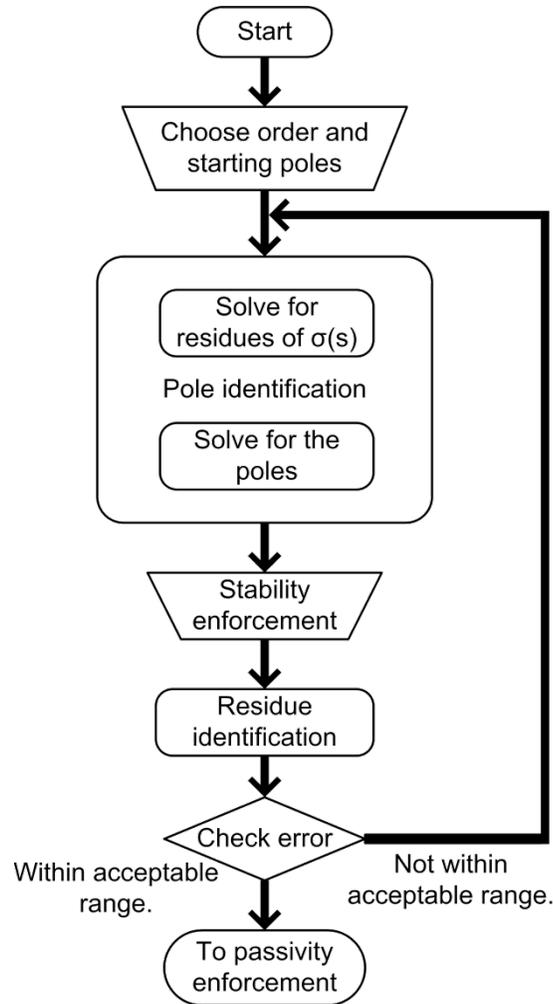# ECE 546
Lecture -15
Circuit Synthesis

Spring 2026

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jesa@illinois.edu

MOR via Vector Fitting

- Rational function approximation:

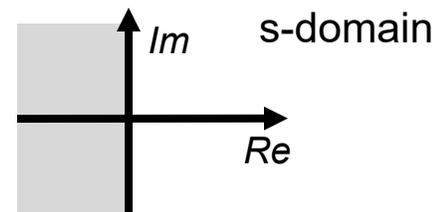$$f(s) \approx \sum_{n=1}^{N} \frac{c_n}{s - a_n} + d + sh$$

- Introduce an unknown function $\sigma(s)$ that satisfies:

$$\begin{bmatrix} \sigma(s)f(s) \\ \sigma(s) \end{bmatrix} \approx \begin{bmatrix} \sum_{n=1}^{N} \dfrac{c_n}{s - \tilde{a}_n} + d + sh \\ \sum_{n=1}^{N} \dfrac{\tilde{c}_n}{s - \tilde{a}_n} + 1 \end{bmatrix}$$

- Poles of $f(s)$ = zeros of $\sigma(s)$:

$$f(s) \approx \frac{\sum_{n=1}^{N} \dfrac{c_n}{s - \tilde{a}_n} + d + sh}{\sum_{n=1}^{N} \dfrac{\tilde{c}_n}{s - \tilde{a}_n} + 1} = \frac{\prod_{n=1}^{N+1} (s - z_n)}{\prod_{n=1}^{N} (s - \tilde{z}_n)}$$

- Flip unstable poles into the left half plane.

s-domain

Blackbox Formulation

Transfer function is approximated

$$H(\omega) = d + \sum_{k=1}^{L} \frac{c_k}{1 + j\omega / \omega_{ck}}$$

Using curve fitting technique (e.g. vector fitting)
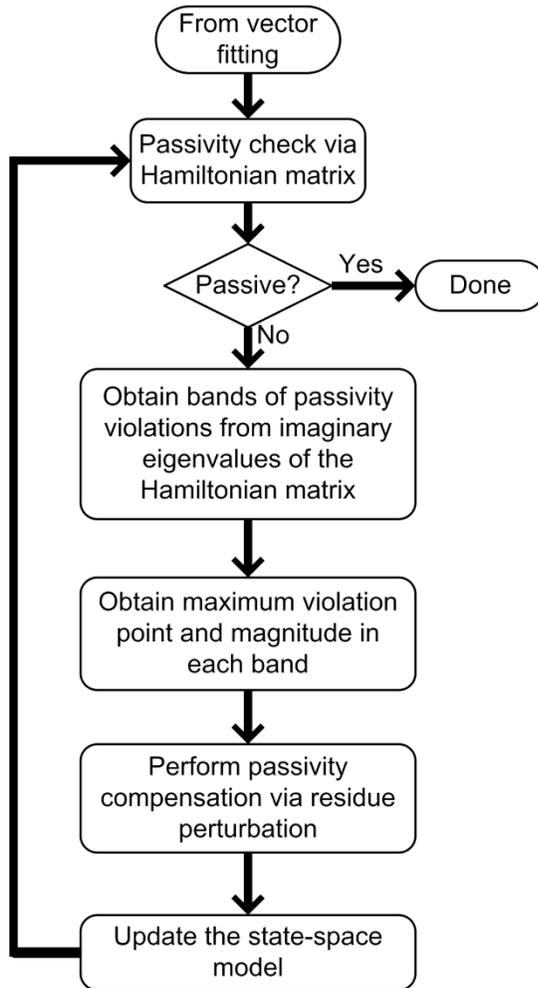
In the time domain, recursive convolution is used

$$y(t) = dx(t-T) + \sum_{k=1}^{L} y_{pk}(t)$$

where

$$y_{pk}(t) = a_k x(t-T)\left(1 - e^{-\omega_{ck}T}\right) + e^{-\omega_{ck}T} y_{pk}(t-T)$$

Recursive convolution is fast

ECE·ILLINOIS

Electrical and Computer Engineering
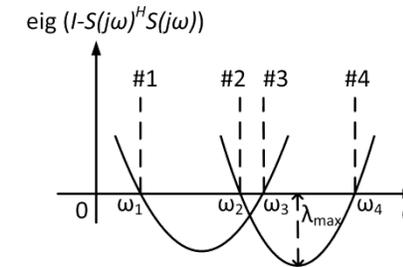University of Illinois at Urbana-Champaign

Passivity Enforcement

- State-space form:
$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

- Hamiltonian matrix:
$$M = \begin{bmatrix} A + BKD^T C & BKB^T \\ -C^T LC & -A^T - C^T DKB^T \end{bmatrix}$$

$$K = \left(I - D^T D\right)^{-1} \quad L = \left(I - DD^T\right)^{-1}$$

- Passive if M has no imaginary eigenvalues.

- Sweep: $eig\left(I - S(j\omega)^H S(j\omega)\right)$

- Quadratic programming:
  - Minimize *(change in response)* subject to *(passivity compensation)*.

$$\min\left(vec(\Delta\mathbf{C})^{\mathsf{T}} \mathbf{H}\, vec(\Delta\mathbf{C})\right) \quad \text{subject to} \quad \Delta\lambda = \mathbf{G} \cdot vec(\Delta\mathbf{C}).$$

Macromodel Circuit Synthesis

Use of Macromodel

- Time-Domain simulation using recursive convolution
- Frequency-domain circuit synthesis for SPICE netlist

Macromodel Circuit Synthesis

Objective: Determine equivalent circuit from macromodel representation

Motivation

- Circuit can be used in SPICE

Goal

- Generate a netlist of circuit elements

Circuit Realization

Circuit realization consists of interfacing the reduced model with a general circuit simulator such as **SPICE**

Model order reduction gives a transfer function that can be presented in matrix form as

$$S(s) = \begin{bmatrix} s_{11}(s) & \cdot & s_{1N}(s) \\ \cdot & \cdot & \cdot \\ s_{N1}(s) & \cdot & s_{NN}(s) \end{bmatrix}$$

or

$$Y(s) = \begin{bmatrix} y_{11}(s) & \cdot & y_{1N}(s) \\ \cdot & \cdot & \cdot \\ y_{N1}(s) & \cdot & y_{NN}(s) \end{bmatrix}$$

Method 1: Y-Parameter/MOR*

Each of the Y-parameters can be represented as

$$y_{ij}(s) = d + \sum_{k=1}^{L} \frac{a_k}{s - p_k}$$

where the a_k's are the residues and the p_k's are the poles. d is a constant

*Giulio Antonini "SPICE Equivalent Circuits of Frequency-Domain Responses", IEEE Transactions on Electromagnetic Compatibility, pp 502-512, Vol. 45, No. 3, August 2003.

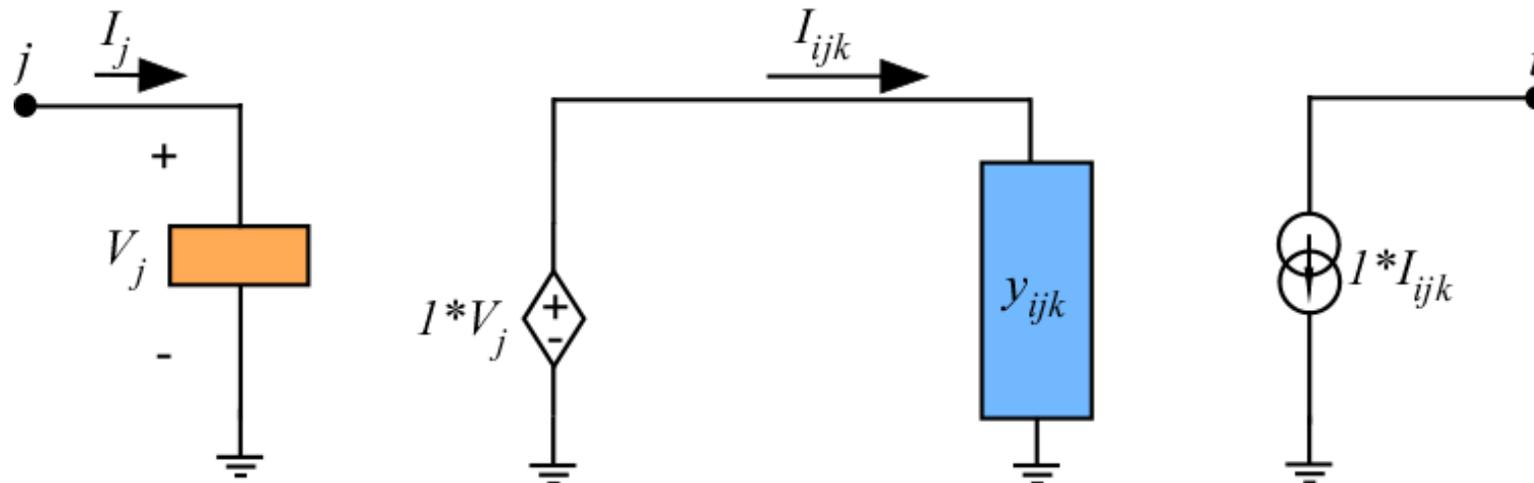# Equivalent-Circuit Extraction

Macromodel is curve-fit to take the form

$$Y(s) = d + \sum_{k=1}^{L} \frac{r_k}{s - p_k}$$

Need to find equivalent circuit associated with

- Constant term d

- Real Poles

- Complex Poles

Y-Parameter - Circuit Realization

Each of the I_{ij} can be realized with a circuit having the following topology:

The resulting current sources can then be superposed for the total current I_i leaving port i

