Loewner Matrix Interpolation for Noisy S-parameter Data

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***Abstract*—Loewner Matrix (LM) interpolation technique was proposed as an efficient macromodeling approach compared to state of the art technologies. However, the method becomes inaccurate in presence of noise as it interpolates noise itself. In this paper, we propose a LM interpolation technique suitable for extracting an accurate and passive macromodel from noisy S-parameter data. An order searching algorithm to find the most accurate model maintaining stability is proposed first. Then we propose a least-square approximation based correction on the macromodel. Finally, the passivity of the model is ensured by using a Hamiltonian Matrix Pencil perturbation scheme. The advantages of the proposed approach is illustrated using one full-wave example.**

***Index Terms*—Loewner Matrix, Macromodeling, Least Square Approximation**

I. Introduction

Obtaining physics based models for Electromagnetic (EM) structures is often challenging. However, such structures can be accurately characterized in frequency-domain using S- or Y-parameters which can be obtained via full-wave simulation. Loewner Matrix (LM) based interpolation techniques [1], [2] were proposed to generate a SPICE-compatible time-domain macromodel from S-parameter data. LM based techniques were shown to be more accurate and efficient in [1], [3] compared to state of the art technique such as Vector Fitting [4]. However, these techniques become inaccurate, non-passive or both in presence of noise [5]. The main reason is the fact that LM method interpolates the noise which causes two major challenges for extracting an accurate and passive macromodel: a) An order preserving passivity of the macromodel is difficult to determine, b) Even if an appropriate order is found, the frequency response becomes inaccurate. We propose a LM interpolation technique suitable for noisy frequency response in this paper. First, we propose a searching algorithm to obtain an order for which the model is the most accurate while still stable. Next, an approach based on least-square approximation is proposed to compute the matrices $\mathbf{B}$ and $\mathbf{C}$ using all the available data points. Finally, passivity of the model is checked and a passivity enforcement technique is applied if required. The overall objective of the paper is introduced in Sec. II, the proposed approach is presented in Sec III, and the simulation results are provided in Sec. IV

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II. Problem Formulation

A p-port EM structure is described in frequency-domain using a number of S-parameter data

$$\{s_l, \mathbf{S}(s_l)\}; \quad 1 \leq l \leq N \tag{1}$$

the number of data points N is very large, typically 1000. A user-controlled random noise is added to the data in (1) to emulate a noisy frequency response,

$$\tilde{\mathbf{S}}(s_l) = \mathbf{S}(s_l) + \tilde{\mathbf{N}}(s_l) \tag{2}$$

where $\tilde{\mathbf{N}}$ is a random noise [5], [6] defined as

$$\tilde{\mathbf{N}}(s_l) = \mathbf{S}(s_l) \times 10^{-SNR/10} \times \left(\tilde{\mathbf{N}}_r + j\tilde{\mathbf{N}}_i\right) \tag{3}$$

$\mathbf{N}_r$ and $\mathbf{N}_i$ are random Gaussian noise. The noise level can be adjusted by varying the Signal to Noise Ratio (SNR). We want to extract a time-domain macromodel

$$\begin{aligned} \mathbf{E}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{a}(t), \\ \mathbf{b}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{a}(t) \end{aligned} \tag{4}$$

from the data in (2), however, matching the noise-free response in (1) as close as possible using the transfer function

$$\mathbf{H}(s_l) = \mathbf{C}\left(s_l\mathbf{E} - \mathbf{A}\right)^{-1}\mathbf{B} + \mathbf{D}. \tag{5}$$

Where $\mathbf{E}, \mathbf{A} \in \mathbb{R}^{m\times m}$, $\mathbf{B} \in \mathbb{R}^{m\times p}$, $\mathbf{C} \in \mathbb{R}^{p\times m}$ and $\mathbf{D} \in \mathbb{R}^{p\times p}$ represent the macromodel of order m.

III. Proposed Loewner Matrix (LM) Interpolation

In presence of noise the macromodel obtained using LM method proposed in [2] becomes inaccurate. The inaccuracy comes from two main sources. Firstly, the order becomes smaller than necessary as the noise level increases [5]. Secondly, the model exhibits some unexpected large resonances near the highest frequency point. The inaccuracy sometimes makes the model non-passive which causes the algorithm in [2] to fail. In this paper, we propose an algorithm to search for necessary order as outlined in Sec. III-A. Next we propose a technique to compute new $\mathbf{B}$ and $\mathbf{C}$ matrices using least-square approximation as outlined in Sec. III-B. Finally a passivity checking and enforcing is performed as shown in Sec. III-C.

