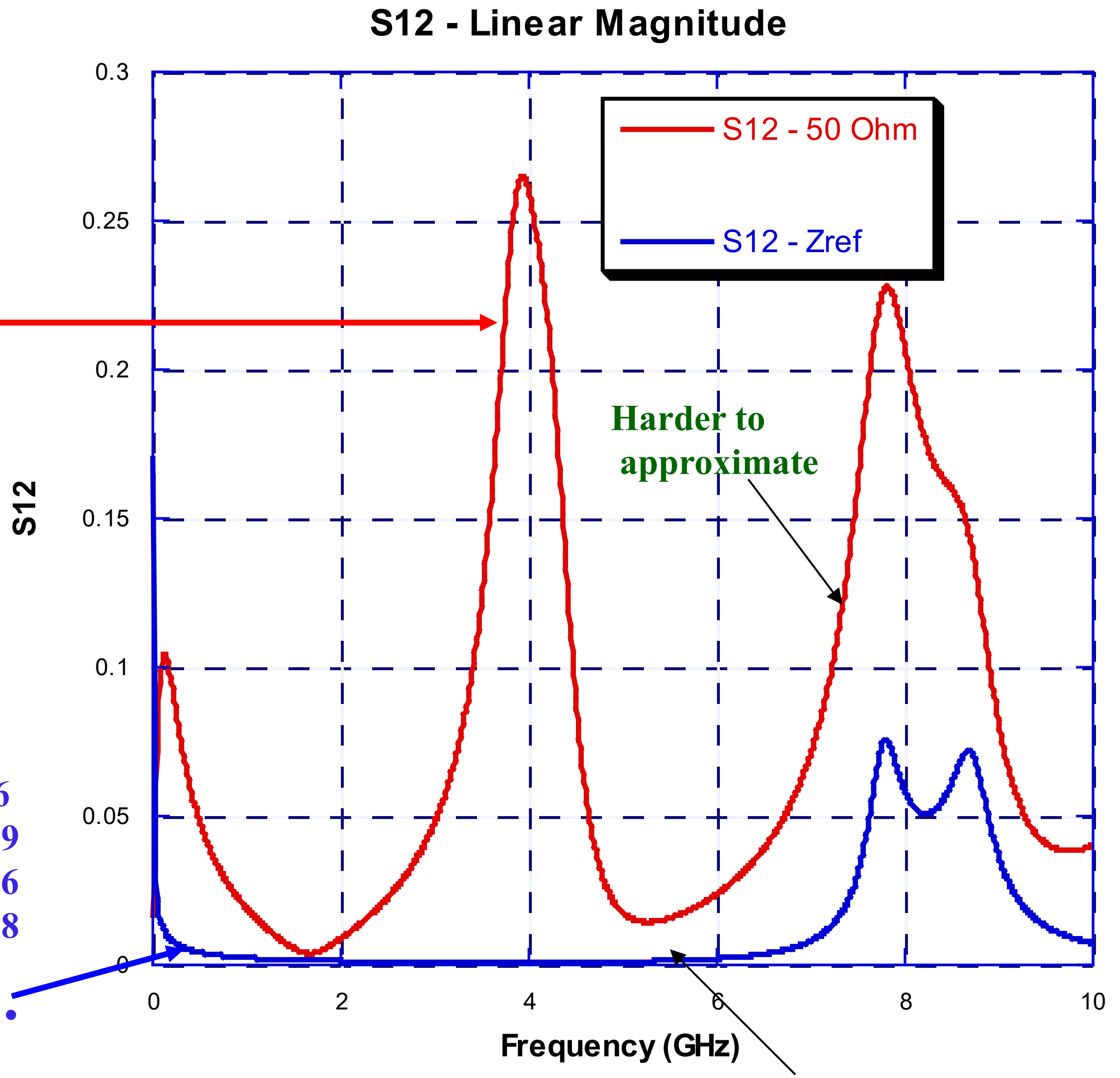
$$Z_o = \begin{matrix} 50.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 50.0 \end{matrix}$$

as reference…

using

$$Z_o = \begin{matrix} 328.0 & 69.6 & 328.9 & 69.6 \\ 69.6 & 328.8 & 69.6 & 328.9 \\ 328.9 & 69.6 & 328.8 & 69.6 \\ 69.6 & 328.9 & 69.6 & 328.8 \end{matrix}$$

as reference…



Easier to approximate (up to 6 GHz)