All Y parameters are treated as if they were <u>one-port</u> Y parameters

Y-Parameter - Circuit Realization

We try to find the circuit associated with each term:

$$y_{ij}(s) = d_{ij} + \sum_{k=1}^{L} \frac{a_{ijk}}{s - p_{ijk}}$$

For a given port *i*, the total current due to all ports with voltages V_j (*j=1,…,P*) is given by

$$I_i = \sum_{j=1}^{P} y_{ij} V_j = \sum_{j=1}^{P} d_{ij} V_j + \sum_{j=1}^{P} V_j \sum_{k=1}^{L} \frac{a_{ijk}}{s - p_{ijk}}$$

For each contributing port, with voltage *i*, the total current due to a voltage V_j at port *j* is given by

$$I_{ij} = d_{ij} V_j + V_j \sum_{k=1}^{L} \frac{a_{ijk}}{s - p_{ijk}}$$

Y-Parameter - Circuit Realization

We try to find the circuit associated with each term:

$$y_{ij}(s) = d + \sum_{k=1}^{L} \frac{a_k}{s - p_k}$$

1. Constant term d

$$y_{ijd}(s) = d$$

2. Each pole-residue pair

$$y_{ijk}(s) = \frac{a_k}{s - p_k}$$

Circuit Realization – Constant Term
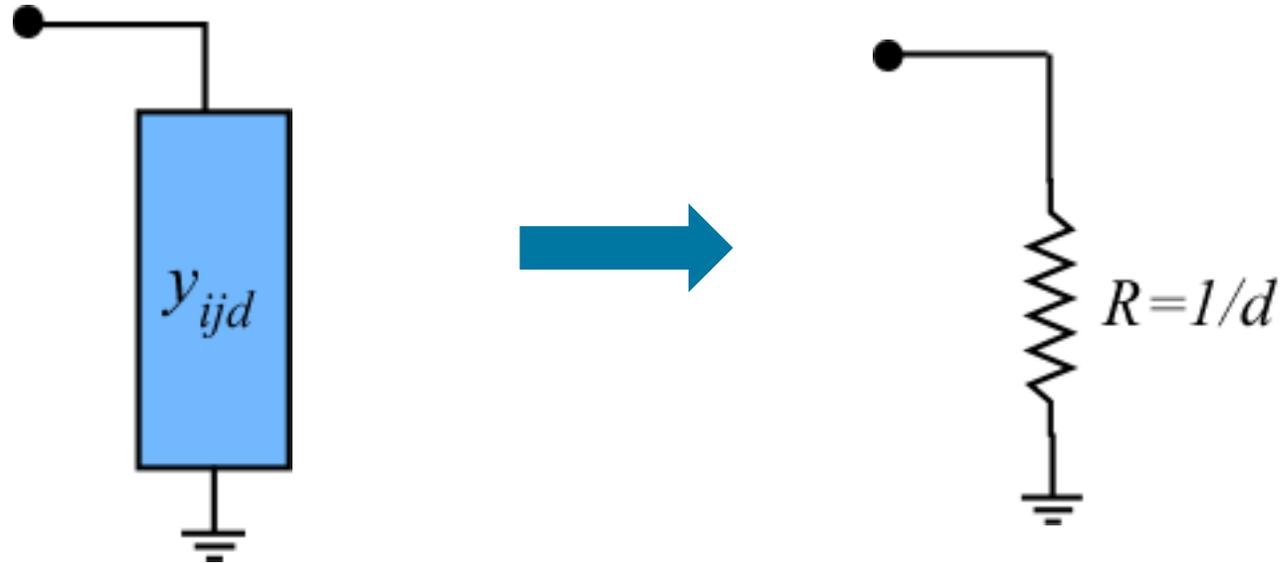
$$R = \frac{1}{d}$$

Circuit Realization – Pole/Residue

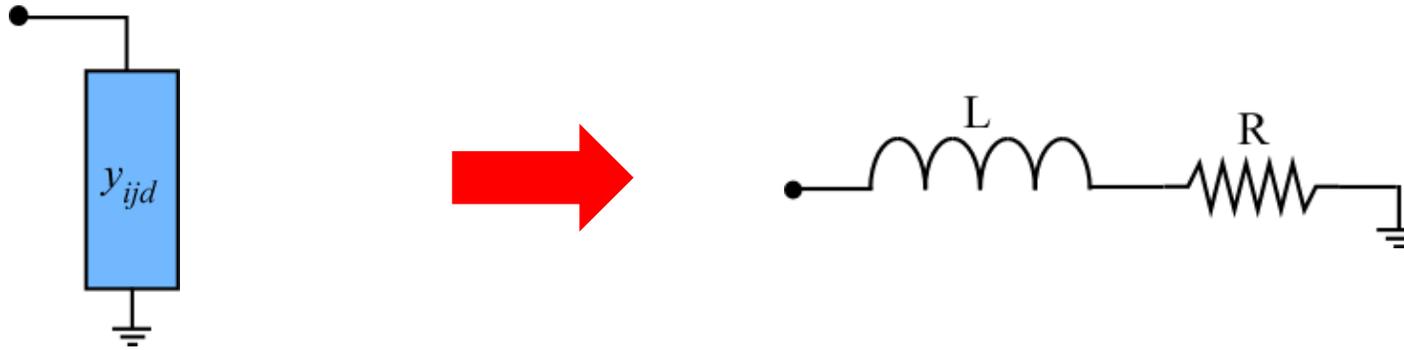**In the pole-residue case, we must distinguish two cases**

(a) Pole is real
$$y_{ijk}(s) = \frac{a_k}{s - p_k}$$

(b) Complex conjugate pair of poles

$$y_{ijk}(s) = \frac{\alpha_k + j\beta_k}{s - \sigma_k - j\omega_k} + \frac{\alpha_k - j\beta_k}{s - \sigma_k + j\omega_k}$$

In all cases, we must find an equivalent circuit consisting of lumped elements that will exhibit the same behavior

Case (a) - Real Pole

Consider the circuit shown above. The input impedance Z as a function of the complex frequency s can be expressed as:

from circuit

from pole and residue

$$Z = sL + R \qquad Y(s) = \frac{1/L}{s + R/L} \qquad \hat{Y}(s) = \frac{a_k}{s - p_k}$$

Comparing $Y(s)$ and $\hat{Y}(s)$ yields the solution

$$L = 1/a_k \qquad R = -p_k/a_k$$

Circuit Realization - Complex Poles

Each term associated with a complex pole pair in the expansion gives:

$$\hat{Y} = \frac{r_1}{s - p_1} + \frac{r_2}{s - p_2}$$

Where r_1, r_2, p_1 and p_2 are the complex residues and poles. They satisfy: $r_1 = r_2*$ and $p_1 = p_2*$

It can be re-arranged as:

$$\hat{Y} = (r_1 + r_2) \frac{\left[s - (r_1 p_2 + r_2 p_1)/(r_1 + r_2) \right]}{s^2 - s(p_1 + p_2) + p_1 p_2}$$

ECE·ILLINOIS

Electrical and Computer Engineering
University of Illinois at Urbana-Champaign

Circuit Realization - Complex Poles

The pole/residue representation of the Y parameters is given by:

$$\hat{Y} = (r_1 + r_2)\frac{\left[s - (r_1 p_2 + r_2 p_1)/(r_1 + r_2)\right]}{s^2 - s(p_1 + p_2) + p_1 p_2}$$

DEFINE

$p = p_1 p_2$ **product of poles**

$a = r_1 + r_2$ **sum of residues**

$g = p_1 + p_2$ **sum of poles**

$x = r_1 p_2 + r_2 p_1$ **cross product**

$$\hat{Y} = a\frac{[s - x/a]}{s^2 - sg + p}$$

Circuit Realization - Complex Poles
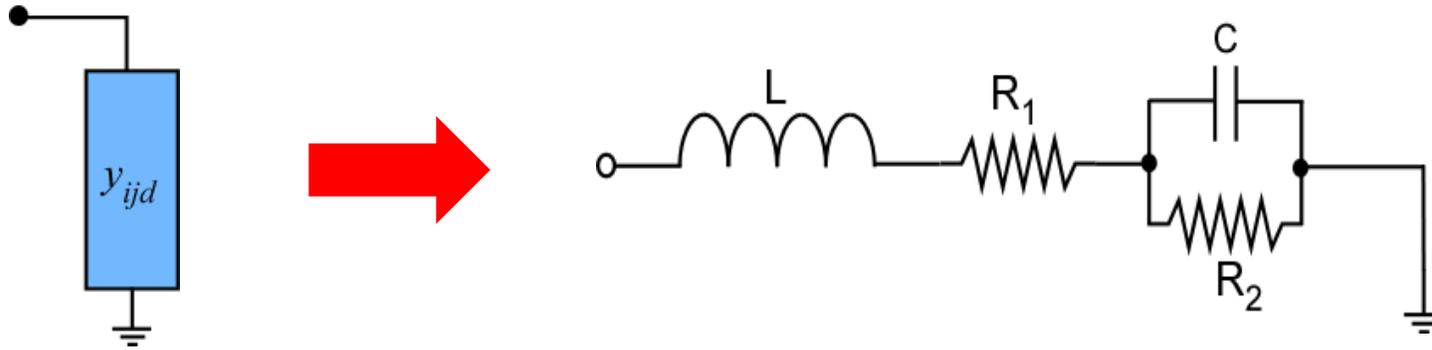
Consider the circuit shown above. The input impedance Z as a function of the complex frequency s can be expressed as:

$$Z = sL + R_1 + \frac{1}{1/R_2 + sC} = sL + R_1 + \frac{R_2}{1 + sCR_2}$$

$$Z = \frac{(R_1 + sL)(1 + sCR_2) + R_2}{1 + sCR_2}$$

Circuit Realization - Complex Poles
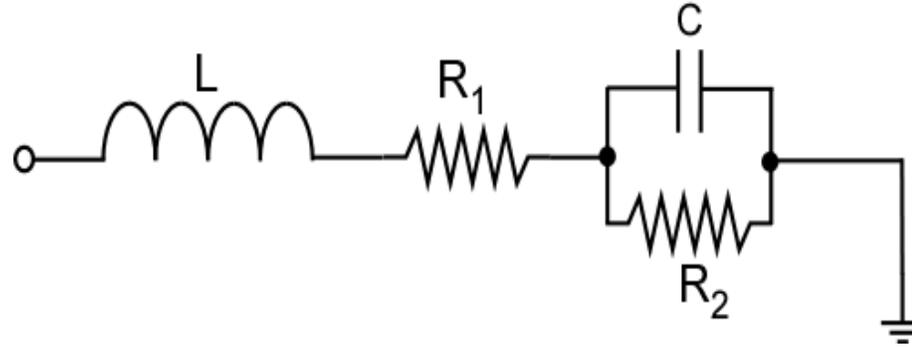
$$Y = \frac{CR_2\left(s + 1/CR_2\right)}{LCR_2\left[s^2 + s\left(\dfrac{L + CR_1R_2}{LR_2C}\right) + \dfrac{\left(R_1 + R_2\right)}{LR_2C}\right]}$$

$$Y = \frac{1}{L}\frac{\left(s + 1/CR_2\right)}{\left[s^2 + s\left(\dfrac{L + CR_1R_2}{LR_2C}\right) + \dfrac{\left(R_1 + R_2\right)}{LR_2C}\right]}$$

Circuit Realization - Complex Poles

Comparing

$$Y = \frac{1}{L} \frac{\left(s + 1/CR_2\right)}{\left[s^2 + s\left(\dfrac{L + CR_1R_2}{LR_2C}\right) + \dfrac{(R_1 + R_2)}{LR_2C}\right]} \quad \textbf{with} \quad \hat{Y} = a\frac{[s - x/a]}{s^2 - sg + p}$$

