Lossy Transmission Line Model Based on the Generalized Method of Characteristics

Mark Keran
Department of Electrical and Computer Engineering
Western University
London, Canada
mkeran2@uwo.ca

Anestis Dounavis
Department of Electrical and Computer Engineering
Western University
London, Canada
adounavis@eng.uwo.ca

***Abstract*—In this paper, an approach for obtaining the rational approximations used in the generalized Method of Characteristics (MoC) transmission line model is developed. Vector fitting is done offline and the proposed model is obtained analytically using predetermined rational coefficients, the per-unit-length parameters, and the line length. The algorithm is developed for two-conductor transmission line structures with frequency-independent per-unit-length parameters with negligible conductance effects. It is shown that the proposed methodology can provide more accurate results for challenging line modelling problems, such as very long transmission lines, when compared with HSPICE's W-Element.**

***Index Terms*—circuit simulation, interconnect modeling, method of characteristics, transient analysis**

I. Introduction

Modeling transmission lines accurately is important for transient analysis when determining near and far-end voltage measurements for power application, on-chip, and printed circuit board interconnect modelling. Transmission line transfer function solutions are easily calculated in the frequency domain; however, the difficulties in finding corresponding time domain solutions lead to challenges in finding accurate time domain models.

Generalized Method of Characteristic (MoC) approaches [1–6] use delay extraction to obtain computationally efficient models. Although this approach is efficient for electrically long lines, as the line length or losses of the line increase, the implementation of the generalized MoC becomes challenging due to the difficulties in obtaining the rational approximations for the model.

In order to develop a more accurate generalized MoC model, this paper provides an analytic formulation based on [5, 6] to efficiently obtain the model using predetermined rational coefficients, the per-unit-length (p.u.l.) parameters, and line length. To obtain analytic expressions without having to perform curve fitting whenever the p.u.l. parameters and line length are changed, vector fitting [7–9] is performed on key functions offline and the rational coefficients are stored. These functions are normalized based on the p.u.l. parameters and can be linearly scaled with the line length. The accuracy of the curve fitting can be verified offline to ensure accurate time domain results. The numerical example shows that the proposed approach can provide more accurate results for challenging line modeling problems such as very long transmission lines when compared to HSPICE's W-Element [10].

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II. Review of the Generalized Method of Characteristics

Distributed transmission line circuits with a line length l and per-unit-length resistance, inductance, conductance, and capacitance, defined by $R(s)$, $L(s)$, $G(s)$, and $C(s) \in \mathbb{R}$, respectively, are modelled using Telegrapher's equations which can be expressed in the Laplace domain as [1–4]

$$I_1(s) = Y_0(s)V_1(s) - J_1(s) \tag{1}$$

$$I_2(s) = Y_0(s)V_2(s) - J_2(s) \tag{2}$$

$$J_1(s) = H(s)(Y_0(s)V_2(s) + I_2(s)) \tag{3}$$

$$J_2(s) = H(s)(Y_0(s)V_1(s) + I_1(s)) \tag{4}$$

$$H(s) = e^{-\gamma(s)l} \tag{5}$$

where s is the Laplace variable. $Y_0(s)$ is the characteristic admittance and $\gamma(s)$ is the propagation function as defined by

$$Y_0(s) = \sqrt{(G(s) + sC(s))(R(s) + sL(s))^{-1}} \tag{6}$$

$$\gamma(s) = \sqrt{(G(s) + sC(s))(R(s) + sL(s))}. \tag{7}$$

$V_1(s)$, $V_2(s)$, $I_1(s)$, and $I_2(s)$ represent the terminal voltages and currents at the near and far end.

Since the solution of (1)-(5) does not have a Laplace Inverse representation, rational approximations are needed for time domain analysis. To obtain an efficient model for electrically long lines, the generalized MoC algorithms perform delay extraction on the propagation operator $H(s)$ as

$$H(s) = e^{-sTl}Q(s) \tag{8}$$

where Tl is the extracted delay and rational approximations are required for $Y_0(s)$ and $Q(s)$. However, obtaining the rational approximation of $Q(s)$ can lead to difficulties as the line length and losses increase.

