On the Accuracy of Cross-Talk Modeling in High-Speed Digital Circuits Using the Accelerated Boundary Element Method

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***Abstract* — This paper presents an investigation on how numerical thresholds impact the accurate modeling of cross-talk phenomena for typical interconnect structures in the context of the accelerated boundary element method. Furthermore, a canonical scenario is presented to expose the noise floor of the methodology and its relation to the accuracy controls involved in the solver.**

Keywords—Cross-talk, fast iterative solvers, noise floor.

I. INTRODUCTION

Modern trends in electronic design call for 3D full-wave modeling of the interconnects in integrated circuit packages and printed circuit boards. Moreover, the co-existence of high-speed digital interconnects and noise-sensitive RF structures mandate an accurate characterization of cross-talk down to a very low dB level. The boundary element method is especially suited for accurate characterization of this type of phenomenon since it does not require the finite discretization of the space between the aggressor and the victim device. On the other hand, traditional techniques using direct solvers are limited by their prohibitive cost, mandating the need for fast-iterative solvers to analyze complex structures. In this paper, we consider a few typical scenarios in interconnect analysis where we need to accurately model low dB cross-talk. Depending on the coupling mechanism, we demonstrate how different aspects of error control mechanisms in an accelerated boundary element solver play a significant role.

II. METHOD OF MOMENTS

The Boundary Element (BE) or Method of Moments (MoM) is a widely used numerical algorithm for boundary element based electromagnetic analysis of Integrated Circuits (ICs) and Printed Circuit Boards (PCBs) [1]. The MoM, in conjunction with the Dyadic Green's Functions (DGFs), allows numerical modeling of ground planes, dielectric interfaces, and the surrounding environments without explicitly discretizing them.

The spatial domain DGFs can be expressed in the form of a Sommerfeld Integral (SI) as reported in (1):

$$G^{EJ}(\rho, z, z') = \frac{1}{2\pi}\int_0^{\infty} \tilde{G}^{EJ}\left(k_\rho, z, z'\right) \cdot J_n\left(k_\rho \rho\right) k_\rho^{n+1} dk \quad (1)$$

where ρ, z, z' and J_n are the locations of the source, observation points in a cylindrical coordinate system, and the *n-th* Bessel function of the first kind, respectively. The SI does not have a closed-form and its numerical solution presents some challenges [2]. Over the last decade, several approaches have been proposed to speed-up the SI calculation [3],[4]. It is worthwhile to emphasize that an optimal compromise must be found between accuracy and numerical efficiency.

Once the Dyadic Green's Functions have been evaluated, a matrix equation is solved for the weight coefficients associated with the basis functions used to represent the induced currents. Since the majority of the DGF terms are characterized by a singular behavior when the distance between source and observation point becomes small; the related matrix elements require a special numerical treatment [5]. It is then evident how an additional source of inaccuracy in the Method of Moments can be identified in the finite quadrature orders and singularity extraction techniques required to evaluate the reaction integrals.

The application of the conventional MoM formulation with subsectional basis functions becomes quite inefficient when the structure is electrically large [1], or geometrically complex such as interconnects in IC packages or printed circuit boards. This, in turn, increases both the MoM matrix generation time and *LU* factorization, which present a quadratic and cubic complexity, respectively. A variety of iterative approaches have been deeply investigated over the last decade [6],[7]. These "fast-solver" techniques, able to reduce the setup and solve time complexity to *O(NlogN)* in their multilevel form, are based on an advantageous representation of the far-field interactions, in conjunction with the use of dedicated iterative solvers. However, a numerically efficient error control mechanism is necessary, both for the iterative solution of the associated linear system of equations, as well as for far-field calculations, to guarantee good accuracy and reasonable solution time.

III. BOUNDARY ELEMENTS AND CROSS TALK

When considering MoM in conjunction with DGFs, all the employed unknowns are associated with current density induced over the conducting structure. Accurate characterization of cross-talk between two traces depends on

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how rigorously the spatial variation of the electric and magnetic field is captured in the space between the two conductors, which is modeled analytically in the Green's function without any explicit discretization of the domain.

With no loss of generality, let us consider the cross-talk between the two differential via transitions shown below (Fig. 1). As we can see, other than the traces themselves, the ground planes are also discretized with a coarser and finer mesh. It is worthwhile noticing how the far (FEXT) and near end cross-talk results (NEXT) are not sensitive to the discretization of the ground planes between the two differential pairs, as long as there is a sufficient amount of mesh elements below the traces to capture the return current accurately.