We can identify the circuit elements

$$L = 1/a$$

$$R_1 = \frac{x}{a^2} - \frac{g}{a}$$

$$R_2 = -\frac{p}{x} - \frac{x}{a^2} + \frac{g}{a}$$

$$C = \frac{pa}{x^2} + \frac{1}{a} - \frac{g}{x}$$

Circuit Realization - Complex Poles

$$L = 1/a$$
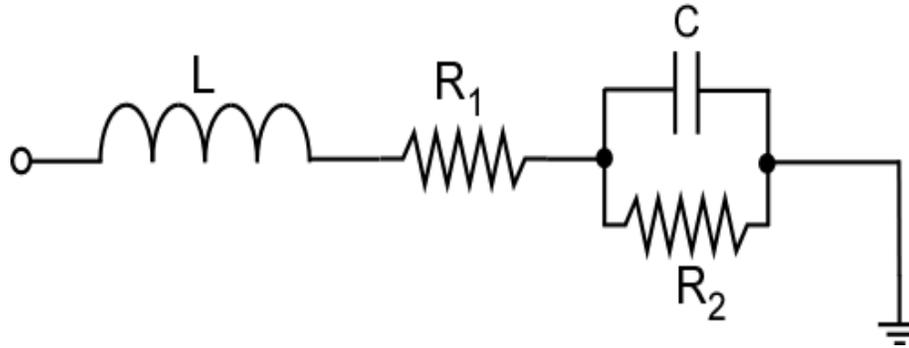
$$R_1 = \frac{x}{a^2} - \frac{g}{a}$$

$$R_2 = -\frac{p}{x} - \frac{x}{a^2} + \frac{g}{a}$$

$$C = \frac{pa}{x^2} + \frac{1}{a} - \frac{g}{x}$$

Negative Elements

In the circuit synthesis process, it is possible that some circuit elements come as negative. To prevent this situation, we add a contribution to the real parts of the residues of the system. In the case of a complex residue, for instance, assume that

$$\hat{Y} = \frac{r_1}{s - p_1} + \frac{r_2}{s - p_2}$$

$$\hat{Y} = \underbrace{\frac{r_1 + \Delta}{s - p_1} + \frac{r_2 + \Delta}{s - p_2}}_{\textit{Augmented Circuit}} - \underbrace{\left(\frac{\Delta}{s - p_1} + \frac{\Delta}{s - p_2} \right)}_{\textit{Compensation Circuit}}$$

Can show that both augmented and compensation circuits will have positive elements

Why S Parameters?

Y-Parameter

Test line: Zc, γ

short
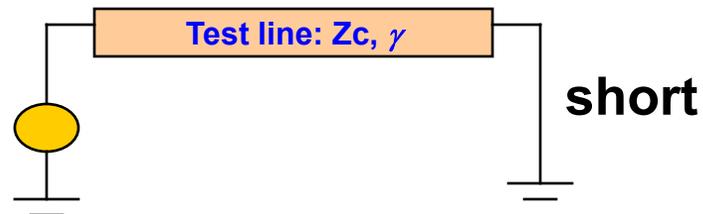
$$Y_{11} = \frac{1 + e^{-2\gamma l}}{Z_c(1 - e^{-2\gamma l})}$$

Z_c : microstrip characteristic impedance
γ : complex propagation constant
l : length of microstrip

Y_{11} can be unstable

S-Parameter

Reference Line   Test Line   Reference Line

Zo   **Test line: Zc, γ**   **Zo**

$$S_{11} = \frac{(1 - e^{-2\gamma l})\Gamma}{1 - \Gamma^2 e^{-2\gamma l}}$$

$$\Gamma = \frac{Z_c - Z_o}{Z_c + Z_o}$$

S_{11} is always stable

Y Versus S Parameters

S (red line)
Y (blue line)

Impulse Responses
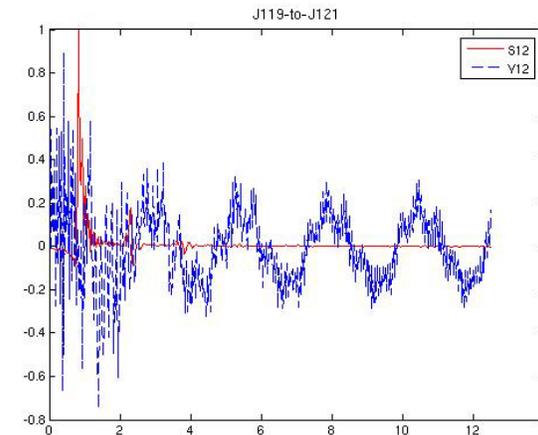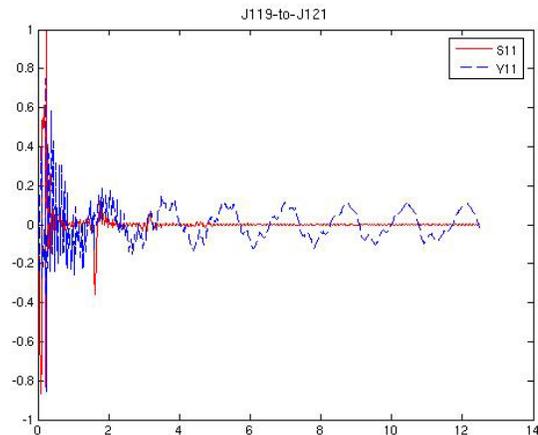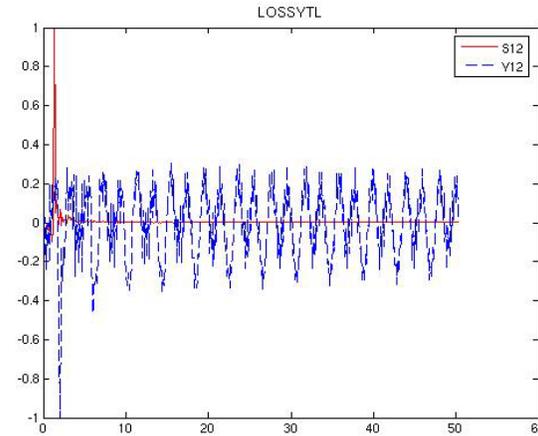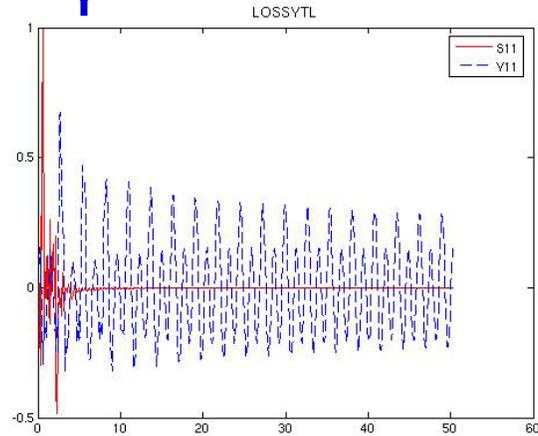
Observation: S-parameters decay rapidly; Y parameters do not.

ECE·ILLINOIS
Electrical and Computer Engineering
University of Illinois at Urbana-Champaign

Optimizing Reference System

$$S = \left[ZZ_o^{-1} + I \right]^{-1} \left[ZZ_o^{-1} - I \right]$$

$$Z = \left[I + S \right] \left[I - S \right]^{-1} Z_o$$

using

$$Z_o = \begin{matrix} 50.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 50.0 \end{matrix}$$

as reference…

using

$$Z_o = \begin{matrix} 328.0 & 69.6 & 328.9 & 69.6 \\ 69.6 & 328.8 & 69.6 & 328.9 \\ 328.9 & 69.6 & 328.8 & 69.6 \\ 69.6 & 328.9 & 69.6 & 328.8 \end{matrix}$$

as reference…

S11 - Linear Magnitude

— S11 - 50 Ohm
— S11 - Zref

S11 vs Frequency (GHz)
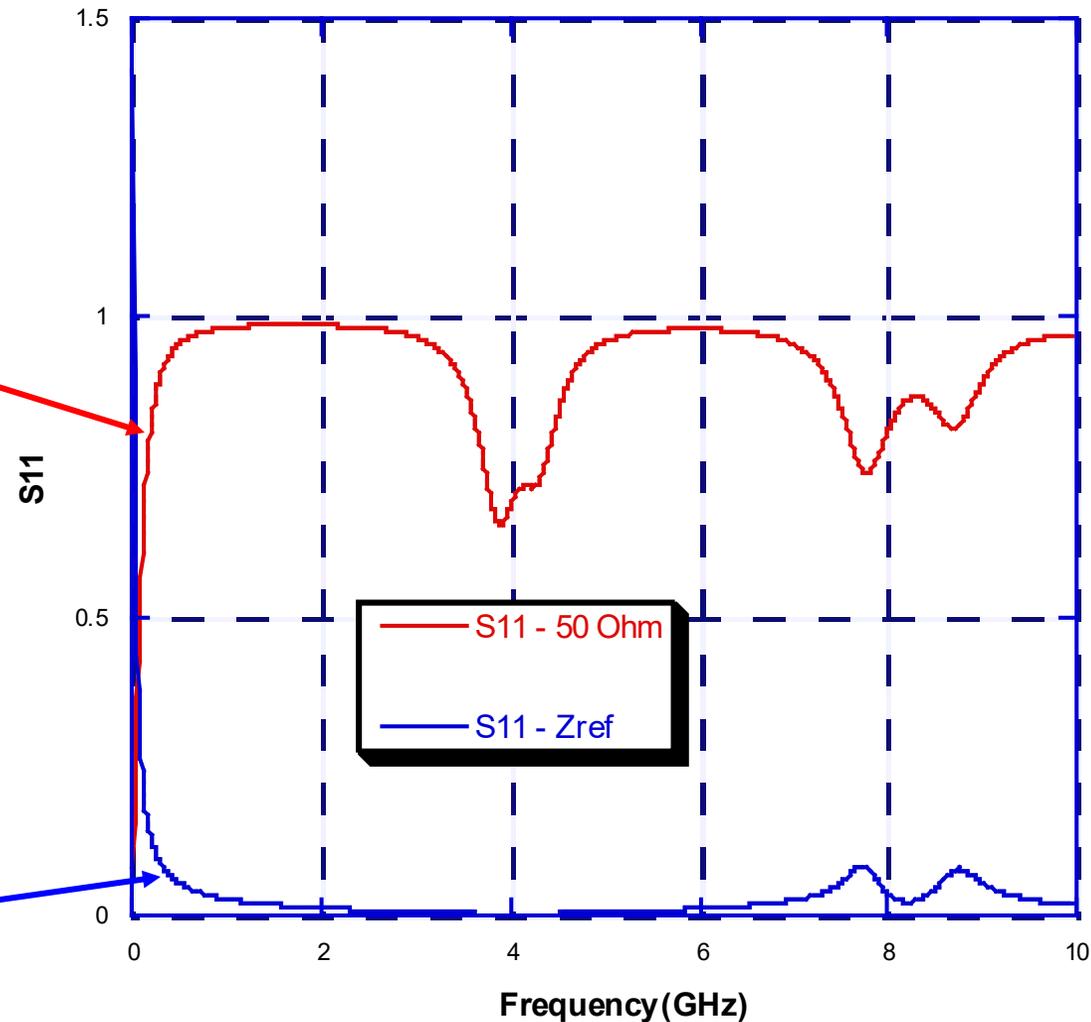
Method 2: S-Parameter /MOR
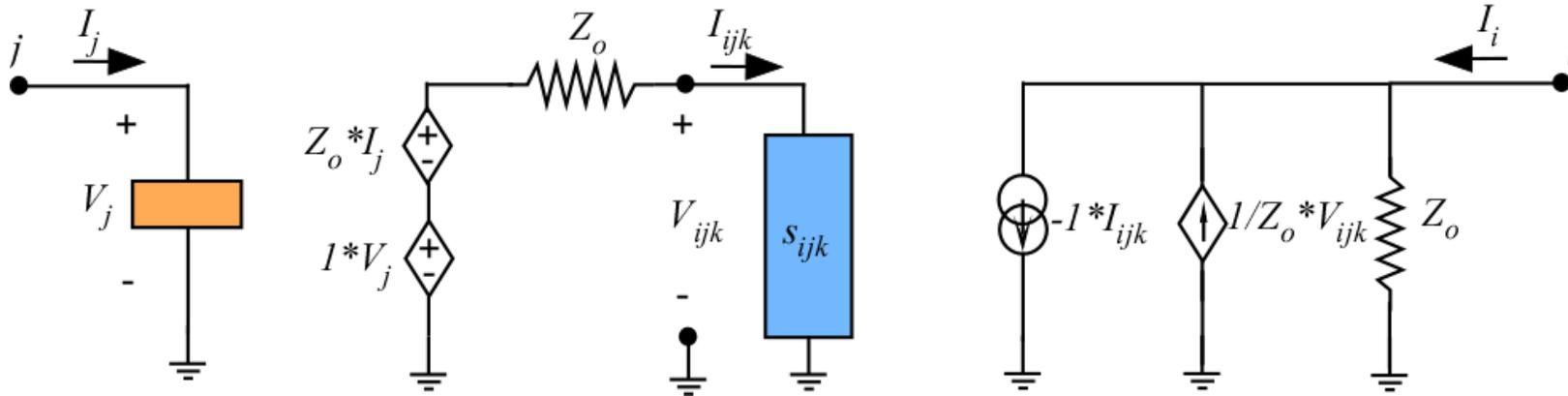
$$A_i(\omega) = \frac{1}{2}\left[V_i(\omega) + Z_o I_i(\omega)\right]$$

