Causal Transmission Line Geometry Optimization for Impedance Control in PCBs

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***Abstract*—PCB transmission lines are designed with broadband impedance matching using differential evolution while including causal dispersion and roughness. Results for striplines show that an optimized geometry can be found with differential evolution.**

***Index Terms*—Board-level interconnects, methodologies and algorithms for modeling, simulation and optimization.**

I. Introduction

TRANSMISSION line behavior is commonplace in printed circuit boards (PCBs) operating with ultrahigh speed digital signals (~10 GHz bandwidth) and analog signals with mmWave frequencies. Dispersion is problematic at these frequencies, and minor impedance mismatches along an interconnect will produce reflections and distortion in various portions of the signal bandwidth. In addition, roughness of copper conductors on PCB laminates creates an additional source of dispersion and losses [1]. These problems of dispersion and roughness lead to signal distortion. A proper description of signal behavior requires enforcing causality in descriptions of dispersion in the substrate's dielectric function $\varepsilon(\omega)$ and copper roughness using Kramers-Kronig relations [2], which is required in equivalent circuit models describing interconnects, e.g., in PCB transmission lines for many high speed standards [2] (e.g., USB 4.0) and 100 Gb/s Ethernet in the IEEE P802.3bj Task Force proposal [3].

A transmission line is designed by adjusting its geometry in CAD software so that its impedance at a specific frequency takes a target value. This method does not consider broadband impedance matching, losses, velocity dispersion, and/or signal distortion. Solving such a design problem is difficult as any change in the line's per-unit-length (p.u.l.) capacitance, inductance, or resistance must be compensated by a change in one of the other quantities, each of which is related to the line's impedance. A simpler method is to use a existing model for the characteristic impedance and incorporate dispersion and roughness throughout the signal bandwidth analytically, which forms a complex nonlinear optimization problem.

As a class of heuristics, evolutionary algorithms can be used to solve nonlinear optimization problems in electromagnetics [4]. This class of algorithms uses iterative solution generation techniques to search a solution space for the global optimal solution. Multi-objective problems can be addressed by solving one objective while holding the others as constraints, and a Pareto surface can be generated through successive iteration.

Previous work has focused on determining circuit model values or dielectric properties using measured S-parameters [2] rather than optimizing the line's geometry. Therefore, a method for designing PCB transmission lines with dispersion and roughness to a target impedance is presented in this paper. Differential evolution was used to determine the geometry that minimizes deviations from a target impedance throughout the signal bandwidth. Analytical expressions for the characteristic impedance of a stripline and a causal model for $\varepsilon(\omega)$ were used to minimize deviations from a target impedance while including dispersion and roughness up to 20 GHz [2, 5]. The method can be applied to other geometries, used with other optimization methods, or reformulated as a multi-objective problem to balance many signal integrity metrics. Some additional objectives include signal distortion metrics, S-parameter values, and crosstalk in coupled transmission lines. The procedure could also be used in modern CAD applications for accurate transmission line design over a broad bandwidth.

II. Theoretical Model and Methods

A. Causality, Dispersion, and Copper Roughness

Building a causal model for a PCB interconnect requires enforcing causal representations for $\varepsilon(\omega)$, conductor roughness, and the line's transfer function [1, 2]. Typical causal dispersion models for PCB substrates are the Lorentzian or wideband Debye models [2]. The RLGC(*f*) model can be used to describe a line's characteristic impedance in terms of lumped circuit elements with dispersion as long as $\varepsilon(\omega) = \varepsilon_R(\omega) + i\varepsilon_I(\omega)$ has a causal representation. The following causal Lorentzian model for striplines on FR4 is valid up to 20 GHz [2, 5]:

$$\begin{aligned}\varepsilon_R(\omega) &= \left(\varepsilon_\infty + \frac{\varepsilon_{s1}-\varepsilon_\infty}{1+(\omega\tau_1)^2} + \frac{\varepsilon_{s2}-\varepsilon_\infty}{1+(\omega\tau_2)^2}\right)\varepsilon_0 \\ \varepsilon_I(\omega) &= \omega\left(\frac{(\varepsilon_{s1}-\varepsilon_\infty)\tau_1}{1+(\omega\tau_1)^2} + \frac{(\varepsilon_{s2}-\varepsilon_\infty)\tau_2}{1+(\omega\tau_2)^2}\right)\varepsilon_0.\end{aligned} \tag{1}$$

Typical values of ε_{s1}, ε_{s2}, ε_∞, τ_1, and τ_2 can be found in [5]. The conductor's RMS roughness H_{RMS} modifies the dielectric constant to $\varepsilon(\omega) \rightarrow \frac{\varepsilon(\omega)T}{T-2H_{RMS}}$ [1]. Dispersion can be placed in

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standard equations in the causal RLGC(*f*) model [2] and analytical equations for transmission line impedance determined using conformal mapping [6].