Choice of Reference

using

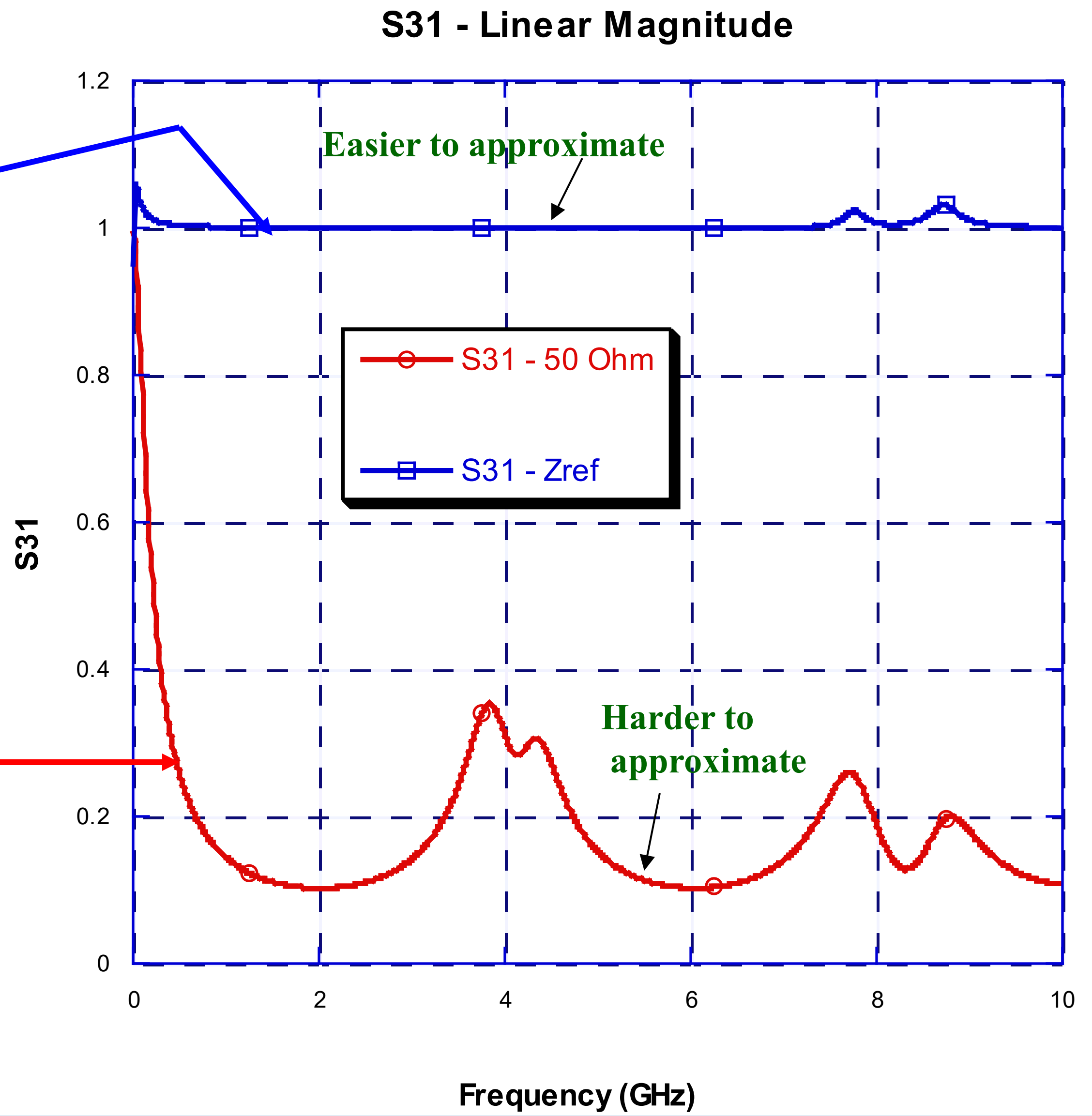
$$Z_o = \begin{matrix} 328.0 & 69.6 & 328.9 & 69.6 \\ 69.6 & 328.8 & 69.6 & 328.9 \\ 328.9 & 69.6 & 328.8 & 69.6 \\ 69.6 & 328.9 & 69.6 & 328.8 \end{matrix}$$