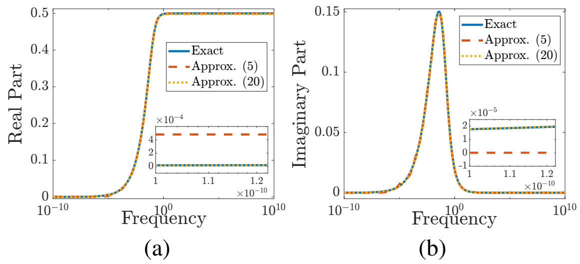


Fig. 1. Frequency response of the (a) real and (b) imaginary part of $X_1(s')$ for a rational approximation using 5 and 20 poles compared with the accurate frequency response with zoomed-in portions of the low-frequency regions

III. Proposed Approach

In this work, the proposed generalized MoC model is developed for two-conductor transmission lines where the p.u.l. parameters are frequency independent and $G(s) = 0$. To make the fitting scalable with the line length, the rational approximation is performed on $\gamma(s) - sT$ instead of $Q(s)$. The function $\gamma(s) - sT$ is expressed as

$$P(s) = \gamma(s) - s\sqrt{LC} = s\sqrt{LC}\left(\sqrt{\frac{\frac{L}{R}s + 1}{\frac{L}{R}s}} - 1\right) \quad (9)$$

where $T = \sqrt{LC}$. By setting

$$s' = (L/R)s \quad (10)$$

$P(s)$ can be represented as

$$\gamma(s') - R\sqrt{C/L}s' = R\sqrt{C/L}s'\left(\sqrt{(s'+1)/s'} - 1\right). \quad (11)$$

The function

$$X_1(s') \approx s'\left(\sqrt{(s'+1)/s'} - 1\right) \quad (12)$$

corresponds to a normalized function which is independent of the p.u.l. parameters. Rational approximations using vector fitting [7–9] for 5, 10, 15, and 20 poles are performed for (12) offline and stored. Fig. 1 shows the accuracy of the fitted response of $X_1(s')$ using vector fitting formulation with 5 poles and 20 poles. The 5-pole approximation is sufficient to obtain the generalized shape of $X_1(s')$; however, there are issues in the low-frequency range as shown in the zoomed-in portion of Fig. 1 and can lead to inaccurate approximations of $Q(s)$ as the line length increases. Depending on the p.u.l. parameters and line length, the appropriate order of the rational approximation for $X_1(s')$ needs to be determined to ensure accurate time domain results of the proposed MoC model. A rational approximation of $P(s)$ can now be obtained using (12) as

$$\hat{P}(s) = R\sqrt{(C/L)}X_1(Ls/R). \quad (13)$$

To approximate $Q(s)$, the Padé representation of e^{-s} can be utilized as

$$e^{-s} \approx \frac{\sum_{i=0}^{M} \frac{(2M-i)!M!}{(2M)!i!(M-i)!}(-s)^i}{\sum_{i=0}^{N} \frac{(2N-i)!N!}{(2N)!i!(N-i)!}(s)^i} \quad (14)$$

where M/N corresponds to the order of the Padé approximation. By replacing s with the rational approximation of $\hat{P}(s)l$, an approximation of $Q(s)$ is realized as

$$\hat{Q}(s) = \frac{\sum_{i=0}^{M} \frac{(2M-i)!M!}{(2M)!i!(M-i)!}(-\hat{P}(s)l)^i}{\sum_{i=0}^{N} \frac{(2N-i)!N!}{(2N)!i!(N-i)!}(\hat{P}(s)l)^i} \quad (15)$$

The order of $\hat{Q}(s)$ is dependent on M/N and the order of $X_1(s')$. These can be determined by ensuring that the absolute difference, of both the real and imaginary parts, between the accurate solution of $Q(s)$ and its approximation, is within a certain error tolerance for the frequency range of interest. The Padé order M/N of the exponential is obtained using the exact function $P(s)$ and comparing the approximation with the accurate solution of $Q(s)$. The exact exponential of the approximation of $-\hat{P}(s)l$ is used to determine the order of $X_1(s')$.

In order to obtain an analytic expression for $Y_0(s)$, (6) can be rearranged and using (10) it can represented as

$$Y_0(s') = \sqrt{C/L}\sqrt{s'/(s'+1)}. \quad (16)$$

Note that the normalized function

$$X_2(s') \approx \sqrt{s'/(s'+1)} \approx s'/(X_1(s') + s') \quad (17)$$

is not dependent on the p.u.l. parameters and can be fitted offline or determined using the approximation of $X_1(s')$ as shown in (17). In this work, the rational approximation of $X_1(s')$ that was used to determine $\hat{Q}(s)$ was also used to obtain an approximation for $Y_0(s)$ since this resulted in accurate frequency domain results of the Y-parameters of the circuit. A rational approximation of $Y_0(s)$ can now be obtained using (17) as

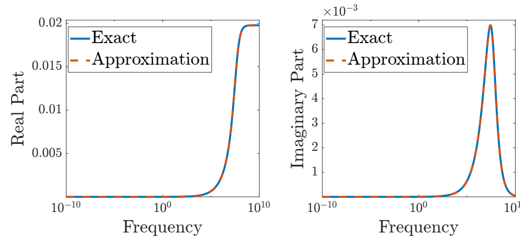
$$\hat{Y}_0(s) = \sqrt{(C/L)}X_2(Ls/R). \quad (18)$$

The rational approximations of $\hat{Y}_0(s)$ and $\hat{Q}(s)$ can then be implemented in HSPICE [10] to simulate the Norton equivalent MoC model [5]. Note that with the rational coefficients of (12), (14), and (17), no additional curve fitting is required. Future work will involve the extension of the proposed algorithm to include frequency-dependent p.u.l. resistance and inductance parameters as well as evaluating the rational approximations to ensure passivity of the overall circuit.