$$B_i(\omega) = \frac{1}{2}\left[V_i(\omega) - Z_o I_i(\omega)\right]$$

Need equivalent circuit for S_{ijk}

All S parameters are treated as if they were <u>one-port</u> S parameters

Strategy

For a given circuit, a relationship between the input admittance $Y_{ijk}(s)$ of the circuit and the associated one-port S-parameter representation $S_{ijk}(s)$ can be described by

$$S_{ijk}(s) = \frac{Y_o - Y_{ijk}(s)}{Y_o + Y_{ijk}(s)} \qquad Y_{ijk}(s) = Y_o \frac{1 - S_{ijk}(s)}{1 + S_{ijk}(s)}$$

Y_o is the reference admittance

Equivalent-Circuit Extraction

Macromodel is curve-fit to take the form

$$S(s) = d + \sum_{k=1}^{L} \frac{r_k}{s - p_k}$$

Need to find equivalent circuit associated with

- Constant term d

- Real Poles

- Complex Poles

Constant Term

$$R = \left(\frac{1 - S_{ijk}}{1 + S_{ijk}} \right) Y_o = \left(\frac{1 - d}{1 + d} \right) Y_o$$

S_{ijk} $\longrightarrow$ $R = Y_o(1-d)/(1+d)$

S- Circuit Realization – Constant Term

s_{ijd}
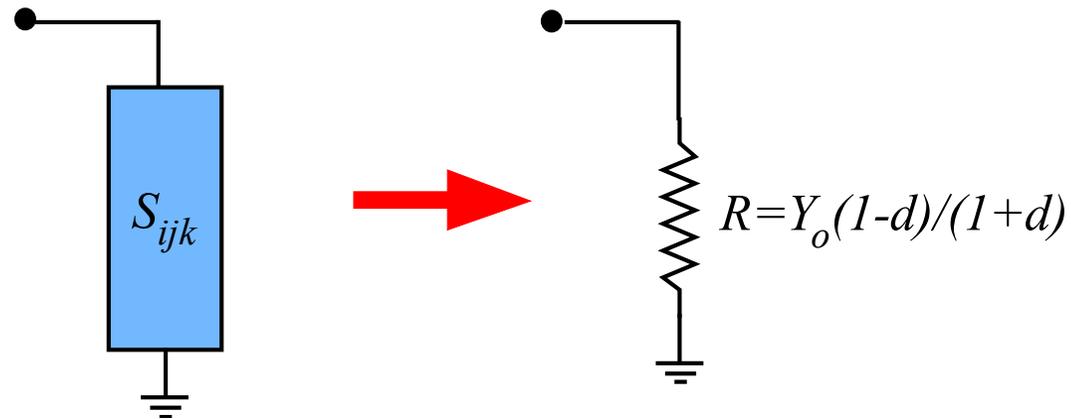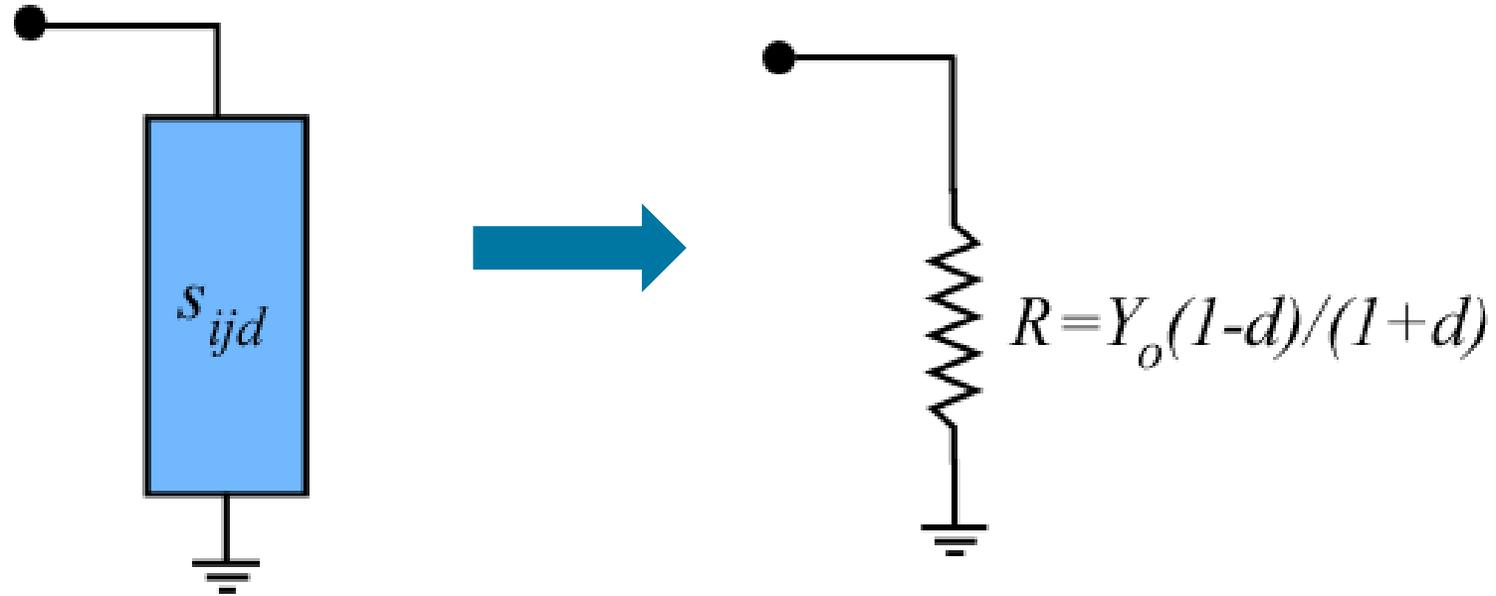
$R = Y_o(1-d)/(1+d)$

$$R = Y_o \left(\frac{1-d}{1+d} \right)$$

S-Parameters - Poles and Residues

In the pole-residue case, we must distinguish two cases

(a) Pole is real

$$s_{ijk}(s) = \frac{a_k}{s - p_k}$$

(b) Complex conjugate pair of poles

$$s_{ijk}(s) = \frac{\alpha_k + j\beta_k}{s - \sigma_k - j\omega_k} + \frac{\alpha_k - j\beta_k}{s - \sigma_k + j\omega_k}$$

In all cases, we must find an equivalent circuit consisting of lumped elements that will exhibit the same behavior

S-Realization – Real Poles

*Proposed
Circuit Model*
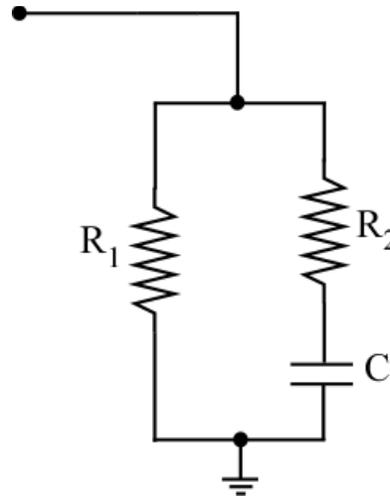
Admittance of proposed model is given by:

$$Y = \frac{\left(R_1 + R_2\right)}{R_1 R_2}\left[\frac{s + \dfrac{1}{\left(R_1 + R_2\right)C}}{s + \dfrac{1}{R_2 C}}\right]$$
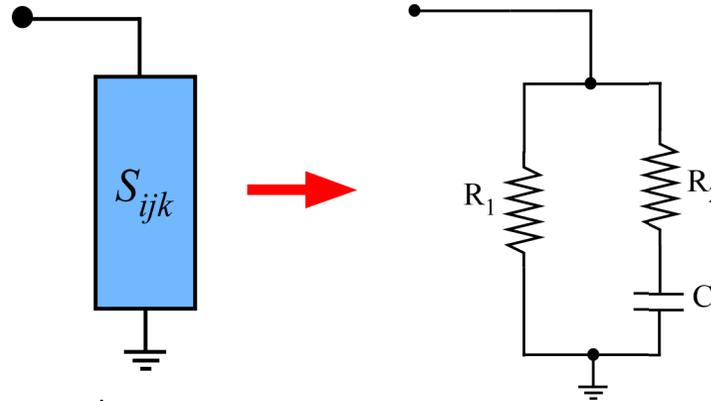
Real Poles

$$Y = \frac{\left(R_1 + R_2\right)}{R_1 R_2} \left[\frac{s + \dfrac{1}{\left(R_1 + R_2\right)C}}{s + \dfrac{1}{R_2 C}} \right]$$

from circuit

$$S_{ijk}\left(s\right) = \frac{a_k}{s - p_k}$$

$$\hat{Y} = Y_o \left(\frac{1 - S_{ijk}}{1 + S_{ijk}} \right) = Y_o \left(\frac{1 - \dfrac{r}{s - p}}{1 + \dfrac{r}{s - p}} \right) = Y_o \left(\frac{s - a}{s + a} \right)$$

