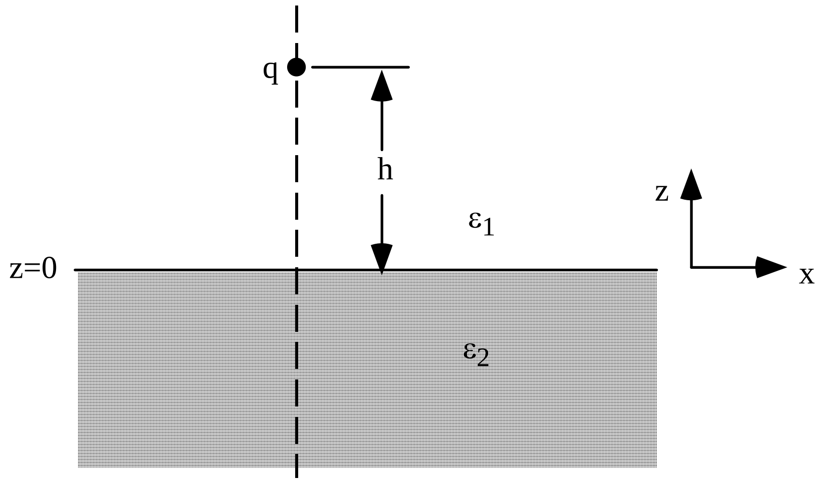
ECE 546 HOMEWORK No 1 Due Wednesday, February 3, 2016



Problem 1

1. In the calculation of capacitance coefficient for complex interconnect structures, it is often desired to determine the 3D Green's function for the electrostatic potential. The Green's function is the potential due to a point source as observed at a specific location.

Consider a point charge *q* located in medium 1 at a height *h* away from a flat boundary of medium 2 as illustrated above. Medium 1 and 2 are characterized by permittivities ε_1 and ε_2 respectively.

(a) Find the electrostatic potential measured by an observer P_1(x, y, z) located in medium 1.

(b) Find the electrostatic potential measured by an observer P_2(x, y, z) located in medium 2.

Potential in air is:

$$\phi^{(a)} = \left[\frac{q}{\sqrt{x^2+y^2+(z-h)^2}} - \frac{q_b}{\sqrt{x^2+y^2+(z+h)^2}}\right]\frac{1}{4\pi\varepsilon_1}$$

$$\phi^{(d)} = \left[\frac{q_a}{\sqrt{x^2+y^2+(z-h)^2}}\right]\frac{1}{4\pi\varepsilon_1}$$

Applying the boundary condition $\phi^{(a)}\Big|_{z=0} = \phi^{(d)}\Big|_{z=0}$

Gives:

$$q - q_b = q_a \qquad (1)$$

Moreover, applying the condition:

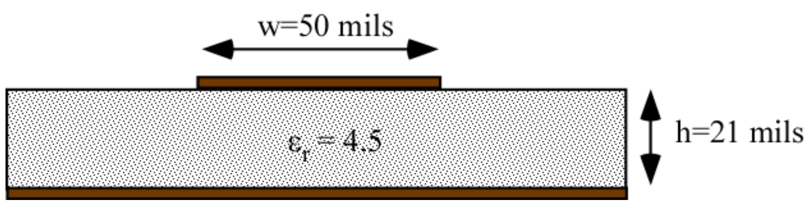
$$\hat{z}\cdot\left(\varepsilon_1\vec{E}^{(a)} - \varepsilon_2 E^{(d)}\right) = 0 \;\; or \;\; \hat{z}\cdot\left(\varepsilon_1 \frac{\partial\phi^{(a)}}{\partial z}\hat{z} - \varepsilon_2 \frac{\partial\phi^{(d)}}{\partial z}\hat{z}\right) = 0$$

$$\varepsilon_1 q + \varepsilon_1 q_b = \varepsilon_2 q_a \qquad (2)$$

Combining (1) and (2) gives

$$q_a = \frac{2q}{\left(\frac{\varepsilon_2}{\varepsilon_1}+1\right)} \quad \text{and} \quad q_b = q\frac{\left(\frac{\varepsilon_2}{\varepsilon_1}-1\right)}{\left(\frac{\varepsilon_2}{\varepsilon_1}+1\right)}$$

2. Consider a microstrip line with width 50 mils and dielectric height 21 mils. Since the strip is relatively wide, you can ignore the fringing fields and model the microstrip as a parallel metal plate waveguide. The dielectric is of FR4 material (ε_r = 4.5).



Problem 2

(a) What is the highest order mode that can propagate in the microstrip at a frequency of

i) 50 GHz TEM or TM_0 mode

h=21 mils=0.5334 × 10^{-3} m

$$v_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{0.3\times10^{+9}}{\sqrt{4.5}} = 0.1414\times10^{+9}\ m/s$$

$$f_{CTE1} = \frac{v_p}{2h} = \frac{0.1414\times10^{+9}}{2\times0.5334\times10^{-3}} = 135\,GHz$$

ii) 150 GHz ➔ TEM, TE_1, TM_1 modes

(b) What is the phase velocity of the lowest order propagating mode at

i) 50 GHz $v_p = 0.1414\times10^{+9}\ m/s$

ii) 150 GHz $v_p = 0.1414 \times 10^{+9}\ m/s$

(c) What is the phase velocity, of the highest order propagating mode at

i) 50 GHz $v_p = 0.1414 \times 10^{+9}\ m/s$

ii) 150 GHz $$v_{pz} = \frac{v_p}{\sqrt{1-\frac{f_C^2}{f^2}}} = \frac{0.1414\times 10^{+9}}{\sqrt{1-\frac{135^2}{150^2}}} = \frac{0.1414\times 10^{+9}}{\sqrt{1-\frac{135^2}{150^2}}} = 0.3244\times 10^{+9}\ m/s$$

(d) From your analysis, what is the bandwidth of the microstrip line?

The bandwidth of the microstrip line is about 135 GHz. Above that frequency, it cannot be used to support a single mode of propagation.