Rather than assume all circuit parameters in the causal RLGC(*f*) model are known *a priori*, the characteristic impedance $Z_0(\omega)$, DC resistance R_0, and causal $\varepsilon(\omega)$ are considered known in the method shown here. These are used to determine the geometry that most closely matches the line impedance with dispersion and roughness $Z(\omega)$ to a target interconnect impedance Z_T throughout the relevant bandwidth.

In the causal RLCG(*f*) model, the characteristic impedance $Z_0 = \sqrt{\frac{R+i\omega L}{G+i\omega C}}$ is defined in terms of lumped circuit elements [2]:

$$\begin{array}{ll} R(\omega) = R_0 + \sqrt{\omega}R_s & L(\omega) = L_\infty + \omega^{-1/2}R_s \\ G(\omega) = \omega C(\omega)\varepsilon_0 \tan\delta(\omega) & C(\omega) = K_g \varepsilon_0 \varepsilon_R(\omega) \end{array}. \quad (2)$$

where R_s is the p.u.l. skin-depth resistance, K_g is a geometry factor, and $\tan\delta(\omega) = \frac{-\varepsilon_I(\omega)-\sigma_{sub}}{\varepsilon_R(\omega)}$ is the frequency-dependent loss tangent [2, 5]. Note that the conductance G_0 of typical PCB substrates is $G_0 \sim 10^{-11}$ S/m, so G_0 and σ_{sub} are ignored in $\tan\delta$. L_∞ is the p.u.l. inductance as $\omega \to \infty$ and is taken as a constant as PCB laminates are non-magnetic [2]. These circuit parameters are related to the characteristic impedance as follows:

$$Z_0(\omega) = \sqrt{\frac{[R(\omega)+i\omega L(\omega)]}{\omega C(\omega)(i+\tan\delta(\omega))}}. \quad (3)$$

where the propagation constant is written as $\gamma(\omega) = \sqrt{C(\omega)[\omega R(\omega) + i\omega^2 L(\omega)][i + \tan\delta(\omega)]}$.

Accounting for copper roughness on the PCB laminate is accomplished by using a causal roughness correction factor $K(\omega)$. This is incorporated into $R(\omega)$ by applying the linear transformation $R_s \to K(\omega)R_s$. Causal models for $K(\omega)$ (e.g., Hammerstad and Cannonball-Huray models) are found in [1].

B. Geometry Optimization with Differential Evolution

Dispersion and roughness were considered while designing to a target characteristic impedance using the L^2 norm of $Z(\omega) - Z_T$ in ω as the objective function:

$$\min\left[\int_{\omega_1}^{\omega_2} \left|\left|Z(\omega) - Z_T\right|\right|^2 d\omega\right]^{0.5}, \quad \text{subject to: } J < W/H < K, \quad (4)$$

This formulation is equivalent to minimizing the mean-squared error between $Z(\omega)$ and Z_T. Let W, T, and H be the width, thickness, and distance to the reference plane, respectively; these are the variables used to minimize $L^2(Z(\omega) - Z_T)$, and J and K are constants. The number of variables is reduced from 3 to 2 when W/H and T/H are used as optimization variables. T/H is normally fixed based on the PCB stackup, which reduces the number of variables to 1. This ensures the line is designed within practical manufacturing constraints on laminate thickness and copper weight.

Algorithm 1 shows pseudocode for the differential evolution algorithm used to solve (4). Differential evolution proceeds by randomly generating a feasible initial solution; trial solutions are generated randomly, and the trial solution that moves the objective function value closer to an optimum in the solution space is accepted as the new solution. Constraint checking for inequalities is implemented using the ConstraintCheck and ConstraintMod functions [7]. GenerateNew mutates the current W/H value using the standard algorithm in [8].