A. Searching for an Appropriate Order

In this step, a LM based algorithm similar to [2] is formulated to determine an initial order m_{init}. Note that only a small subset of the data in (2)

$$\{s_k, \tilde{\mathbf{S}}(s_k)\}; \quad 1 \leq k \leq M; \quad M << N \tag{6}$$

is used in LM algorithm. The algorithm searches for an order m_{init} by examining the 10 largest singular value drops [2]. First, macromodels are extracted at the indices of the drops. Then the accuracy and stability of the macromodels are checked and finally the order for which the model is the most accurate while still stable is selected as m_{init}. Note that unlike LM method in [2] passivity is not checked in this step.

Typically m_{init} results in an accurate macromodel for noise-free data. However, extracting final macromodel as $m = m_{init}$ is not sufficient for noisy data [5]. In this paper, we formulate an algorithm to search for a higher order by increasing it linearly using a step size of two. At each searching point stability and accuracy of the model is checked, the order m and error ε is saved for further use if the model is stable. The error with respect to the data in (6) ε is measured in terms of entry-wise vector norms [6]. The search algorithm stops if five consecutive models become unstable. Finally, the three most accurate models are selected

$$\{m^{(1)}, m^{(2)}, m^{(3)}\};\ \ where\ \{\varepsilon^{(1)} < \varepsilon^{(2)} < \varepsilon^{(3)}\} \tag{7}$$

$m^{(1)}$ represents the order for which the model is the most accurate while still stable.

B. Least-Square Approximation

The final macromodel is computed iteratively among the three orders in (7). Model $\{\mathbf{E}, \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}^{(1)}$ is extracted as [2] using $m = m^{(1)}$ in the first iteration. Then we compute new $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{C}}$ matrices by applying a least square approximation using all the data in (2). At iteration i the following system of equations is formed first to solve for $\tilde{\mathbf{B}}_{i+1}$

$$\begin{bmatrix} \tilde{\mathbf{C}}_i \left(s_1\mathbf{E} - \mathbf{A}\right)^{-1} \\ \vdots \\ \tilde{\mathbf{C}}_i \left(s_N\mathbf{E} - \mathbf{A}\right)^{-1} \end{bmatrix} \tilde{\mathbf{B}}_{i+1} = \begin{bmatrix} \tilde{\mathbf{S}}(s_1) - \mathbf{D} \\ \vdots \\ \tilde{\mathbf{S}}(s_N) - \mathbf{D} \end{bmatrix} \tag{8}$$

where $()_i$ represents the iteration number $\tilde{\mathbf{C}}_0 = \mathbf{C}$, $\tilde{\mathbf{S}}(s_l)$ are the noisy data in (2). A least square approximation is used to solve (8). A similar system of equation is then formed to solve for $\tilde{\mathbf{C}}_{i+1}$

$$\begin{aligned} \tilde{\mathbf{C}}_{i+1} \left[\ \left(s_1\mathbf{E} - \mathbf{A}\right)^{-1} \tilde{\mathbf{B}}_{i+1} \quad \ldots \quad \left(s_N\mathbf{E} - \mathbf{A}\right)^{-1} \tilde{\mathbf{B}}_{i+1}\ \right] \\ = \left[\ \tilde{\mathbf{S}}(s_1) - \mathbf{D} \quad \ldots \quad \tilde{\mathbf{S}}(s_N) - \mathbf{D}\ \right] \end{aligned} \tag{9}$$

(9) is also solved using least square approximation. The iteration stops if $norm(\Delta\tilde{\mathbf{B}}, \Delta\tilde{\mathbf{C}}) < 0.005$ or $\#iteration > 20$. Typically the scheme converges within five to nine iterations. The new model

$$\{\mathbf{E}, \mathbf{A}, \mathbf{B} = \tilde{\mathbf{B}}, \mathbf{C} = \tilde{\mathbf{C}}, \mathbf{D} = \mathbf{0}\} \tag{10}$$

is more accurate and the unexpected resonances are reduced.

