Efficient and Robust Dyadic Green's Function Evaluation Algorithm for the Analysis of IC Packages and Printed Circuit Boards

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***Abstract*— This paper presents an efficient and accurate methodology for the layered media Green's function evaluation in fully 3D electromagnetic scenarios such as of IC packages and printed circuit boards.**

Keywords—Layered media Green's functions, Method of Moments, Mixed Potential Integral Equation, interpolation, singularities

I. INTRODUCTION

The Method of Moments (MoM) is a widely used numerical algorithm for the electromagnetic analysis of structures embedded in layered dielectric media such as Integrated Circuits (ICs) and Printed Circuit Boards (PCBs) [1]. In particular, among different types of available Integral Equations (IEs), the Mixed-Potential formulation (MPIE) is one of the most appealing for the low order type of singularities involved in the associated reaction kernels. Furthermore, the MPIE approach, thanks to the use of the Dyadic Green's Function (DGFs), does not require to model explicitly the ground planes and the dielectric interfaces, leading to a remarkable reduction in the total number of mesh elements with virtually no accuracy penalty in the final results [2].

Without loss of generality, let us consider a dielectric medium where the stratification is normal to the *z*-axis. To evaluate the associated DGFs, a z-directed equivalent Transmission Line (TL) is introduced and each element of the dyad is expressed, in the wave number domain, in terms of currents and voltages defined over the line. Then, the spatial domain Green's function can be cast in the form of a semi-infinite Sommerfeld Integral (SI) as reported in (1):

$$f(\rho,z,z') = \frac{1}{2\pi}\int_0^\infty \tilde{f}(k_\rho,z,z')J_n(k_\rho\rho)k_\rho^{\,n+1}dk_\rho \quad (1)$$

where ρ, z, z' and J_n are the horizontal distance in a cylindrical reference system, the vertical location of the observation point, the vertical location of the source point and the first kind Bessel function of order n, respectively.

The mentioned approach, even if general and rigorous, can be numerically demanding due to the slowly converging and rapidly varying nature of the Bessel function [3]. Having in mind that the process of filling up the MoM matrix can require several thousand of source-observation interactions, it is not realistic to calculate a new SI every time a Green's function is required. Based on this background, it is not surprising that an efficient and fast procedure for the SI evaluation is of crucial relevance. In the past decade, several approaches have been proposed to accomplish this goal and, among them, the Discrete Complex Image Method (DCIM) is a very popular solution [4]. The main idea behind the DCIM is to express the spectral domain Green's function kernel as a sum of special functions whose integration admits an analytical solution. The DCIM, even if very fast for 2D and 2.5D type of problems, is not always robust and requires the extraction of the surface wave poles in order to reduce the uncertainties about its operational range and accuracy. Furthermore, the DCIM has to be repeated each time the vertical location of the source and/or observation point is changed, leading to a severe degradation of the algorithm performance for fully 3D scenarios where z and z' can assume a continuous range of values. For these reasons, the pure numerical solution of the SI, which represents a safer scheme, is still attracting a lot of attention [5]. In fact, if the latter numerical algorithm is efficient enough, it can be used directly as part of a complete Method of Moment-based software tool. In order to accomplish this task, one of the most common approach is to interpolate a set of pre-calculated Sommerfeld Integrals defined over an appropriated spatial grid. In [6], the Green's function interpolation has been applied to the case of planar structures. In [7], a fully 3D approach, based on a barycentric coordinate system, has been presented for a direct interpolation of the MPIE Green's function. Although this approach does not present any restriction on the number of layers which the scatters may penetrate, it does not provide any kind of error control mechanism able to guarantee a pre-defined level of accuracy.

This paper introduces an efficient SI-based interpolation algorithm for the evaluation of the spatial domain dyadic Green's function arising in fully 3D electromagnetic problems. In particular, the proposed technique is able to

speed-up the MoM matrix filling time with a guaranteed level of accuracy thanks to a spatial domain adaptive algorithm.