from pole and residue

Solution for Real Poles
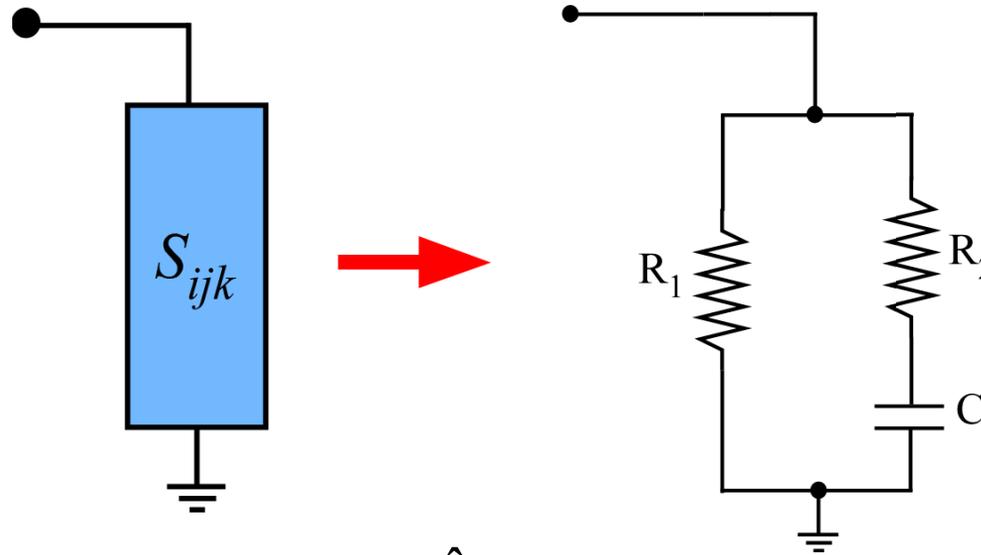
Comparing $\hat{Y}(s)$ with $Y(s)$ gives

$$C = -\frac{(b-a)}{b^2 Z_o}$$

$$R_2 = \frac{-1}{bC}$$

$$R_1 = -R_2 - \frac{1}{aC}$$

where $\quad a = p_k + r_k, \quad$ and $\quad b = p_k - r_k$

Realization – Complex Poles

From the S-parameter expansion, the complex pole pair gives:

$$\hat{S} = \frac{r_1}{s - p_1} - \frac{r_2}{s - p_2} = \frac{s(r_1 + r_2) - (r_1 p_2 + r_2 p_1)}{s^2 - s(p_1 + p_2) + p_1 p_2}$$

which corresponds to an admittance of:

$$\hat{Y} = Y_o \left(\frac{1 - \hat{S}}{1 + \hat{S}} \right) = \left(\frac{1 - \dfrac{sa - x}{s^2 - sg + p}}{1 + \dfrac{sa - x}{s^2 - sg + p}} \right) Y_o$$

Electrical and Computer Engineering
University of Illinois at Urbana-Champaign

Realization – Complex Poles

The admittance expression can be re-arranged as

$$\hat{Y} = \left(\frac{s^2 - sg + p - sa + x}{s^2 - sg + p + sa - x} \right) Y_o = \left(\frac{s^2 - s(g+a) + p + x}{s^2 - s(g-a) + p - x} \right) Y_o$$

WE HAD DEFINED

$p = p_1 p_2$ **product of poles**

$a = r_1 + r_2$ **sum of residues**

$g = p_1 + p_2$ **sum of poles**

$x = r_1 p_2 + r_2 p_1$ **cross product**

Electrical and Computer Engineering
University of Illinois at Urbana-Champaign

Realization – Complex Poles

$$\hat{Y} = \left(\frac{s^2 - sg + p - sa + x}{s^2 - sg + p + sa - x} \right) Y_o = \left(\frac{s^2 - s(g+a) + p + x}{s^2 - s(g-a) + p - x} \right) Y_o$$

This can be further rearranged as

$$\hat{Y} = \left(\frac{s^2 + sA + B}{s^2 + sD + F} \right) E$$

in which

$$A = -(g+a), \;\; B = p+x, \;\; D = -(g-a), \;\; F = p-x$$

and $\;\; E = Y_o = \dfrac{1}{H}$

ECE·ILLINOIS
Electrical and Computer Engineering
University of Illinois at Urbana-Champaign

Realization – Complex Poles

There are several circuit topologies that will work

Model 1

Model 9

Model 13

Model 12

ECE·ILLINOIS
Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
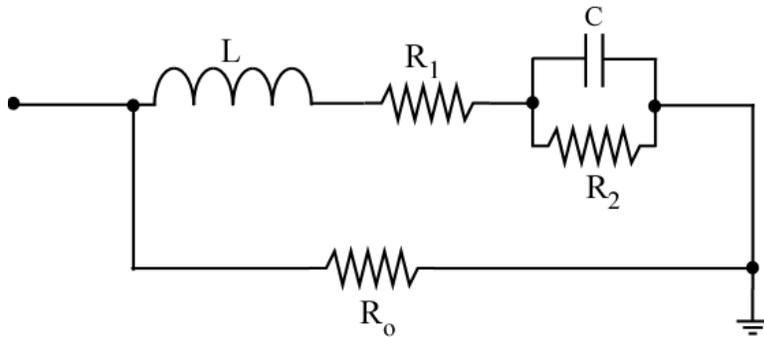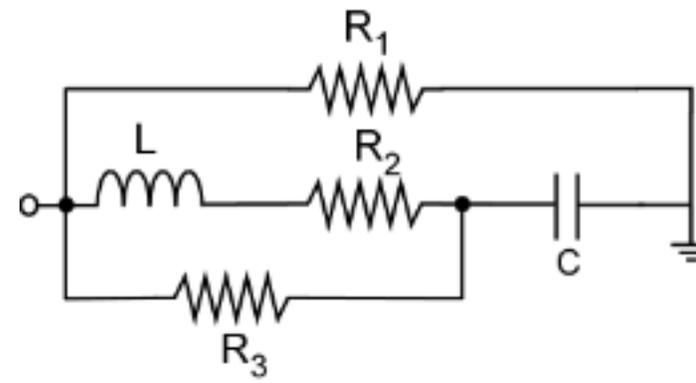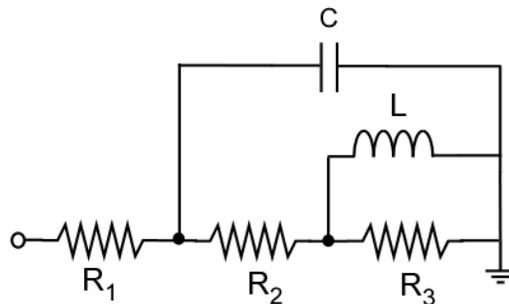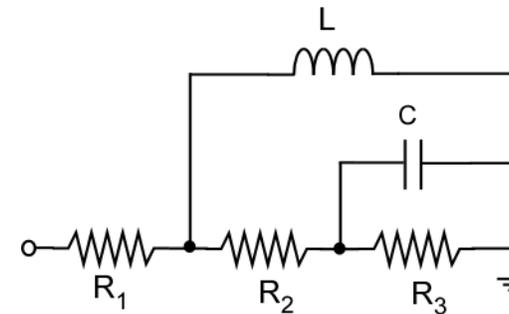
Realization – Complex Poles

More circuit topologies that will work

Model 10
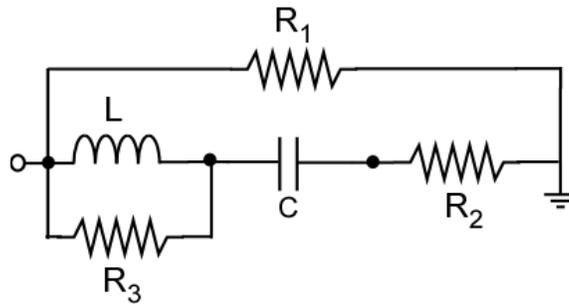
Model 11
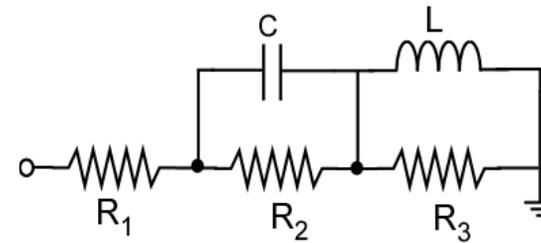
Model 8
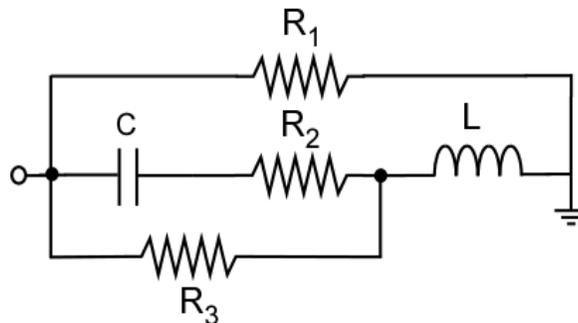
Model 12
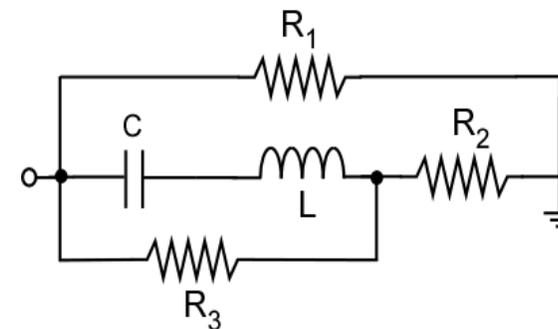
Complex Poles – Model 1

In particular, if we choose model 1
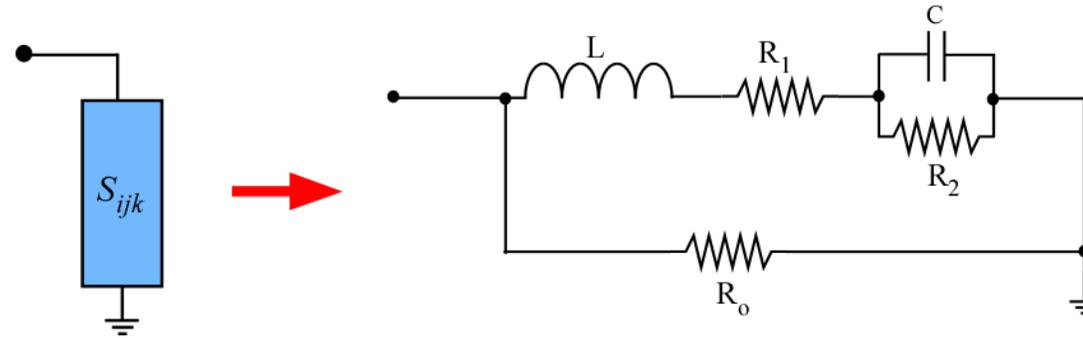
$$Y = \frac{1}{R_o}\left[\frac{s^2 + s\left(\dfrac{L + R_1 R_2 C + R_o R_2 C}{LCR_2}\right) + \dfrac{R_o + R_1 + R_2}{LCR_2}}{s^2 + s\left(\dfrac{L + R_1 R_2 C}{LCR_2}\right)_o + \dfrac{R_1 + R_2}{LCR_2}}\right]$$

Must be matched with

$$\hat{Y} = Y_o\left(\frac{1 - S_{ijk}}{1 + S_{ijk}}\right) = \left(\frac{1 - \dfrac{sa - x}{s^2 - sg + p}}{1 + \dfrac{sa - x}{s^2 - sg + p}}\right)Y_o$$