IV. Numerical Example

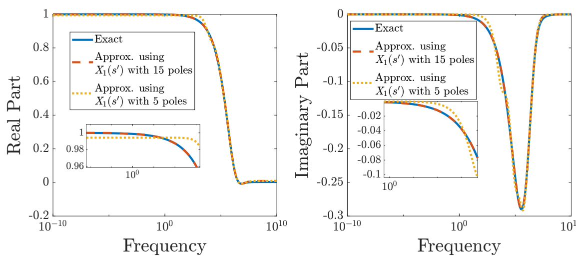
To illustrate the advantages of this approach, the proposed algorithm is compared with HSPICE's W-Element, HSPICE with lumped resistive-inductive-capacitive (RLC) elements, and performing IFFT on the frequency solution of the circuit. The rational approximations for the proposed model are realized in MATLAB R2022a and implemented in HSPICE (Version 2022.06-SP2-2) using Foster elements [10].

A cable with $R = 60.3\Omega$/m, $L = 211$nH/m, $C = 82.3$pF/m, a 50Ω driving resistor attached in series to the near end, and a 1fF load capacitor attached to the far end is analyzed. A linear piecewise ramped input source with a rise time of 0.1ns



(a) (b)

Fig. 2. Frequency response of the (a) real and (b) imaginary part of the approximation $\hat{Y}_0(s)$ using 15 poles for $X_1(s')$ compared with the accurate frequency response $Y_0(s)$



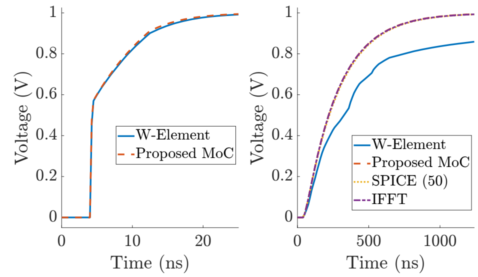
(a) (b)

Fig. 3. Frequency response of the (a) real and (b) imaginary part of the approximation of $\hat{Q}(s)$ using 5 and 15 poles for $X_1(s')$ compared with the accurate $Q(s)$ for a 10m line with zoomed-in sections

and an amplitude of 1V is attached in series with the driving resistor.

By setting the threshold of the error criteria for both the real and imaginary portions of $Q(s)$ to 0.001 and fitting the response up 10GHz, $M/N = 1/3$ for the exponential function and 10 poles for $X_1(s')$ are required for a 1m line, while $M/N = 2/4$ for the exponential function and 15 poles for $X_1(s')$ are required for a 10m line. Fig. 2 and Fig. 3 show the $\hat{Y}_0(s)$ and $\hat{Q}(s)$ rational approximations for the 10m line compared to the exact frequency response. Fig. 3 shows that for the 10m line, the 5-pole approximation of $X_1(s')$ was not sufficient to satisfy the error criteria of $\hat{Q}(s)$ and 15-poles were required.

Fig. 4 shows the far-end time domain voltage response for line lengths of 1m and 10m for both W-Element and the proposed methodology. For the 1m line, both W-Element and the proposed methodology are in agreement. However, as the line length increases to 10m, W-Element has difficulty in modelling the frequency response of the transmission line leading to inaccurate time domain solutions as shown in Fig. 4b. In Fig. 4b the lumped model, the proposed model, and the solution using IFFT were in agreement. By determining and verifying the accuracy of the coefficients of $X_1(s')$ and $X_2(s')$ offline and selecting the appropriate order of M/N, the proposed methodology can create the model analytically in a manner which is less prone to numerical errors. As a result, the proposed methodology provides a more accurate time domain response than the W-Element for the 10m case. For the 10m line, HSPICE's W-Element, HSPICE with 50 lumped RLC sections, and the proposed methodology required about 0.05,



(a) (b)

Fig. 4. Far-end voltage response for a line length of (a) 1m and (b) 10m comparing HSPICE's W-Element, the proposed MoC methodology, HSPICE with 50 lumped elements, and IFFT

0.05, and 0.06 s, respectively.

V. Conclusion

In this paper, an approach for time domain modeling of two-conductor transmission line structures with frequency-independent p.u.l. parameters is developed to avoid fitting each time the line length or p.u.l. parameters are altered. By performing vector fitting offline, the accuracy of the rational approximations can be verified and the MoC model is obtained using predetermined rational coefficients, the p.u.l. parameters, and the length of the line. This approach can provide more accurate responses than HSPICE's W-Element for challenging line modelling problems such as lines with large resistive losses and very long lines.

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