Simple Extraction of the First Resonant Frequency via Integral Equation Method

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***Abstract*—The accurate determination of the first resonant frequency (FRF) is important for the design of electromagnetic (EM) devices and for establishing safety margins for their operation. However, the numerical extraction of the FRF typically involves a costly parametric sweep across operational frequencies. This article introduces a method for extracting the FRF based on energetic considerations and the PEEC scheme for the solution of EM problem. Despite its simplicity, the proposed approach significantly reduces the computational cost while maintaining a good accuracy compared to conventional methods. This advancement promises to enhance the efficiency and accuracy of FRF extraction in EM device characterization.**

***Index Terms*—integral equations (IEs), parasitic extraction, resonant frequency, partial element equivalent circuit (PEEC).**

I. Introduction

The extraction of equivalent electric circuit parameters of electromagnetic (EM) devices holds significant importance in defining concise SPICE models for the analysis and design of interconnects [1], printed circuit boards (PCB) [2], [3], inductive power transfer (IPT) devices [4], and electromagnetic interface (EMI) filters [5], [6]. A simple example of circuit representation for a power inductor is given in Fig. 1.

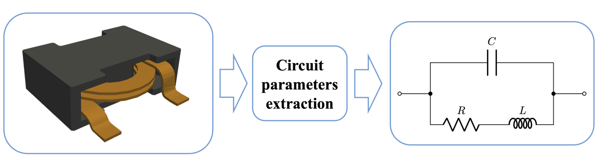

Fig. 1. Simple electric circuit representation of a power inductor.

However, except for elementary geometries for which analytical expressions of equivalent series resistance (ESR), inductance L, and capacitance C are available, in most cases, the parameter extraction is assisted by numerical simulation [7]. Despite the remarkable performance of the major commercial software,the accurate extraction of parameters with computed aided engineering (CAE) tools are still non-trivial.

The need to analyze open (radiation) boundary problems pushed toward the development of numerical methods avoiding the discretization (mesh) of the air/vacuum parts. The solution of the EM problem for example for interconnects, integrated circuit (IC) packages, or IPT devices is usually performed with integral equation methods (IEMs) such as the surface integral equation (SIE) [8], [9] and the volume integral equation (VIE) [10], [11], instead by using the standard FEM approach. It is worth mentioning that the main drawback of IEMs is the dense pattern of the arising matrices, which could rapidly saturate the computer memory. However, ad-hoc methods were proposed in the literature to reduce computational burden [12]. This article proposes a method for evaluating the first resonant frequency (FRF), based on energetic consideration and integral equation method for the solution of the EM problem, which, to the author's knowledge has never been proposed before. The method allows for a simple and rapid evaluation of the FRF, without the need of performing a computationally demanding parametric sweep.

The paper is organized as follows: in Section II the basic concept of VIE for the solution of the EM problem is given. In Section III the novel method for the evaluation of the FRF is described. In Section IV the capability of the method is numerically verified. The conclusions are given in Section V.

II. Electromagnetic Analysis

The starting point for the EM analysis in $\Omega \in \mathbb{R}^3$, based on the VIE method, neglecting the presence of magnetic materials, is the augmented electric-field integral equation (A-EFIE) in the frequency domain [13]–[17]:

$$\mathbf{E}(\mathbf{r}) = -i\omega\mathbf{A}(\mathbf{r}) - \nabla\varphi(\mathbf{r}) + \mathbf{E}_{inc}, \tag{1}$$

and the current continuity equation:

$$\nabla \cdot \mathbf{J}(\mathbf{r}) = -i\omega\varrho(\mathbf{r}), \tag{2}$$

where $\omega = 2\pi f$ is the angular frequency, $\mathbf{E}$ and $\mathbf{E}_{inc}$ the total and incident electric fields, $\mathbf{A}$ the vector potential, and φ the scalar potential. Vector and scalar potential are related to current density $\mathbf{J}$ and charge density ϱ through integral expressions [18]. Within conductive and dielectric media:

$$\mathbf{E} = \rho_c\mathbf{J}_c, \quad \mathbf{E} = \rho_d\mathbf{J}_d, \tag{3}$$

where ρ_c is the electric resistivity of the conductor, $\mathbf{J}_c$ the conduction current density, ρ_d the equivalent resistivity of dielectric, and $\mathbf{J}_d$ is the polarization current density [18].