as reference…

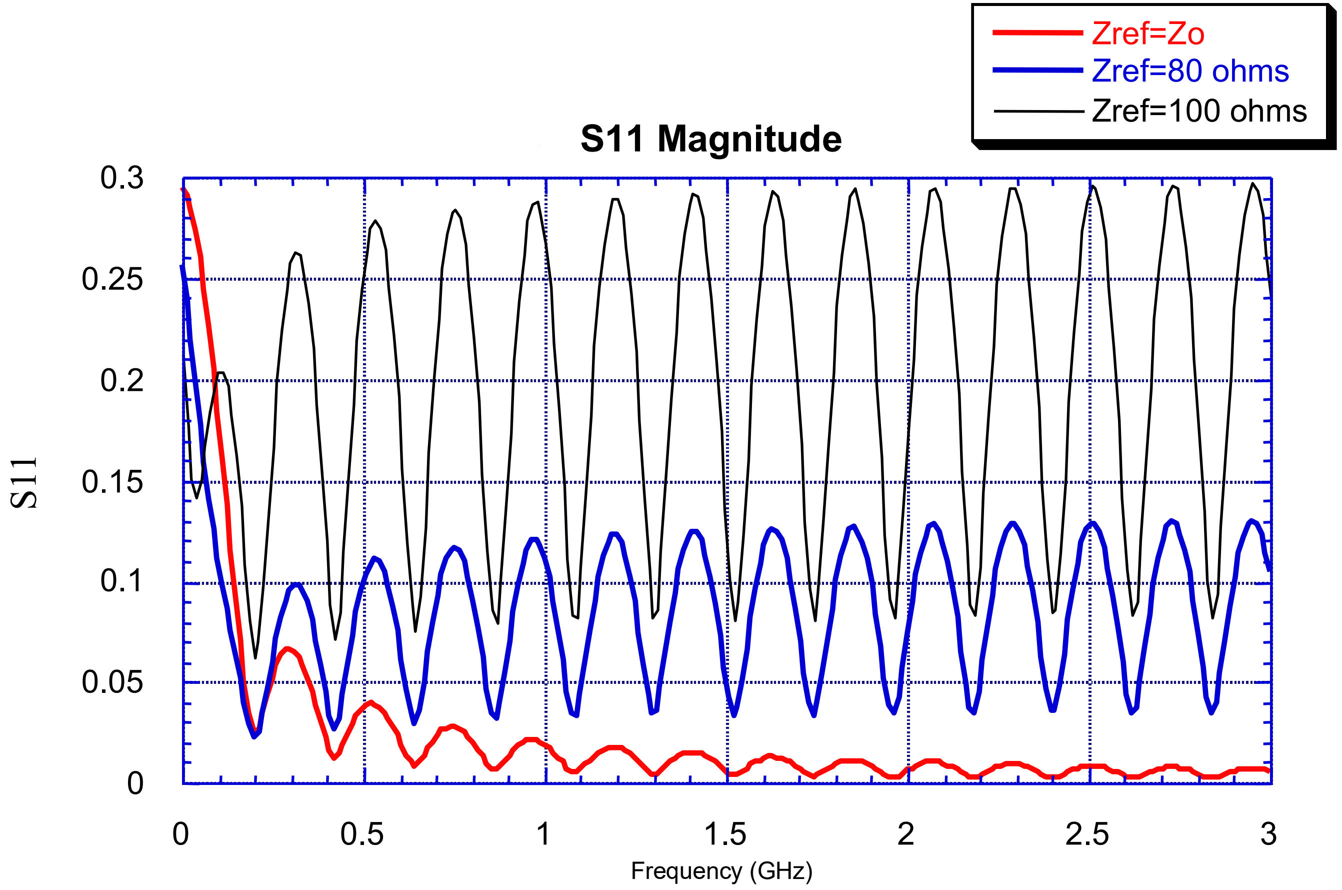
using

$$Z_o = \begin{matrix} 50.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 50.0 \end{matrix}$$