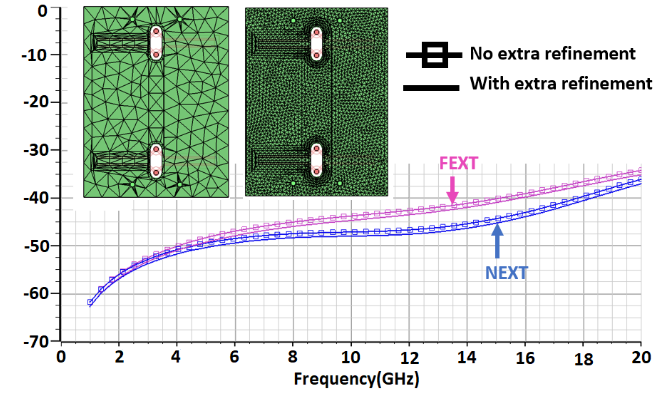

Fig 1. Cross-talk coupling for the differential via transitions.

To further demonstrate the mentioned advantage associated with the boundary element method, we present the field variation between two differential strip-lines (Fig. 2). It is remarkable noticing how even if there is no additional discretization for the ground planes between the differential traces, a smooth variation both for the electric field (b) as well as for the magnetic field (c) is accurately captured.

IV. NUMERICAL RESULTS

A. Different cross-talk creation and mitigation mechanisms

To study the impact of possible numerical tolerances (section-II) on cross-talk calculations, a few accuracy settings have been summarized in Table I with a progressively conservative target. A saturated configuration, '**Ref**', is assumed to be the golden standard for this convergence study.

Three different mechanisms of low-level cross-talk creation are investigated in this paper. Table II summarizes the required settings to achieve convergence at a certain dB level.

Case 1: ***Low far-end cross-talk due to electrically large interconnects***. For this experiment, we have analyzed two pairs of 12'-long differential microstrip lines up to 50 GHz. In this scenario, the FEXT, due to the conduction and roughness loss mechanism, is very low and needs accurate modeling to maintain adequate isolation at the far end. We find that this scenario requires relatively less stringent error thresholds in quadrature, far-field threshold, and iterative solver (Table II), because here the cross-talk is low due to high loss in the conductor, which at high frequency is modeled with surface impedance model, that accurately does not need to stress the numerical thresholds significantly.

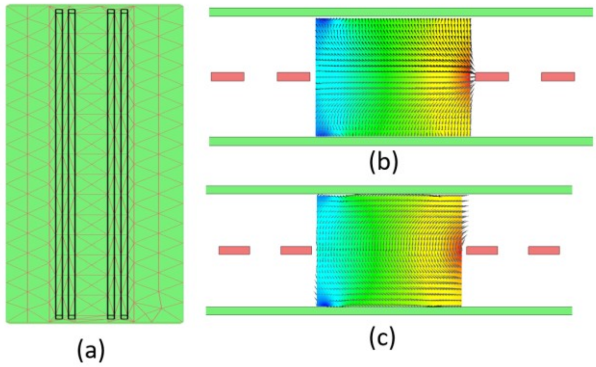

Fig 2. Strip-line differential pair geometry (a), electric field (b), and magnetic field (c) in the space between the differential traces.

Table I. Accuracy settings in Fast Boundary Element Solver

Setting	Quadrature order	Far-field interaction threshold	Iterative solver tolerance
I	1	0.005	$1e^{-3}$
II	2	0.005	$1e^{-3}$
III	2	0.001	$1e^{-4}$
IV	2	0.0005	$1e^{-5}$
V	2	0.0001	$1e^{-6}$
Ref	3	0.00001	$1e^{-8}$

Case 2: ***Cross-talk mitigation by spatial separation.*** In particular, we increase the trace separation distance in parallel and broadside coupled traces to achieve low level of cross-talk. As shown in Table II, more stringent error control is required in this case since the low-value cross-talk is associated with far-field interactions between mesh elements or coupling over a small segment of the traces.

Case 3: ***Cross-talk mitigation obtained with isolation planes or stitching vias***. As can be noticed from Table II, the most stringent error control is required in this case. With no loss of generality, let us consider three mesh elements: A, B, C defined over the aggressor, the shielding object, and the victim, respectively. In particular, the mesh element A creates a direct scattered field over C through the Green's function, but at the same time, induces a current on B which, in turn, creates a "nearly" equal and opposite scatterd field on C that has to cancel the previously described contribution from A.