The paper is organized as follows. In Section II, the properties of proposed technique are presented. Section III provides several examples in order to prove the efficiency and the numerical accuracy of the described approach. Finally, some concluding remarks are included in Section IV.

II. ADAPTIVE INTERPOLATION SCHEME

Consider an arbitrary PEC object embedded in a layered medium. The MPIE expression of the scattered electric field $\underline{E}^s$ can be summarized as:

$$\underline{E}^S = -j\omega\mu_0\left\langle \underline{G}^A, \underline{J}\right\rangle + \frac{\nabla}{j\omega\varepsilon_0}\left\langle K^\phi, (\nabla' \cdot \underline{J})\right\rangle \qquad (2)$$

where $\underline{G}^A$ is the vector potential dyad, K^ϕ is the scalar potential, ω is the angular frequency, μ_0 and ε_0 are the free space magnetic permeability and dielectric permittivity, respectively [2].

A. Green's function regularization

Since both the vector as well as the scalar potential term are characterized by a singular behavior when the distance between source and observation point is small, a regularization technique is required to smooth the SI behavior which then can be interpolated with a desired level of accuracy. As suggested in [8], the DGF regularization can be performed by subtracting from SI kernel the asymptotic k_ρ limit of the correspondent DGF element. It is important to remark that these limits, which can be interpreted as the reflected and transmitted fields from the various dielectric interfaces, can be cast in closed form both in the spectral as well as in the spatial domain. In Fig. 1 we have reported the regularized scalar potential for the four layer stack shown in the caption at a frequency of 30GHz. In order to prove the accuracy of the derived approach, the direct calculation (Direct) has been compared with the results obtained by summing up the regularized Green's function and the associated analytic singular term in the spatial domain (Reconstructed). It is evident that a good agreement has been achieved over the whole analyzed spatial range but, at the same time, the regularized potential is almost three orders of magnitude smaller with respect to its direct calculation.

B. Adaptive interpolation algorithm

As mentioned in the previous paragraphs, interpolation is particularly important for the analysis of 3D geometries, where the Green's function depends on three different variables namely, ρ, z and z'. The proposed approach subdivides the Green's function domain in a certain number of cubical sub-regions based on the wavelength in the correspondent dielectric medium. Each component of the Green's function is then sampled over the vertices of the defined grid. In particular, since the proposed approach implements a second order Lagrange interpolation technique, every cell, defined herein as "root", requires a total number of 27 vertices. In order to avoid any kind of discontinuities in the sampled Green's function which will prevent the convergence of the adaptive algorithm, it is crucial to guarantee that each cell is not crossing any dielectric interface. Finally, within each root, a combination of different tree data structures is employed in order to adaptively refine the initial mesh until the desired accuracy level is reached. In particular, based on the relative error distribution obtained within the cell, the proposed approach can adopt a binary tree, a quad-tree or an oct-tree subdivision. The described adaptive technique, in conjunction with the initial space cell subdivision, has been implemented in order to guarantee an efficient identification of the cell that contains the considered source-observation interaction during the matrix filling stage.

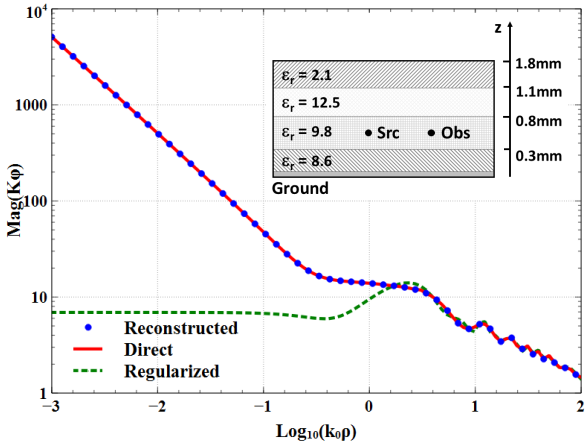

Fig 1. Regularized, direct calculation and reconstructed scalar potential Green's function for the reported four layer stack at 30GHz.