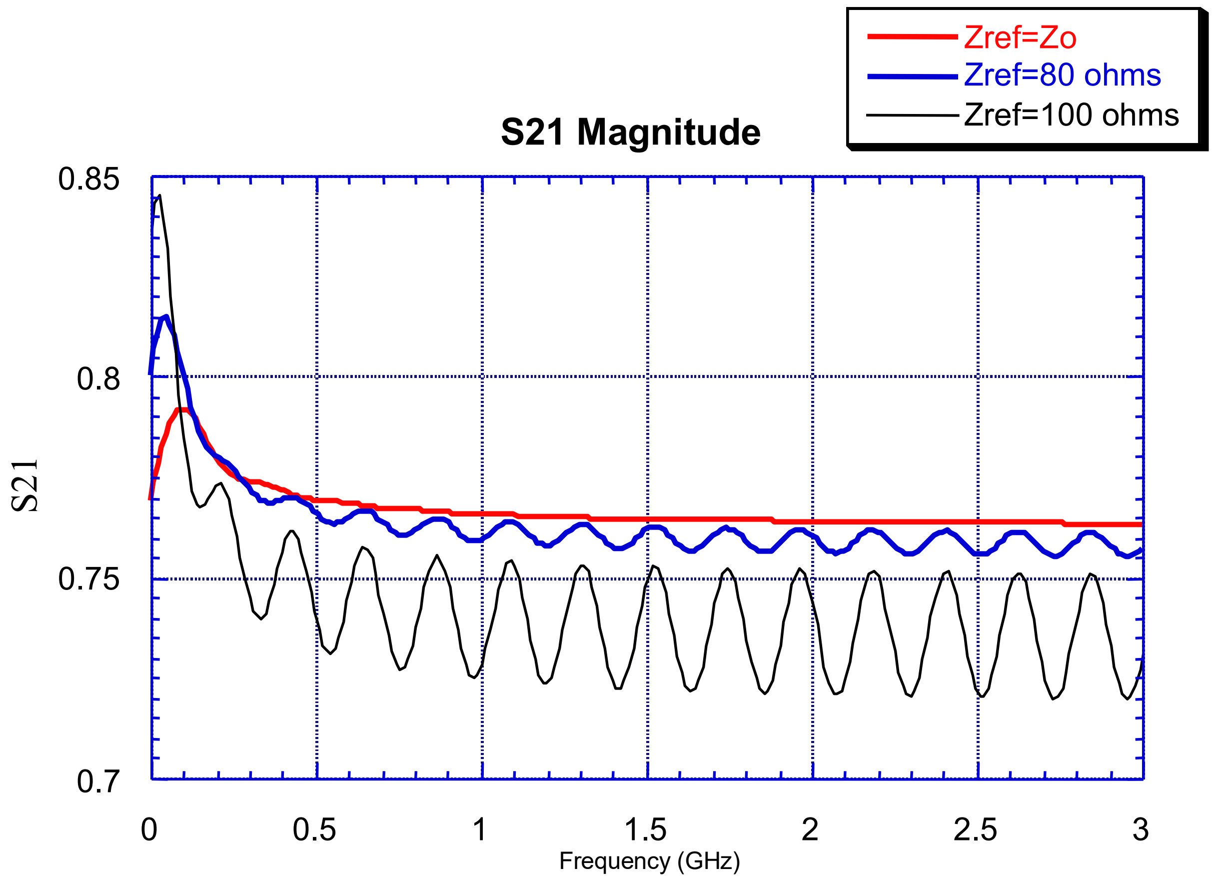
as reference…



Choice of Reference

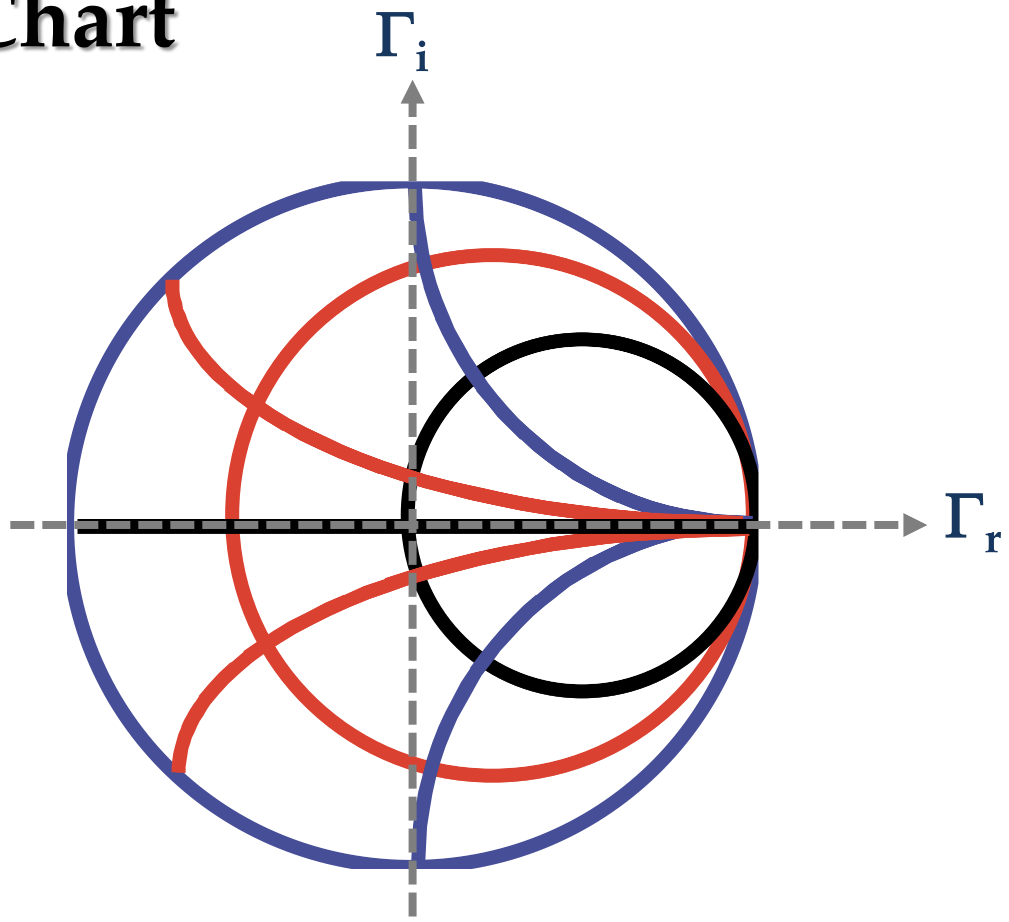


Choice of Reference



The Smith Chart

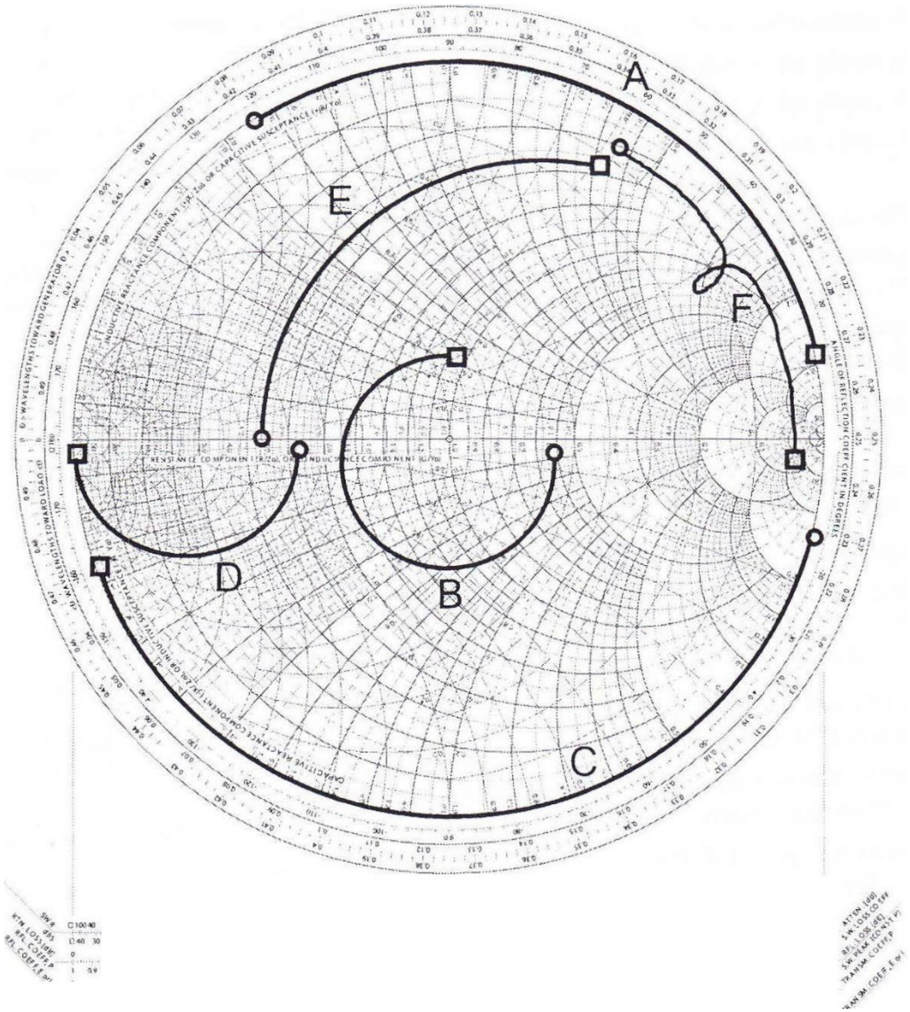
- **Mapping of complex impedance plane on complex reflection coefficient plane**
- **Consists of constant resistance circles and constant reactance arcs**
- **Magnitude limited to 1**
- **3 ways to move on Smith chart**
  - Constant SWR circle ➔displacement along TL
  - Constant resistance circle ➔ addition of reactance
  - Constant reactance arc ➔ addition of resistance