Algorithm 1: Differential evolution pseudocode

Input: K, J, Z_T, N_{max}
00. **While** $N < N_{max}$
01. **Generate initial solution** $Z(\omega)$ and W/H
02. **If** $N < N_{max}$
03. GenerateNew W'/H' and $Z'(\omega)$
04. **If** $L^2(Z'(\omega) - Z_T) < L^2(Z(\omega) - Z_T)$ **and** ConstraintCheck = True
05. $Z(\omega) = Z'(\omega)$, $N = 0$
06. **Go to 02**
07. **Else**
08. ConstraintMod
09. $N \to N + 1$
10. **Go to 03**
11. **Else**
12. **End**

Algorithm 1 was executed in Python 3.7 on a PC with a 1.8 GHz quad-core processor and 8 GB RAM. The time required to solve this problem primarily depends on the level of discretization used in the L^2 norm; 400 data points from 0.01 to 20 GHz were used in this method.

III. Example for a Long Stripline

In this section, the method outlined in Section II is applied to the stripline shown in Figure 1. Striplines with and without dispersion and roughness will be compared here. The stripline without dispersion and roughness has $\varepsilon = 4.300 + i0.0681$, which is taken at 5 GHz. On the rough line, $H_{RMS} = 0.5$ μm was used with average particle size of 0.2 μm. Wadell's equation [6] for a stripline's characteristic impedance $Z_0(\omega)$ is shown in (5):

$$Z_0(\omega) = \frac{60}{\sqrt{\varepsilon_R(\omega)}} \ln\left(1 + \left(\frac{8H}{\pi\widetilde{W}}\right)\left[\left(\frac{16H}{\pi\widetilde{W}}\right) + \sqrt{\left(\frac{16H}{\pi\widetilde{W}}\right)^2 + 6.27}\right]\right). \quad (5)$$

In (5), $\widetilde{W} = W + \left(\frac{T}{\pi}\right)\left[1 - \frac{1}{2}\ln\left(\left(\frac{T}{4H+T}\right)^2 + \left(\frac{\pi(T/H)}{4(W/H + 1.1T/H)}\right)^m\right)\right]$ and $m = 6H/(3H + T)$ [7].

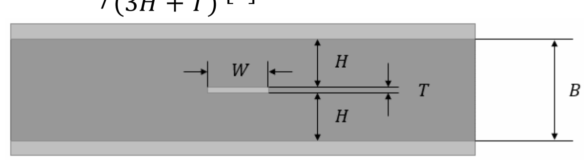

Fig. 1. Stripline transmission line on FR4 showing the definition of the geometric parameters in (5).

Using (3), (5), and the phase velocity $\left(\frac{\omega}{\gamma(\omega)}\right)$ with $R(\omega) = 0$, one can derive equations for $Z(\omega)$ and $\gamma(\omega)$ that include causal dispersion and copper roughness with linear transformations:

$$Z(\omega) = \sqrt{\frac{i\omega Z_0^2\sqrt{\varepsilon_R(\omega)} + Z_0 c_0\left(R_0 + (1+i)K(\omega)R_S\sqrt{\omega}\right)}{i\omega\sqrt{\varepsilon_R(\omega)}(1 - i\tan\delta)}}, \tag{6a}$$

$$\gamma(\omega) = \sqrt{\frac{-\omega^2 Z_0\varepsilon(\omega) + i\omega c_0\sqrt{\varepsilon_R(\omega)}(1 - i\tan\delta)\left(R_0 + K(\omega)(1+i)\sqrt{\omega}R_s\right)}{Z_0 c_0^2}}, \tag{6b}$$

where c_0 is the speed of light in vacuum. These equations are applicable to any transmission line if Z_0 and $\varepsilon(\omega)$ are known. In (6), $R_s \approx \sqrt{\frac{\mu_0}{8\sigma(T+W)^2}}$, and $K(\omega)$ obeys the Cannonball-Huray model [1]. Note that $K(\omega)$ could be defined using any other copper roughness model to match the line's morphology.

For the stripline on FR4 shown in Fig. 1, Table 1 shows the values for the parameters in (1) and the conductivity of copper.

