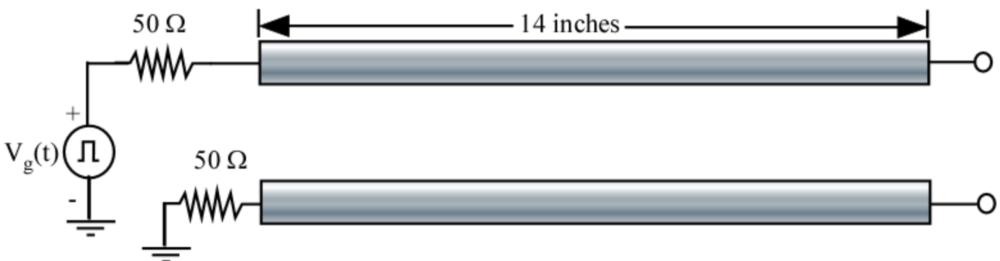
ECE 546 HOMEWORK No 3 SOLUTIONS

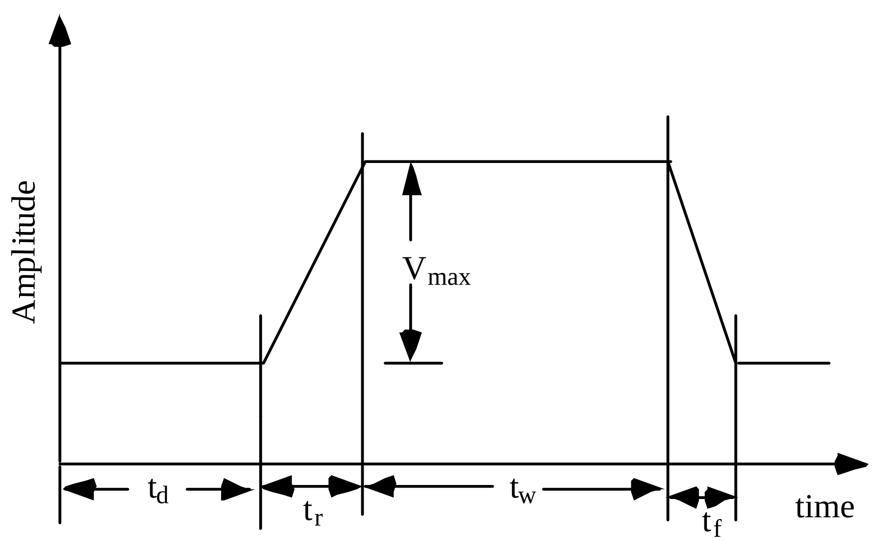
1. Write a program to simulate the response of a lossless coupled-line system with the following L and C matrices:

$$L = \begin{bmatrix} 418 & 150 \\ 150 & 418 \end{bmatrix} (nH/m) \qquad C = \begin{bmatrix} 0.118 & -0.022 \\ -0.022 & 0.118 \end{bmatrix} (nF/m)$$

Test your program using the example shown below. Plot the waveforms at the near and far ends of the sense line.



The pulse characteristics for $V_g(t)$ are as shown in the figure below, with time delay: t_d = 1 ns, rise time: t_r = 1 ns, fall time: t_f = 1 ns, pulse width: t_w = 20 ns, pulse amplitude: V_{max} = 1 volt



Matlab Code

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%This program simulates the response of a coupled line system
% J. Schutt-Aine
%3/11/2009
clear all;
```