C. Passivity Checking/Enforcement

Passivity of the macromodel in (10) is checked using a Hamiltonian Matrix pencil approach [7]. Hamiltonian Matrix pencil $(\mathcal{J}, \mathcal{K})$ is defined as follows

$$\begin{aligned} \mathcal{K} &= \begin{bmatrix} \mathbf{E} & \\ & \mathbf{E}^T \end{bmatrix}, \\ \mathcal{J} &= \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{D}^T\mathbf{S}^{-1}\mathbf{C} & -\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T \\ \mathbf{C}^T\mathbf{S}^{-1}\mathbf{C} & -\mathbf{A}^T + \mathbf{C}^T\mathbf{D}\mathbf{R}^{-1}\mathbf{B}^T \end{bmatrix} \end{aligned} \tag{11}$$

where $\mathbf{R} = \mathbf{D}^T\mathbf{D} - \mathbf{U}$, $\mathbf{S} = \mathbf{D}\mathbf{D}^T - \mathbf{U}$ and $\mathbf{U}$ is a identity matrix. Any passivity violation can be detected by examining the imaginary eigenvalues of pencil $(\mathcal{J}, \mathcal{K})$.

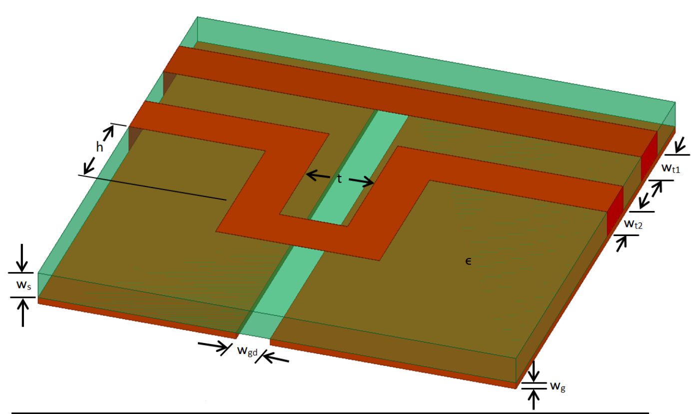
An eigenvalue perturbation scheme [7] is applied to enforce any passivity violation by modifying matrix $\mathbf{C}$ to $\mathbf{C}_p$. After passivity enforcement the error ε of the macromodel

$$\{\mathbf{E}, \mathbf{A}, \mathbf{B}, \mathbf{C} = \mathbf{C}_p, \mathbf{D} = \mathbf{0}\}. \tag{12}$$

is measured. If the error is very high ($\varepsilon > 4\varepsilon_{max}$) the model in (12) cannot be used, in that case another macromodel with $m = m^{(2)}$ is extracted and the algorithm starts from least-square approximation in Sec. III-B and so on. Note that the maximum achievable accuracy ε_{max} can be obtained from the noise variance as outlined in [6].

IV. Simulation Results

We evaluate the performance of the proposed approach compared to LM method [2] using a 4 port 7 × 7 cm microstrip line structure. The structure contains two copper lines, one of which has four 90° bends. Furthermore, the ground is made defective by introducing a small slot and only 5 mm opening for the return current. The detailed dimensions and specifications are provided in Fig. 1. The bend in the line and the defective ground structure cause EM interactions among the lines and the ground plane. The S-parameter data spanning the bandwidth from 1 MHz to 10 GHz is generated by performing full-wave analysis of the structure.



w_t	h_t	w_g	w_s	ϵ	h	t	w_{gd}
7 mm	35 µm	1 mm	4 mm	10	15 mm	10 mm	5 mm

Fig. 1. Four port microstrip line with defective ground structure

Next a synthetic noisy data is generated by adding Gaussian noise to full-wave data as (2). We set up three test cases by adding noise of 30, 25, and 20 dB SNR. Finally, the time-domain macromodel is extracted from the noisy data using the proposed approach and LM method [2]. The robustness study for the macromodeling approaches is performed by

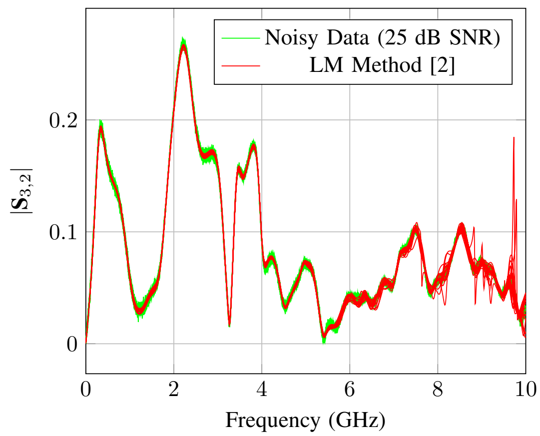


Fig. 2. Sample data for LM method [2] using noisy data with 25 dB SNR.