TABLE I
CAUSAL TWO-TERM LORENTZIAN MODEL PARAMETERS

ε_{s1}	ε_{s2}	ε_{∞}	τ_1 (ps)	τ_2 (ps)	σ ($\Omega^{-1}m^{-1}$)
4.081	4.068	3.95	82.12	5.712	$5.81\cdot10^7$

IV. RESULTS

Fig. 2 shows the impedance spectra of two striplines (25 cm length) designed to a target impedance Z_T = 50 Ω. The load capacitance is $C_L = 1$ pF with parallel termination at its target impedance; these values collectively determine the impedance seen at the input to the load, which then determines the return loss (S_{11}). The optimized geometry is W = 0.1774 mm (6.983 mil) for the rough, dispersive stripline, and W = 0.1741 mm (6.856 mil) for the smooth, non-dispersive stripline. The line is placed on an 8-layer PCB stackup with H = 0.224 mm and T = 17.5 µm (0.5 oz./sq. ft. copper weight).

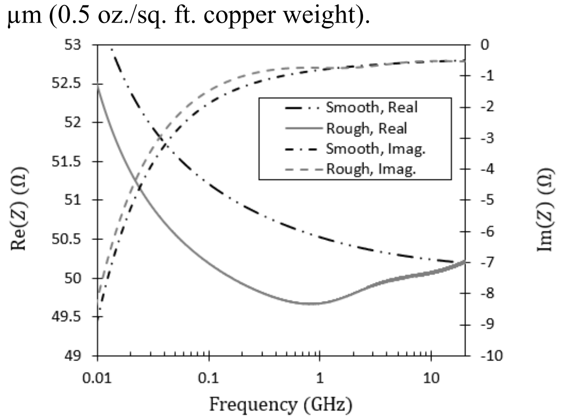

Fig. 2. Optimized impedance spectra for striplines with and without roughness and dispersion.

The proposed procedure gives a design that provides low deviation from the target impedance, despite roughness and dispersion. For electrically short interconnects, return loss is the quantity of concern, while insertion loss is more critical in long interconnects. Fig. 3 compares insertion and return losses on both lines. One can see that making the rough stripline slightly wider provides slightly lower losses than the smooth stripline.

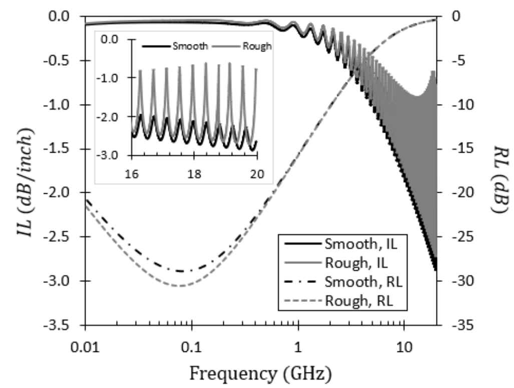

Fig. 3. Insertion loss and return loss in the terminated rough and smooth striplines (1 pF load capacitance). The inset shows a magnified view of insertion loss from 16 to 20 GHz.

Finally, the impulse function can be calculated using the line's causal transfer function for an arbitrary stimulus. Impulse responses for rough and smooth lines are compared in Fig. 4. The input pulse used here matches that used in [2] (see (46)).

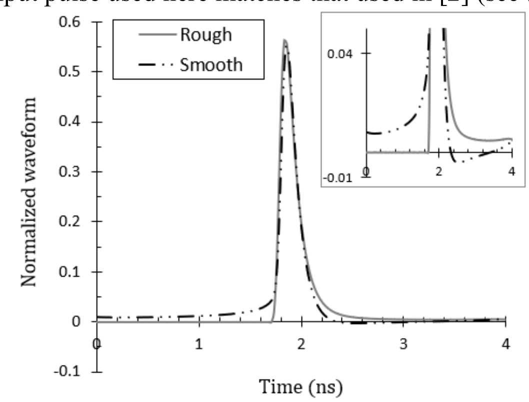

Fig. 4. Impulse responses for rough and smooth transmission lines. The inset shows a magnified view, revealing non-causal artifacts on the smooth transmission line.

By imposing design for manufacturing constraints, the designed stripline meets practical PCB fabrication requirements. Additional objectives could also be used in the L^2 norm in (4), such as wideband S-parameters or insertion loss.

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