```
%-------------------Input Section----------------------
n=2; % Number of lines
lself=418.0; %self inductance(nH/m)
lmut=150.0;% direct mutual capacitance(nH/m)
cself=96.0*0.001; % self capacitance (nF/m)
cmut=22.0*0.001; % mutual capacitance (nF/m)
len=.3665; % length of lines (meters)
    ax=1.0; % pulse delay in ns
    ris=1.0; % pulse rise time in ns
    fal=1.0; % pulse fall time in ns
    wid=20.0; % pulse width in ns
    magn=1.0; % pulse magnitude (V)
    rdload=100000; %load resistance (ohms)
    rdsource=50.0; % source resistance in ohms (Thevenin)
mh=400; % number of simulation points
dur=40.0; % duration of simulation (ns)
%---------------------end-------------------------
cap=[0.118,-0.022;-0.022,0.118];
lind=[418.0,150.0;150.0,418];
zs=[50,0;0,50];
zl=[1000,0;0,10000];
mag(1)=1.0;
mag(2)=0.0;
tstep=dur/mh; % time step
un=eye(n);
% Setting up L and C matrices */
         clm=cap*lind; % calculate CL product
         lcm=lind*cap; % calculate LC product
         clmt=clm;
         lcmt=lcm;
         [eor,eival2]=eig(lcmt); % voltage eigenvalues and eigenvectors
         [hor,eival2]=eig(clmt); % voltage eigenvalues and eigenvectors
eival=eival2^0.5; % eigenvalues of lossless system
         hori=inv(hor); % inverse of current eigenvector (lossless system)
         eori=inv(eor); % inverse of voltage eigenvector (lossless system)
eivali=inv(eival);
zc=eori*eivali*eor*lind;
zci=inv(zc);
ng1=(un+eor*zs*zci*eori);
disp(ng1);
t1=inv(ng1);
gm1=-t1*(un-eor*zs*zci*eori);
ng2=(un+eor*zl*zci*eori);
t2=inv(ng2);
gm2=-t2*(un-eor*zl*zci*eori);
eori=inv(eor);
       h=tstep;
       c9=mh;

for row=1:n
a1h(row,1)=0.0;
a2h(row,1)=0.0;
end
% start time loop for simulation
for cw=1:c9
        c=cw+1;
        tim=cw*tstep; % define time point
        time(c)=tim;
for col=1:n
        ax=1.0;   % pulse parameters
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        ris=1.0;
        fal=1.0;
        wid=20.0;
zet=Sub_timecomp(tim,ax,ris,fal,wid); % get pulse amplitude value
      vs(col)=zet*mag(col); % near-end voltage source vector
      vfar(col)=0.0 ; % far-end voltage source vector
end
g1=eor*vs';
g2=eor*vfar';
bx1= memory(tim,tstep,a2h,eival,len,n);
bx2= memory(tim,tstep,a1h,eival,len,n);
b1=bx1';
b2=bx2';
a1=t1*g1+gm1*b1;
a2=t2*g2+gm2*b2;
vm1=a1+b1;
vm2=a2+b2;
v1=eori*vm1;
v2=eori*vm2;
vn1(c)=v1(1);
vn2(c)=v1(2);
vf1(c)=v2(1);
vf2(c)=v2(2);
for row=1:n
a1h(row,c)=a1(row);
a2h(row,c)=a2(row);
end
end
%plot(time,vn1,'r',time,vn2,'b',time,vf1,'g',time,vf2,'k');
plot(time,vn2,'r',time,vf2,'b');