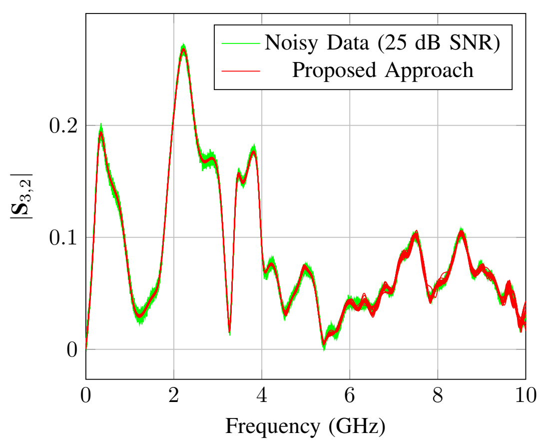


Fig. 3. Sample data for proposed approach using noise of 25 dB SNR.

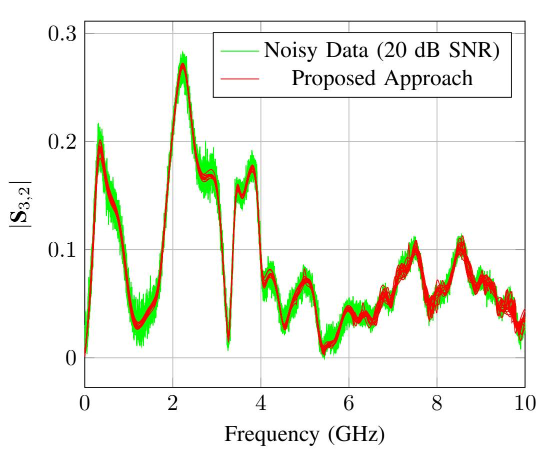


Fig. 4. Sample data for proposed approach using noise of 20 dB SNR.

TABLE I
SIMULATION RESULTS FOR DIFFERENT LEVEL OF RANDOM NOISE

SNR		30 dB		25 dB		20 dB	
Methods		[2]	**Prop.**	[2]	**Prop.**	[2]	**Prop.**
# MC		1000	1000	1000	1000	1000	1000
# Failure		**0**	**0**	**19**	**0**	**999**	**82**
Average	ε_{max}	0.009	0.009	0.03	0.03	-	0.09
	ε	0.03	0.02	0.06	0.04	-	0.11
	Order	152	159	133	136	-	117
	Time	1.2 s	4.4 s	1.07 s	4.23 s	-	7 s

extracting 1000 macromodels each with different random noise of same SNR. The results are summarized in Table I. Sample S-parameter data interpolated using the proposed approach for 25 and 20 dB SNR are presented in Fig. 3 and 4 respectively. A similar plot using LM method [2] for 25 dB SNR is provided in Fig. 2.

As can be seen, both of the approaches are very robust for moderate level of random noise (30 dB SNR). However, LM method [2] loses robustness as the noise level gets higher. Furthermore, the frequency response becomes inaccurate near highest frequency region as can be seen from Fig. 2. The proposed order selection scheme followed by the least-square approximation of $\mathbf{B}$ and $\mathbf{C}$ matrices improve not only the robustness but also the accuracy of LM interpolation based macromodeling algorithm at the expense of slightly higher CPU cost.

V. CONCLUSION

The regular Loewner Matrix (LM) based interpolation technique loses its robustness and accuracy in presence of high level of noise. We propose a new LM interpolation technique suitable for noisy frequency response by searching for a higher order and applying a correction on macromodel by computing new $\mathbf{B}$ and $\mathbf{C}$ matrices. The robustness and accuracy of the proposed approach is illustrated using a full-wave example.

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