Complex Poles – Model 1

Matching the terms with like coefficients gives

$$R_o = \frac{1}{Y_o}$$

$$p + x = \frac{R_o + R_1 + R_2}{LCR_2}$$

$$p - x = \frac{R_1 + R_2}{LCR_2}$$

$$2p = \frac{R_o + 2R_1 + 2R_2}{LCR_2}$$

$$2x = \frac{R_o}{LCR_2}$$

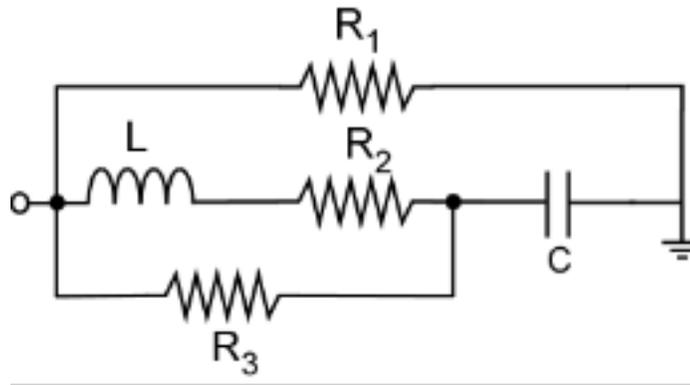# Complex Poles – Model 1
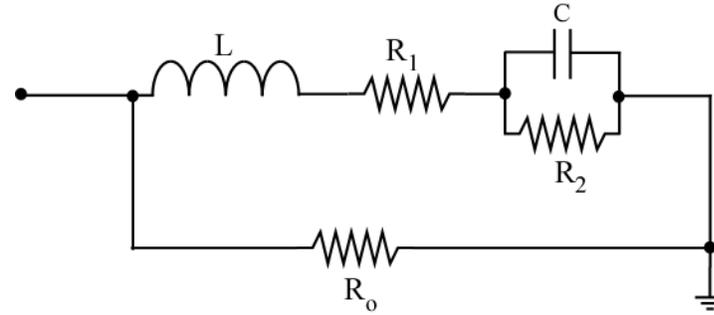
Solving gives

$$L = -\frac{R_o}{2a}$$

$$R_2 = \frac{R_o\left(\dfrac{p}{x} - 1\right)}{2} - R_1$$

$$R_1 = \frac{1}{2}\left(\frac{gR_o}{a} + \frac{2Lx}{a} - R_o\right)$$
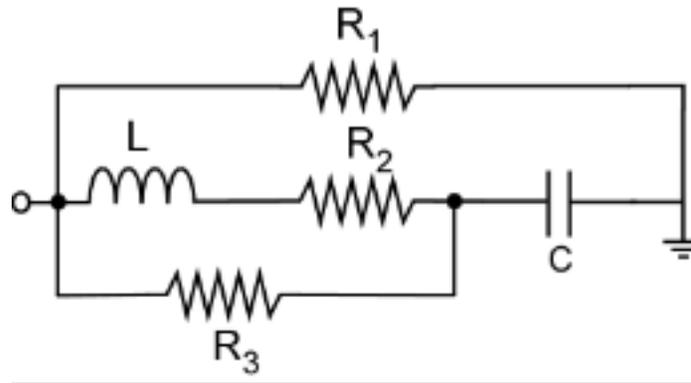
$$C = \frac{-a}{R_2 x}$$

Complex Poles – Model 9

If we choose Model 9

$$Y_9 = \overbrace{\frac{R_1 + R_3}{R_1 R_3}}^{E} \left[\frac{s^2 + s\, \dfrac{\overbrace{L + CR_2 R_3 + CR_1\left(R_2 + R_3\right)}^{A}}{LC\left(R_1 + R_3\right)} + \overbrace{\dfrac{R_2 + R_3}{LC\left(R_1 + R_3\right)}}^{B}}{s^2 + s\, \underbrace{\dfrac{CR_1 R_2 R_3 + LR_1}{LCR_1 R_3}}_{D} + \underbrace{\dfrac{R_1\left(R_2 + R_3\right)}{LCR_1 R_3}}_{F}} \right]$$

from which the circuit elements can be extracted

Complex Poles – Model 9

$$R_1 = \frac{F}{BH}$$

$$C = \frac{\left(-BD + AF\right)H}{F^2}$$

$$R_2 = \frac{\left(-BD^2 + BF + ADF - F^2\right)}{\left(B^2 - ABD + BD^2 + A^2F - 2BF - ADF + F^2\right)H}$$

$$R_3 = \frac{F}{\left(-B + F\right)H}$$

$$L = \frac{-BD + AF}{\left(B^2 - ABD + BD^2 + A^2F - 2BF - ADF + F^2\right)H}$$

Special Case – Model 4

A special case exists when $x=0$

$$\textbf{or} \quad x = r_1 p_2 + r_2 p_1 = 0$$

$$\hat{Y} = \left(\frac{s^2 - s(g+a) + p + x}{s^2 - s(g-a) + p - x} \right) Y_o = \left(\frac{s^2 - s(g+a) + p}{s^2 - s(g-a) + p} \right) Y_o$$

$$\hat{Y} = \left(\frac{s^2 + sA + B}{s^2 + sD + B} \right) E$$

in which

$$A = -(g+a), \quad B = p + x, \quad D = -(g-a), \quad F = p$$

and $\quad E = Y_o = \dfrac{1}{H}$

Special Case – Model 4

The proposed circuit model is

Model 4
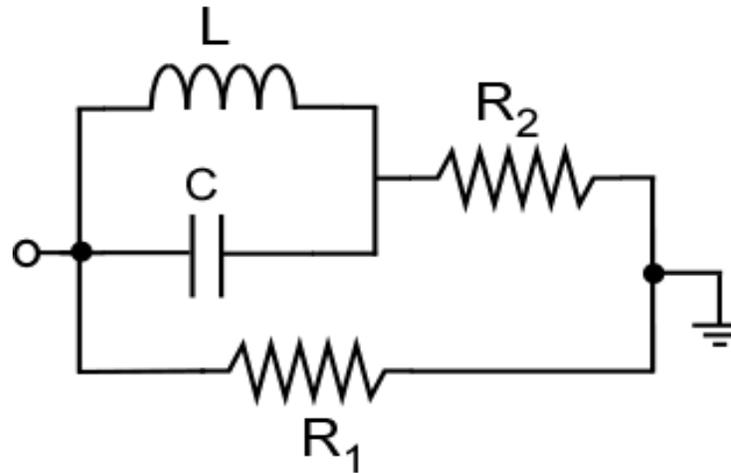
This model has one inductor, one capacitor and two resistors.

Special Case – Model 4
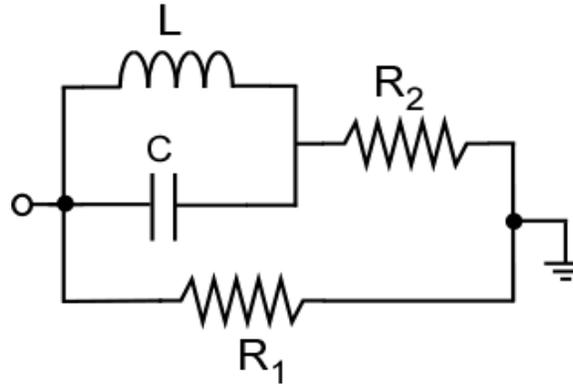
The admittance seen at the input is:

$$Y_4 = \overbrace{\frac{R_1 + R_2}{R_1 R_2}}^{E} \left[\frac{s^2 + s\overbrace{\dfrac{1}{C(R_1 + R_2)}}^{A} + \overbrace{\dfrac{1}{LC}}^{B=F}}{s^2 + s\underbrace{\dfrac{1}{CR_2}}_{D} + \underbrace{\dfrac{1}{LC}}_{F=B}} \right]$$

The element are extracted as

$$R_1 = \frac{DE}{A} \qquad R_2 = \frac{R_1 E}{R_1 - E} \qquad C = \frac{1}{DR_2} \qquad L = \frac{1}{FC}$$

Method 3 - Convolution with S Parameters
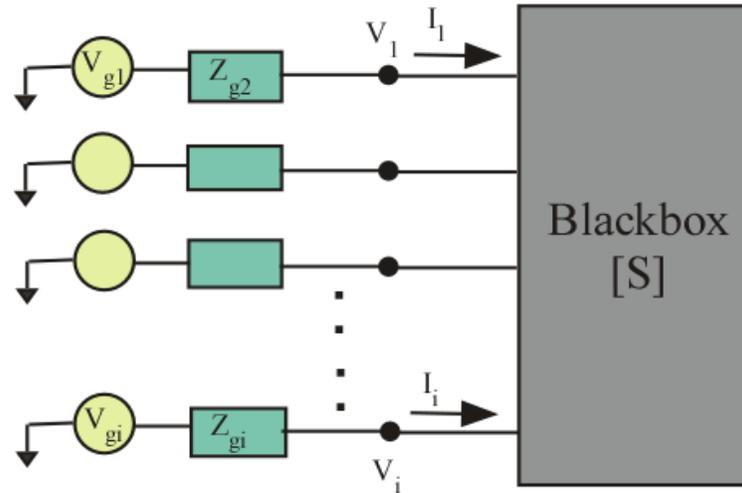
In frequency domain   *B=SA*

In time domain   *b(t) = s(t)*a(t)*

Convolution: $s(t) * a(t) = \int\limits_{-\infty}^{\infty} s(t-\tau) a(\tau) d\tau$

SPICE Macromodel
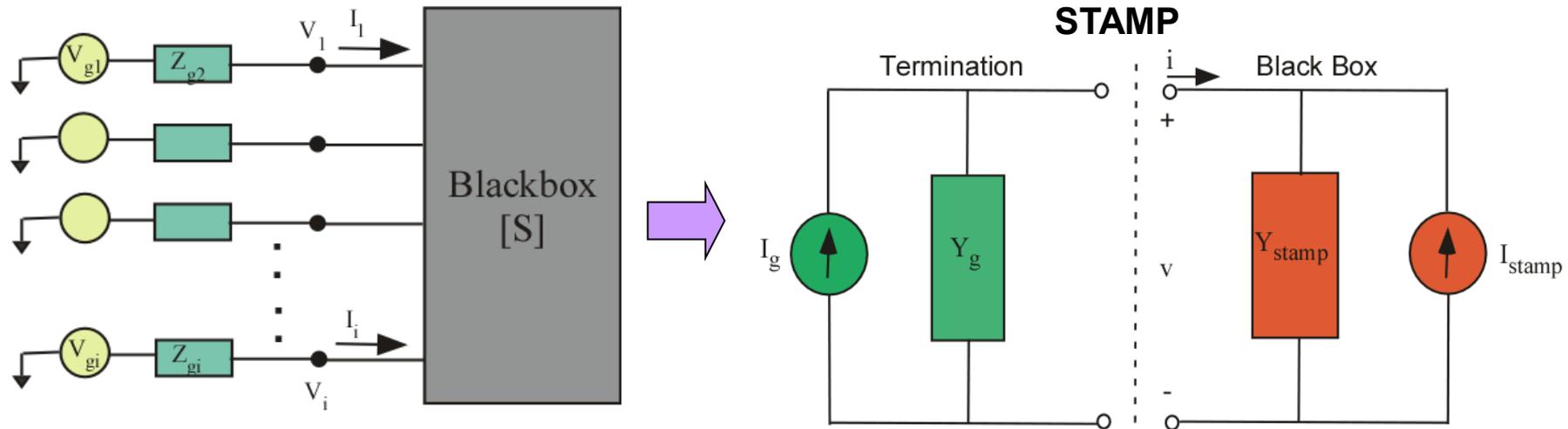
$$Y_{stamp} = Z_o^{-1}\left[1 + s'(0)\right]^{-1}\left[1 - s'(0)\right]$$

$$I_{stamp} = 2Z_o^{-1}\left[1 + s'(0)\right]^{-1} H(t)$$

Most of the computational effort is in the convolution calculation of the history *H(t)* at each time step.