%-------------------------------------
function r= memory(t,h,a,e,l,n)
        for row=1:n
        dely=e(row,row)*l;
        argm=t-dely;
        if (argm < 0)
        r(row)=0.0;
        else
        argu=ceil(argm/h+1);
        r(row)=a(row,argu);
        end
        end
%-----------------------------------------------
function att=Sub_timecomp(t,a,ris,fal,wid)
% This subroutine calculates waveform of a pulse
b = a + ris;
c=b + wid;
d=c + fal;
p = t;
if (p < a)
    att=0;
   else if (p < b)
           att=1/(b - a)*(p - a);
          else if (p < c)
                  att=1;
               else if (p < d)
                       att = 1/(c-d)*(p-d);
                   else att = 0;
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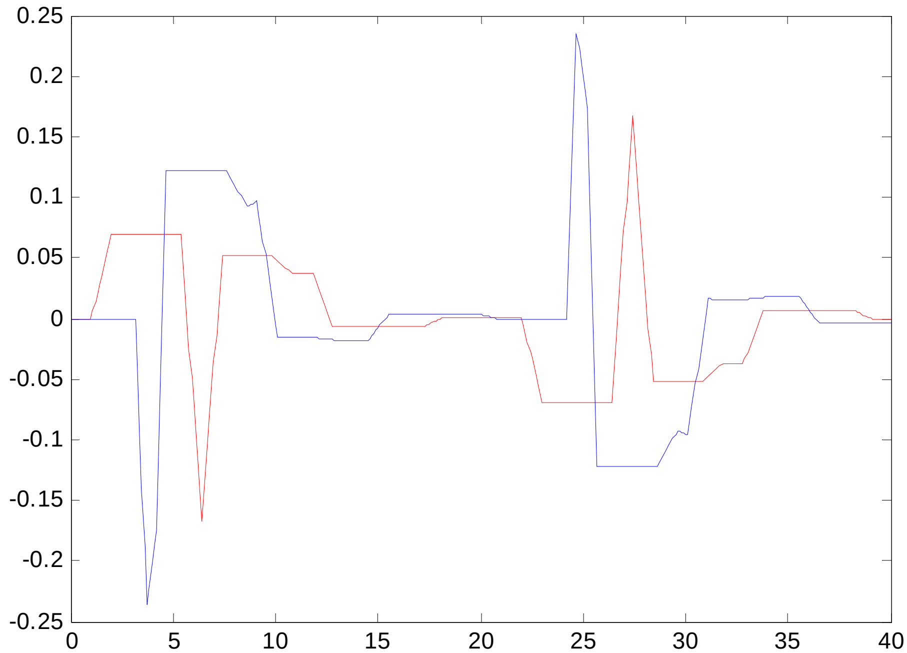
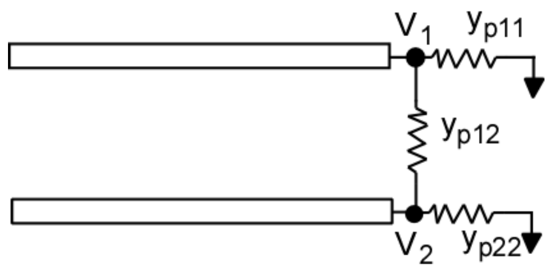
            end
          end
       end
    end
```



2. Using your knowledge of multiconductor transmission lines, determine the load network that will provide an exact match to a coupled line system in terms of the even and odd-mode impedances. Compare your solution with the network in Dally & Poulton Figure 3-24 (b) (section 3.4.3). Will this network provide an exact match? Verify your results using your code from Problem 1.



Consider the termination network above. The nodal equations yield:

$$I_1 = y_{p11}V_1 + y_{p12}(V_1 - V_2) \qquad I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{p22}V_2 + y_{p12}(V_2 - V_1) \qquad I_2 = y_{21}V_1 + y_{22}V_2$$

In order to obtain equivalence between these two formulations, we must have:

$$y_{12} = y_{21} = -y_{p12} \qquad (1)$$

$$y_{11} = y_{p11} + y_{p12} \qquad (2)$$

$$y_{22} = y_{p22} + y_{p12} \qquad (3)$$

The objective of the problem is to determine y_{p11}, y_{p22} and y_{p12}

We can see from (1)-(3)

$$\boxed{\begin{aligned} y_{p12} &= -y_{12} \\ y_{p11} &= y_{11} - y_{p12} = y_{11} + y_{12} \\ y_{p22} &= y_{22} - y_{p12} = y_{22} + y_{12} \end{aligned}}$$

It is best to work with admittances.

For an exact match, we want the load admittance $Y_L = Y_c$ where $Y_c = Z_c^{-1}$ is the line admittance matrix.

$$\text{Since } Z_c = E^{-1} Z_m H$$

$$Y_c = H^{-1} Y_m E^{-1}$$

$$Y_m = \begin{pmatrix} Y_{even} & 0 \\ 0 & Y_{odd} \end{pmatrix} \text{ with } Y_{even} = \frac{1}{Z_{even}} \text{ and } Y_{odd} = \frac{1}{Z_{odd}}$$

$$E = H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \text{ and } H^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\text{Thus } Y_c = \frac{1}{2}\begin{pmatrix} Y_{even} + Y_{odd} & Y_{even} - Y_{odd} \\ Y_{even} - Y_{odd} & Y_{even} + Y_{odd} \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}$$

$$\boxed{\begin{aligned} &\textbf{Resistor to ground} = \frac{1}{y_{p11}} = \frac{1}{y_{11} + y_{12}} = Z_{even} \\ &\textbf{Mutual resistor} = \frac{1}{y_{p12}} = -\frac{1}{y_{12}} = 2\frac{Z_{even} Z_{odd}}{Z_{even} - Z_{odd}} \end{aligned}}$$