Finally, it is then crucial to quantify the relative cost of the described accuracy settings for more complex scenarios. We have then analyzed three different projects: a 10 layer package with 13 ports discretized with ~100k mesh elements (Project 1), a 10 layer PCB with 20 ports discretized with ~150k mesh elements (Project 2), and a 40 layer PCB with 16 ports and ~150k mesh elements (Project 3). The obtained results are summarized in Table III. It is worthwhile noticing how no more than 3x slow-down is generally required by the more conservative settings.

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Table II. Summary of the cross-talk study observations.

Case	-40 dB	-60 dB	-80 dB
FEXT for electrically long lines	I	I	III
Broadside coupling with vertical separation	I	II	III
Differential traces isolated by horizontal separation	I	II	III
Differential vias isolated by stitching vias	I	III	IV
Differential vias isolated by shielding planes	I	III	V

Table III. Relative time penalty as a function of the different accuracy settings for three realistic scenarios.

	Relative Time Penalty		
Setting	**Project 1**	**Project 2**	**Project 3**
I	1.0x	1.0x	1.0x
II	1.4x	1.6x	1.3x
III	1.7x	2.1x	2.1x
IV	2.0x	2.3x	2.8x
V	2.5x	2.8x	6.3x

B. In-pair coupling benchmark

In this section, we will investigate the noise floor associated with a boundary element method by analyzing the differential strip-line shown in Fig. 3. Since the transmission line is a symmetrical structure with homogeneous dielectric, no differential to common mode conversion is expected. The differential to common return/insertion loss can be then considered to be a pure numerical artifact. In particular, four differential strip-line configurations with different coupling levels, namely High Coupling (HC), Medium Coupling (MC), Low Coupling (LC), and Uncoupled (UC) have been analyzed by using HyperLynx Full-Wave Solver [9] and the accuracy setting II (Figs. 3). The authors refer to [10] for all of the physical and electrical dimensions of the different transmission lines. It is remarkable noticing how the noise level is bounded at -60 dB uniformly for all the test cases. Please note that these numerical inaccuracies can be further reduced by increasing the requested level of solution accuracy. To prove that, a lower noise floor is achieved by tuning the accuracy settings to level IV (see Fig. 4). In this case, a -80 dB noise floor level is obtained for the differential to common insertion loss.

V. CONCLUSIONS

The accelerated MoM is a powerful tool to accurately model coupling phenomena down to a very low decibel range. Capturing low-level cross-talk for various scenarios common in modern designs requires different numerical tolerances for some key components of the solver depending on the coupling and isolation mechanism. For a given scenario, it is possible to reduce the noise floor by using more conservative settings in gradual cost vs. accuracy trade-off. Numerical control required to model low level cross-talk accurately, depends on the mechanism of how that low level cross-talk is created.

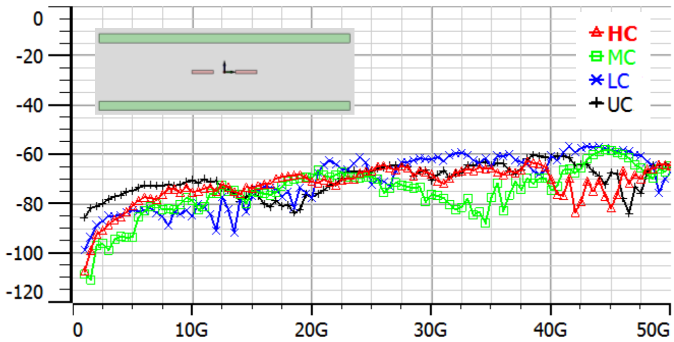

Fig 3. Differential to common return loss conversion.

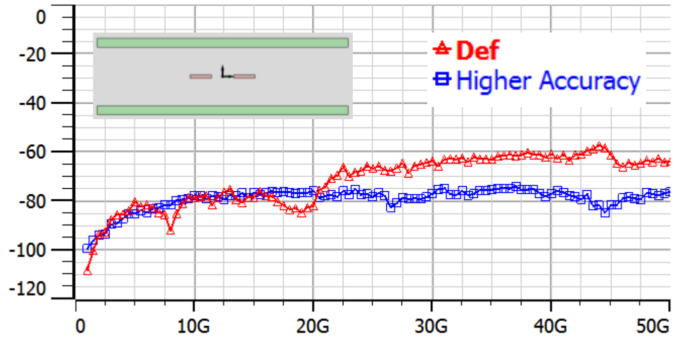

Fig 4. Differential to common insertion loss conversion with higher accuracy level for the HC configuration.

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