$$H(t) = \sum_{\tau=1}^{t-1} s(\tau)a(t-\tau)\Delta\tau$$ ⟵ *This operation can be accelerated*

We make the convolution faster using a δ-function expansion for *s(t)*

S-Parameter Expansion

In the frequency domain, assume that S parameter can be expanded in the form:

$$S(l) = \sum_{k=1}^{M} c_k e^{j2\pi lk}$$

In the time domain, this corresponds to:

$$s(p) = \sum_{k=1}^{M} c_k \delta(p-k)$$

Which is an impulse train of order **M**. Convolution will then give:

$$y(p) = s(p) * x(p) = \left[\sum_{k=1}^{M} c_k \delta(p-k) \right] * x(p) = \sum_{k=1}^{M} c_k x(p-k)$$

If we truncate the summation to the **Q** largest impulses:

$$y(p) = \left[\sum_{k=1}^{Q} c_k \delta(p-k) \right] * x(p) = \sum_{k=1}^{Q} c_k x(p-k)$$

If the reference system is optimized, most of the coefficients c_k will be negligibly small; therefore $\Rightarrow$ **Q** will be a relatively small number

Fast Convolution is achieved by making Q small

Strategy

$$s_{ijk}(u) = c_{ijk}\delta(u - k)$$

Since the S-parameter is a simple delay, the one-port circuit representation is a transmission line with characteristic impedance Z_o and delay k
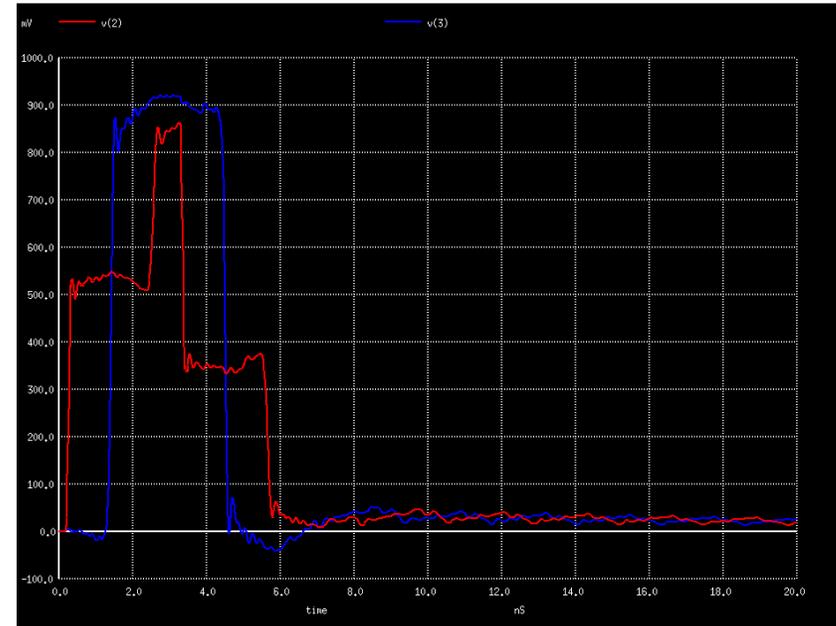
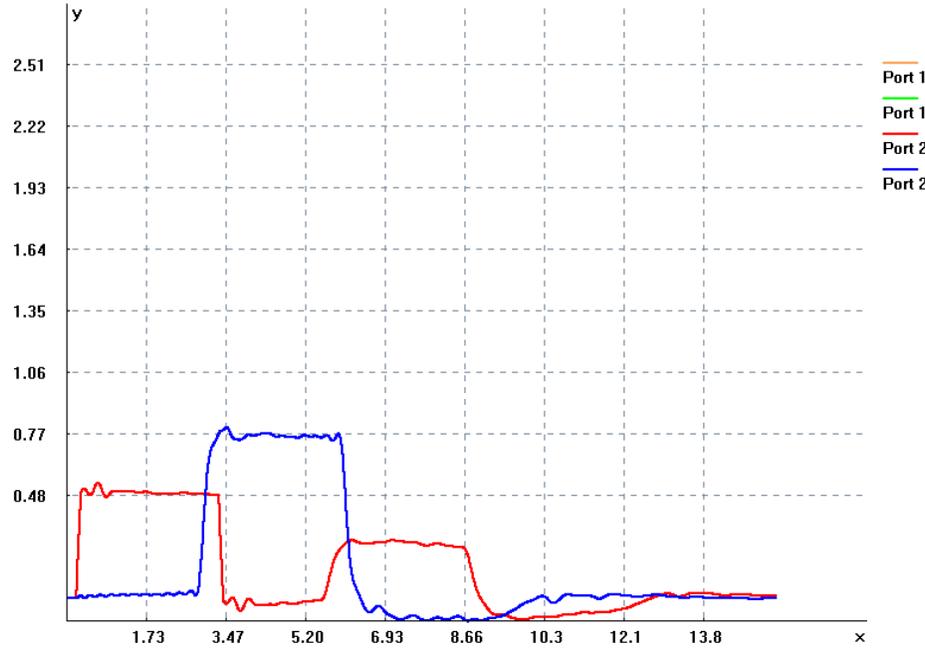# Circuit Interpretation

$$s_{ijk}(u) = c_{ijk}\delta(u-k)$$

$$\tau_d = k\Delta t \qquad \Delta t = \text{time step}$$

ECE·ILLINOIS

Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
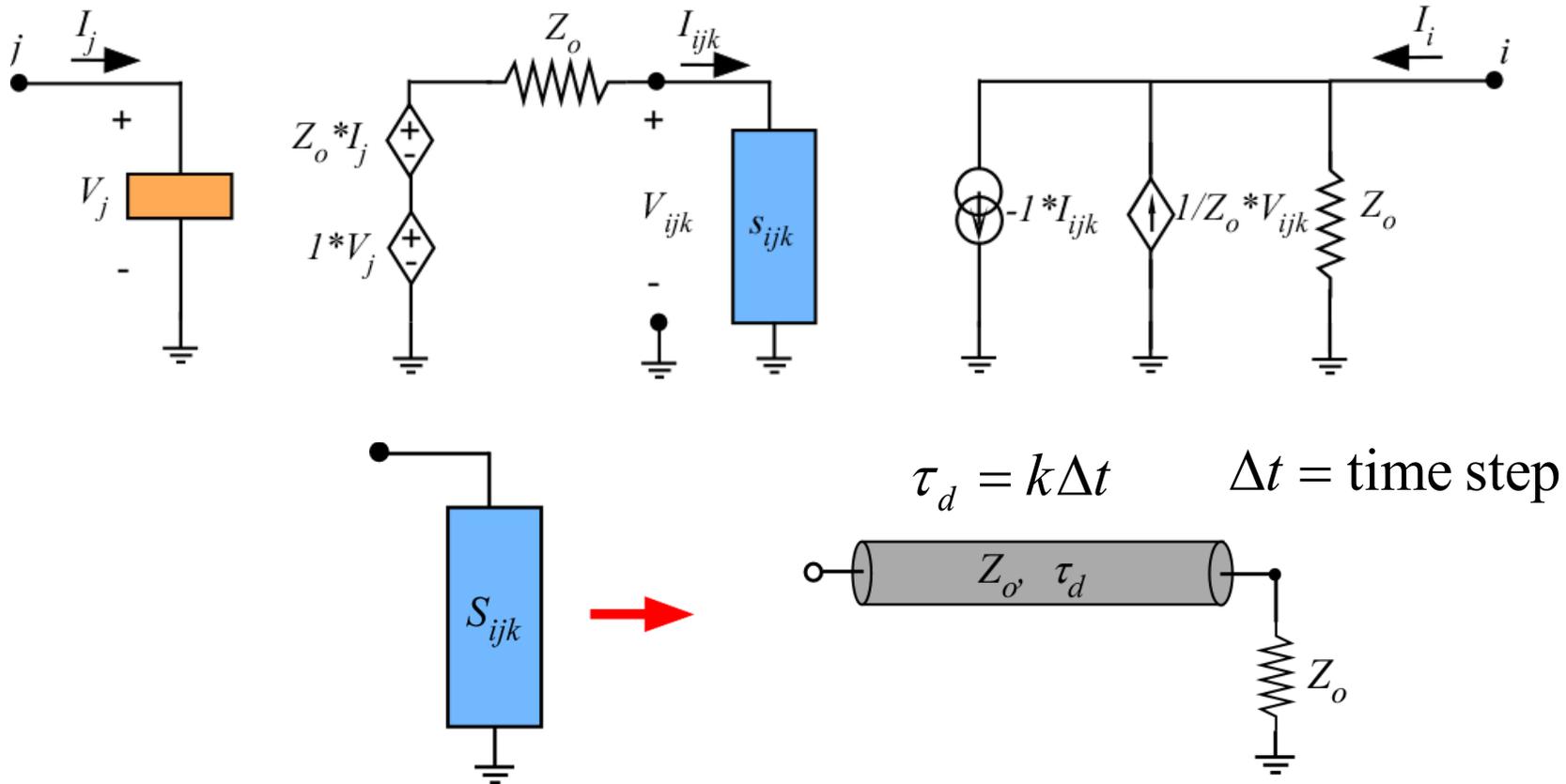
Method 1: Y-Parameters/MOR
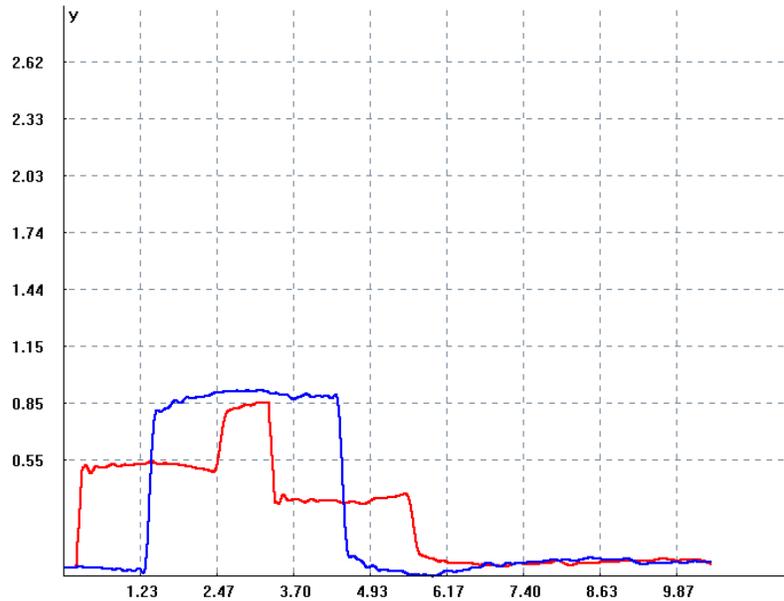
Recursive convolution