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The numerical solution of (1)-(2) follows a Galerkin testing procedure where the computational domain Ω is subdivided into tetrahedral or hexahedral elements, and the unknown current density (conduction or polarization) and electric potential are expanded in terms of vector and pulse basis functions. This leads to the following system of equations:

$$\begin{bmatrix} \mathbf{Z} & \mathbf{D}^\top \\ \mathbf{PD} & i\omega\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{j} \\ \mathbf{\Phi} \end{bmatrix} = \begin{bmatrix} \mathbf{e}_{inc} \\ \mathbf{0} \end{bmatrix}, \tag{4}$$

where the array $\mathbf{j} = [\mathbf{j_c}; \mathbf{j_d}]^\top$ collects the currents degrees of freedom (DoFs) across faces of the mesh, while $\mathbf{\Phi} = [\mathbf{\Phi_c}; \mathbf{\Phi_d}]^\top$, is the array of electric potentials within each element. Subscripts $\mathbf{c}$ and $\mathbf{d}$ indicate that quantities are related to conductors and dielectrics, respectively. Usually, the A-EFIE is written in terms of charges, instead of potentials, recognizing that $\mathbf{\Phi} = \mathbf{Pq}$, and $\mathbf{q}$ is the array related to charges. It is worth noting that the structure of system (4) is the same derived by following the Partial Element Equivalent Circuit (PEEC) method [19], which allows a natural electric circuit interpretation of the EM problem. The PEEC system (4) allows the coupling with lumped circuit elements, for example, voltage excitation applied to model ports [18]. In (4), matrix $\mathbf{Z} = \mathbf{R} + i\omega\mathbf{L}$ is the impedance matrix, the sum of the sparse resistance matrix $\mathbf{R}$ and the dense inductance matrix $\mathbf{L}$, while $\mathbf{P}$ is the potential matrix. Expressions for the entries of such matrices can be found for example in [18]. The sparse matrix $\mathbf{D}$ is the incidence matrix between faces and volumes of the mesh, which can be viewed as the discrete form of the divergence operator [16].

III. First Resonant Frequency Evaluation

Extracting the first resonant frequency (FRF) f_{RF} is crucial for various reasons. It serves as an upper limit for operational frequency, as operating beyond f_{RF} often leads to unexpected device behavior. Additionally, f_{RF} knowledge enables defining a safety margin, accommodating parameter uncertainties. The FRF is formally defined as:

$$f_{RF} = \frac{1}{2\pi\sqrt{L_{eq}C_{eq}}}, \tag{5}$$

where L_{eq} and C_{eq} are the equivalent inductance and capacitance of the device. It is well known that f_{RF} can be extracted from the first root of $\arg(Z(f))$, where $Z(f)$ is the impedance seen at the device terminals. Typically, this is accomplished through a parametric sweep of frequencies or iterative techniques like the bisection method. However, these solutions impose a significant computational burden due to the extensive numerical computations required to solve the EM problem. Here, a novel method for the evaluation of f_{RF} is proposed. From the solution of (4) at the working frequency (which must be sufficiently lower than the FRF), it is possible to extract the capacitive (or electrostatic) energy W_e:

$$W_e = \frac{\mathbf{q_c}^*\mathbf{\Phi_c}}{4}, \tag{6}$$

where $\mathbf{q_c} = -\mathbf{D_c}\mathbf{j_c}/(i\omega)$ (i.e., the continuity equation (2) for conducting domains), and * is the conjugate transpose operator. Then, the equivalent capacitance can be found as:

$$C_{eq} = 4\frac{W_e}{|V_{vs}|^2}, \tag{7}$$

where V_{vs} is the value of the voltage source. Coefficient 4 is used since peak values are always assumed. From the same solution of (4) it is also possible to extract the equivalent inductance as:

$$L_{eq} = \frac{1}{\omega}\Im\Big(\frac{V_{vs}}{I_{vs}}\Big), \tag{8}$$

where I_{vs} is the current flowing in the voltage source. The inductance can be extracted also from the magnetic energy:

$$L_{eq} = 4\frac{W_m}{|I_{vs}|^2}, \tag{9}$$

where, in the discrete setting:

$$W_m = \frac{\mathbf{j_c}^*\mathbf{a_c}}{4}, \tag{10}$$

where $\mathbf{a}_c$ is the array related to the vector potential DoFs of the conductor and $\mathbf{a} = \mathbf{Lj}$ with $\mathbf{a} = [\mathbf{a_c}; \mathbf{a_d}]$. The extracted C_{eq} and L_{eq} are thus used for the evaluation of f_{RF} through (5). It is worth noting that the considerations above can also be extended to the presence of magnetic media by solving MFIE together with EFIE and introducing the contribution of the magnetization to the vector potential into $\mathbf{a}$. Note that it is not possible to extract C_{eq} from an electrostatic simulation since the coil is a single conductor, thus it is not possible to identify the two terminals required by an electrostatic capacitance extraction.

