n-line simulation (lossless)

1) Get $\boldsymbol{L}$ and $\boldsymbol{C}$ matrices and calculate $\boldsymbol{LC}$ product

2) Get square root of eigenvalues and eigenvectors of $\boldsymbol{LC}$ matrix ➔ $\boldsymbol{\Lambda_m}$

3) Arrange eigenvectors into the voltage eigenvector matrix $\boldsymbol{E}$

4) Get square root of eigenvalues and eigenvectors of $\boldsymbol{CL}$ matrix ➔ $\boldsymbol{\Lambda_m}$

5) Arrange eigenvectors into the current eigenvector matrix $\boldsymbol{H}$

7) Invert matrices $\boldsymbol{E}$, $\boldsymbol{H}$, $\boldsymbol{\Lambda_m}$.

6) Calculate the line impedance matrix $\boldsymbol{Z_c}$.

$$Z_C = E^{-1} \Lambda_m^{-1} E L$$

8) Construct source and load impedance matrices $\boldsymbol{Z_s(t)}$ and $\boldsymbol{Z_L(t)}$

9) Construct source and load reflection coefficient matrices $\boldsymbol{\Gamma_1(t)}$ and $\boldsymbol{\Gamma_2(t)}$.

   Indices 1 and 2 refer to near and far ends respectively.

$$\Gamma_1(t) = -\left[1 + E Z_S Z_C^{-1} E^{-1}\right]^{-1} \left[1 - E Z_S Z_C^{-1} E^{-1}\right]$$

$$\Gamma_2(t) = -\left[1 + E Z_L Z_C^{-1} E^{-1}\right]^{-1} \left[1 - E Z_L Z_C^{-1} E^{-1}\right]$$

10) Construct source and load transmission coefficient matrices $\boldsymbol{T_1(t)}$ and $\boldsymbol{T_2(t)}$

$$T_1(t) = \left[1 + E Z_S Z_C^{-1} E^{-1}\right]^{-1}$$

$$T_2(t) = \left[1 + EZ_L Z_C^{-1} E^{-1}\right]^{-1}$$

11) Calculate modal voltage sources $g_1(t)$ and $g_2(t)$

$$g_1(t) = EV_s(t)$$

$$g_2(t) = EV_L(t)$$

12) Calculate modal voltage waves:

$$a_1(t) = T_1(t)\, g_1(t) + \Gamma_1(t)\, a_2(t - \tau_m)$$

$$a_2(t) = T_2(t)\, g_2(t) + \Gamma_2(t)\, a_1(t - \tau_m)$$

$$b_1(t) = a_2(t - \tau_m)$$

$$b_2(t) = a_1(t - \tau_m)$$

where

$$a_i(t - \tau_m) = \begin{bmatrix} a_{i\text{-}mode\text{-}1}(t - \tau_{m1}) \\ a_{i\text{-}mode\text{-}2}(t - \tau_{m2}) \\ \cdot \\ a_{i\text{-}mode\text{-}n}(t - \tau_{mn}) \end{bmatrix}$$

τ_{mi} is the delay associated with mode i. τ_{mi} = length/velocity of mode i. The modal voltage wave vectors $a_1(t)$ and $a_2(t)$ need to be stored since they contain the history of the system.

13) Calculate total modal voltage vectors:

$$V_{m1}(t) = a_1(t) + b_1(t)$$

$$V_{m2}(t) = a_2(t) + b_2(t)$$

14) Calculate line voltage vectors:

$$V_1(t) = E^{-1} V_{m1}(t)$$

$$V_2(t) = E^{-1} V_{m2}(t)$$

Note: subscript 1 and 2 refer to near and far ends respectively.

$$V_1(t) = \begin{bmatrix} V_{near\text{-}line\text{-}1} \\ V_{near\text{-}line\text{-}2} \\ . \\ V_{near\text{-}line\text{-}n} \end{bmatrix} \qquad V_2(t) = \begin{bmatrix} V_{far-line-1} \\ V_{far-line-2} \\ . \\ V_{far-line-n} \end{bmatrix}$$