SPICE realization

Method 2: S-Parameters/MOR

Recursive convolution
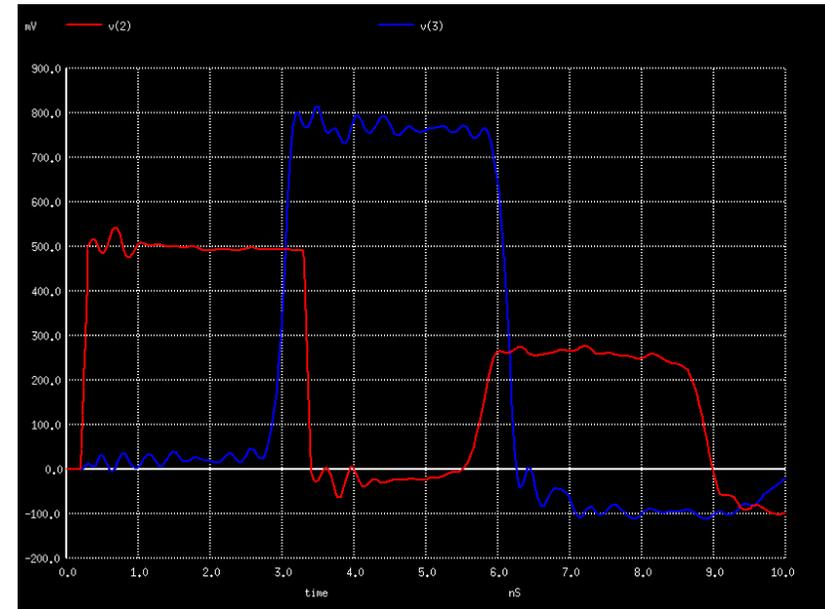
SPICE realization

Comparisons
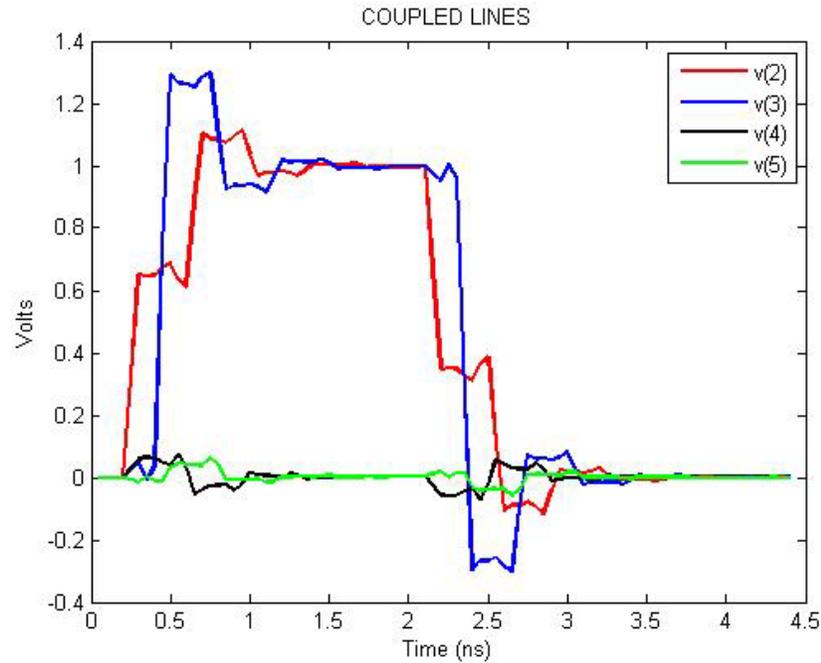
**SPICE simulation
from MOR generated Netlist
(Method 2)**
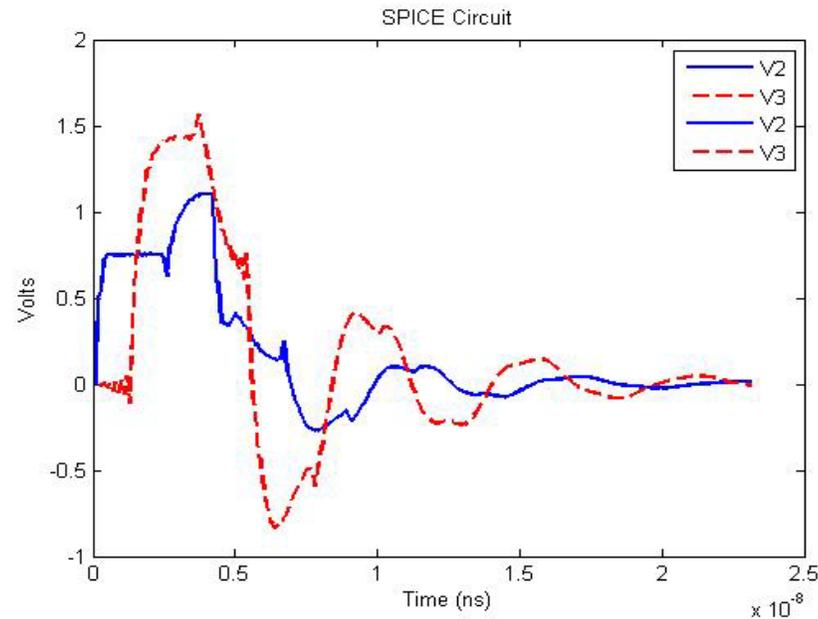
**Direct
Convolution**
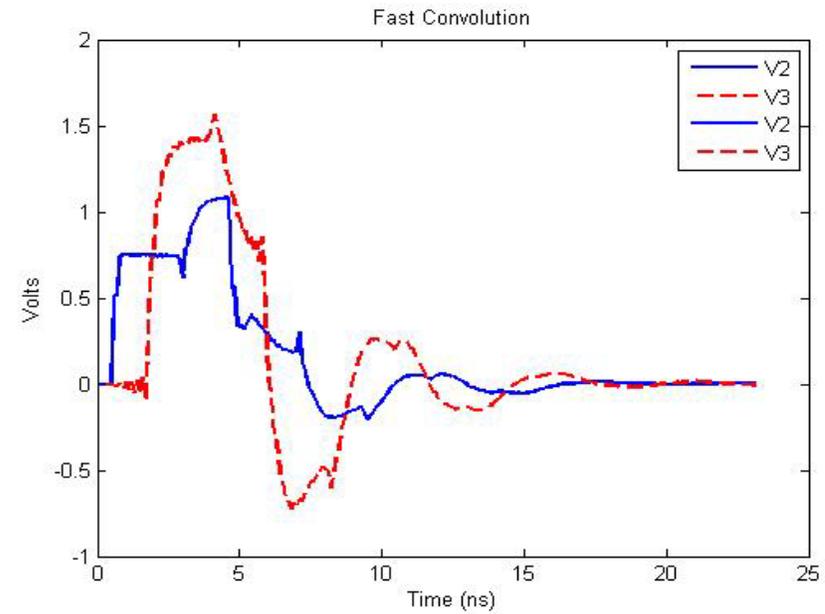
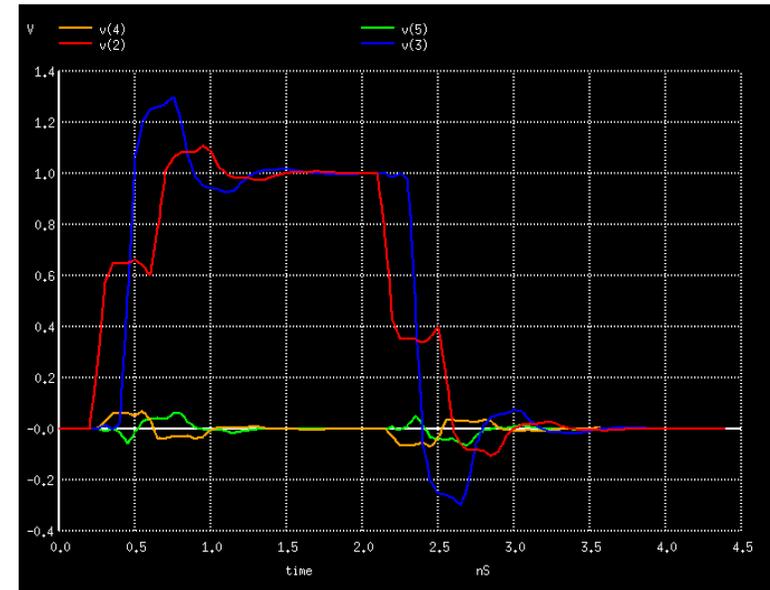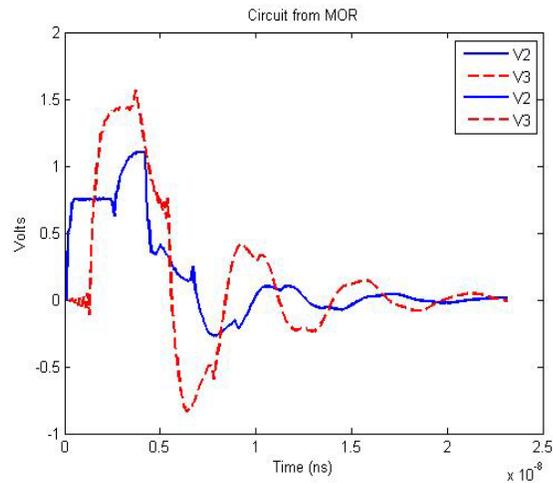# Coupled Lines
(4-port)

Direct

SPICE simulation
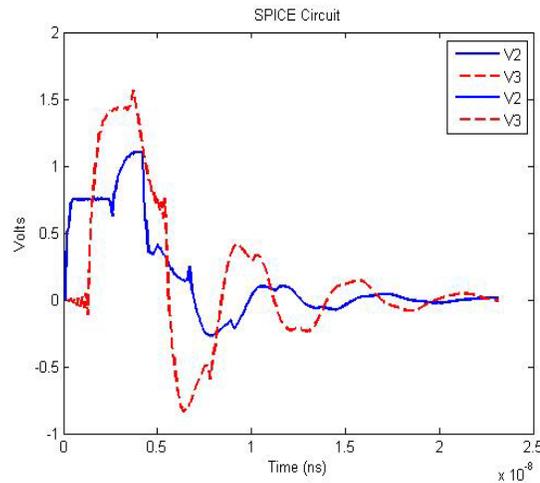**Using generated netlist**
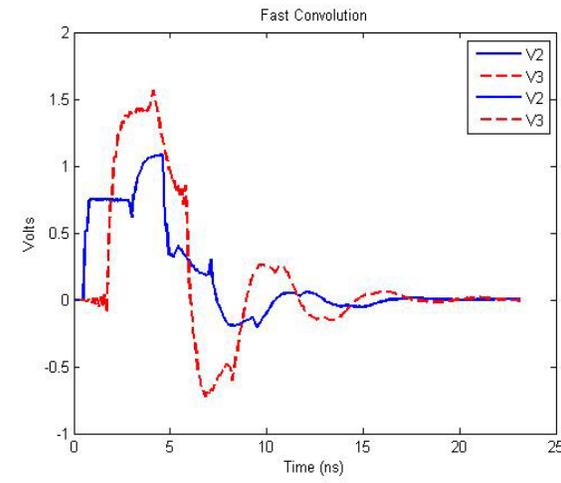**(Method 2)**

Comparison of S-methods

SPICE Netlist from MOR with S (Method 2)

SPICE Netlist from Fast convolution with S (Method 3)

Convolution (no SPICE netlist)

References

1. Giulio Antonini "SPICE Equivalent Circuits of Frequency-Domain Responses", IEEE Transactions on Electromagnetic Compatibility, pp 502-512, Vol. 45, No. 3, August 2003.

2. José Schutt-Ainé, "SPICE circuit synthesis from complex pole-residue S-parameter representation", 2016 Asia-Pacific International Symposium on Electromagnetic Compatibility (APEMC), pp. 1050-105, Shenzhen, May 2016,

3. José Schutt-Ainé, "Circuit synthesis of blackbox macromodels from S-parameter representation", 2018 IEEE 22nd Workshop on Signal and Power Integrity (SPI), Brest, France, May 2018.

4. Chiu-Chih Chou, José E. Schutt-Ainé, "Equivalent Circuit Synthesis of Multiport S Parameters in Pole–Residue Form", IEEE Transactions on Components, Packaging and Manufacturing Technology, Volume 11, Issue: 11, pp. 1971-1979, 2021, November 2021