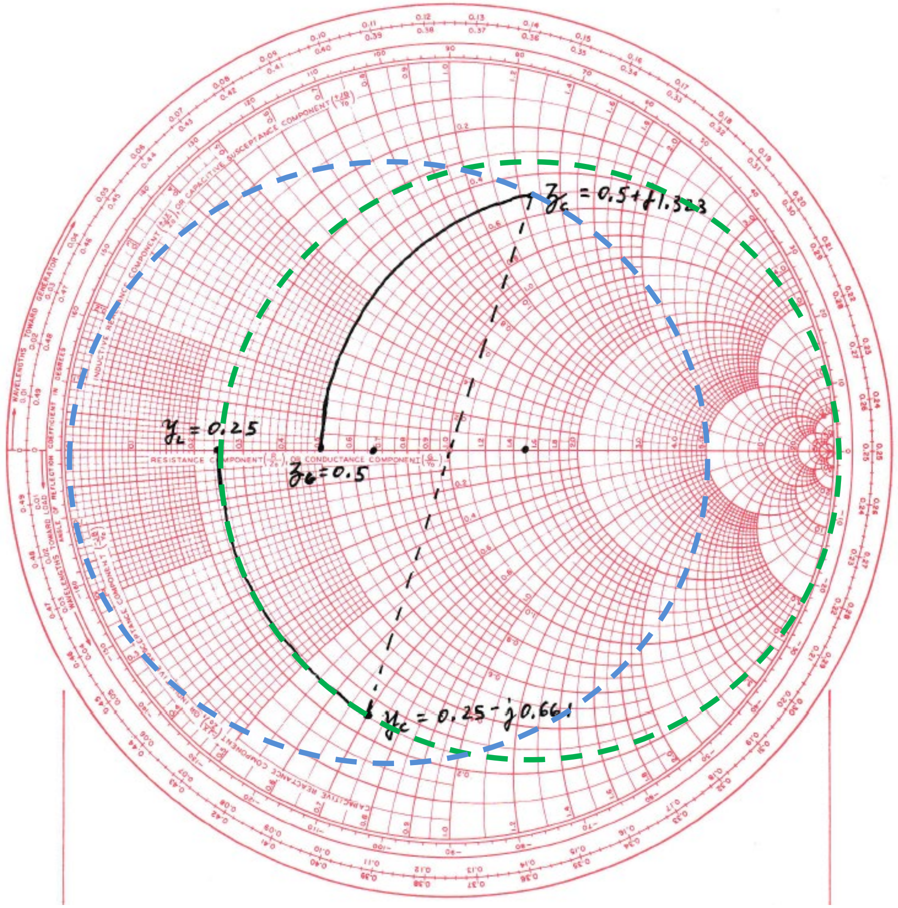
Smith chart can also be used for admittance; going from normalized impedance to normalized admittance corresponds to a 180 degree shift.

Smith Chart Applications

- Frequency dependence of components



- Matching networks.



Network Analyzers