IV. Numerical Result

This section illustrates the accuracy of our proposed model for extracting the FRF by examining a near field communication (NFC) antenna operating at $f = 13.56$ MHz, as shown in Fig. 2. The coil, with trace width of $w = 0.5$ mm, is made of copper ($\sigma = 5.7 \cdot 10^7$ S/m), while the substrate of length $l = 27$ mm and thickness 0.12 mm is made of Polymide, with a complex permittivity $\varepsilon_r = 3.5(1 - i0.008)$. The voltage excitation at the terminals is $V_{vs} = 1$ V, and no external field is present $\mathbf{E}_{inc} = 0$. The FRF is extracted over the range

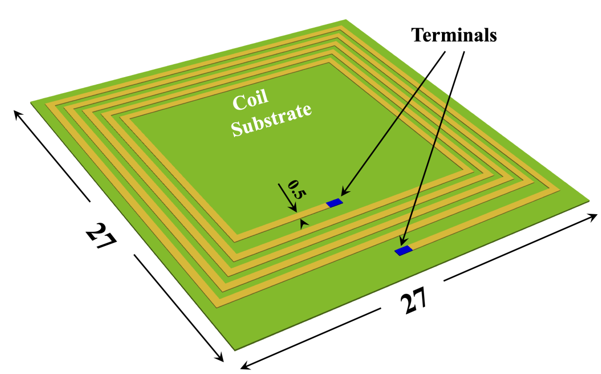

Fig. 2. CAD model of NFC antenna. Units in mm.

$[10, 300]$ MHz by using the iterative bisection method and the proposed approach. A parametric sweep was conducted to extract the trend of the equivalent impedance's module and phase, as shown in Fig. 3. Using iterative bisection, the FRF is at $f_{RF} = 215$ MHz; the proposed approach yields $f_{RF} = 218$ MHz, in good agreement with a $< 2\%$ relative error. Looking at Fig. 4, the proposed approach safely extracts f_{RF} up to one-tenth of the resonant frequency.

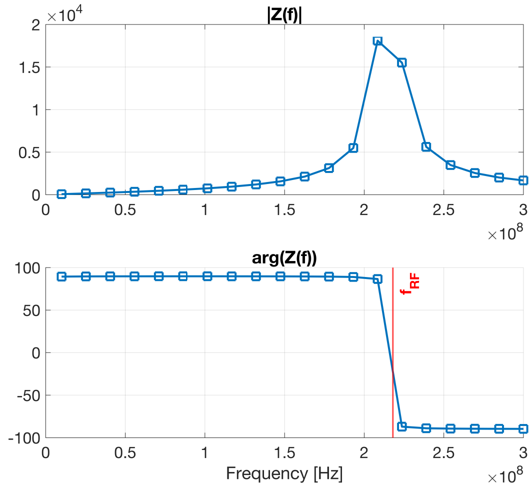

Fig. 3. Equivalent impedance (amplitude and phase) from 10 MHz to 300 MHz. The value of the FRF evaluated with the proposed approach is also reported.

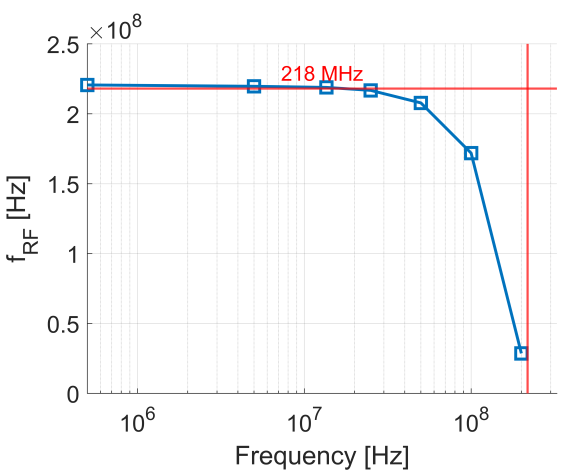

Fig. 4. Trend of extracted f_{RF} as a function of operational frequency.

V. Conclusions

In this paper, a novel method for the extraction of the first self-resonant frequency of electromagnetic devices is proposed. Despite its simple form, the extracted FRF agrees well with that evaluated by standard frequency sweep or by using iterative procedures. The proposed method, based on energetic considerations can be easily applied for the extraction of the FRF with a noticeable reduction of the computational burden, in the presence of conductive, dielectric, and magnetic media.

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