At this point, it worth mentioning that, when the distance between source and observation point is large, a cancellation phenomenon can be observed between the regularized Green's function and the correspondent spatial domain singular term. In other words, the regularized Green's function can be significantly larger than the direct calculation, and this can easily introduce inaccuracies in the SI adaptive evaluation algorithm as well as in the interpolation procedure. One possible solution to the described issue is to reduce the used error criteria to a very low limit but this can severally downgrade the numerical efficiency of the overall algorithm. In other to avoid this phenomenon, the proposed technique adaptively choose the cells where the regularization has to be performed, namely the near-field region, and the ones where the direct calculation can be employed instead (far-field region).

III. NUMERICAL RESULTS

To illustrate the performance of the derived technique, we have analyzed the dielectric stack reported in Fig. 1 at a frequency of 1.0GHz. In order to prove the robustness of the adaptive algorithm, we have compared the scalar potential results provided by the proposed approach with the ones generated by the direct calculation for different configurations of source/observation points. In particular, we have analyzed the case where z and z' belong to the same layer (most demanding scenario) and the one where source and observation points are located in adjacent layers (Fig. 2). It

worth noticing that in both cases an excellent agreement has been achieved.

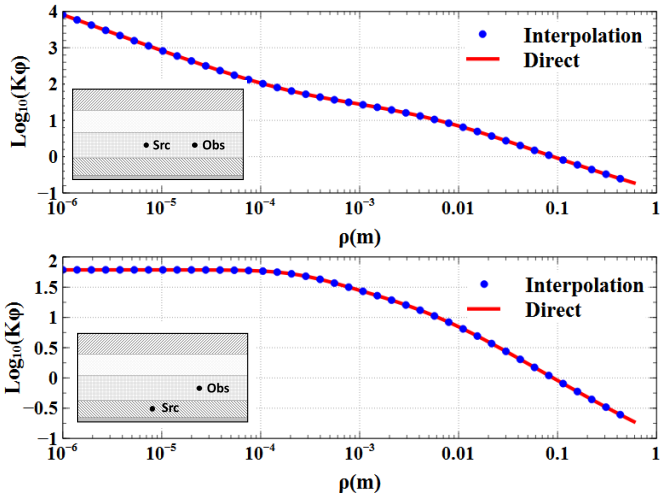

Fig 2. Comparison between the interpolated and directly calculated scalar potential term: (a) same layer case, (b) adjacent layers case.

In Fig. 3 we have reported the relative error for z equals to z' in the middle of the second slab as a function of the horizontal distance ρ. In order to prove the accuracy of the proposed approach, the same test has been repeated by using a static interpolation scheme where the error control mechanism has been disabled.

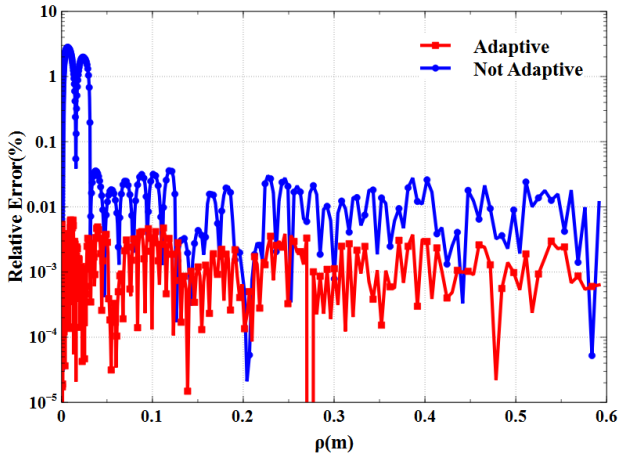

Fig 3. Relative error provided by the adaptive algorithm and by a static interpolator.

It is evident that adaptive algorithm is able to guarantee much more accurate results with respect to the static approach. Finally, in Fig. 4, we have reported the relative time, defined as the ratio between the CPU time required by the described algorithm and that needed by the direct calculation, as a function of the SI calculation number. As expected, the CPU time associated with the interpolation algorithm is almost constant with respect to the number of SI since the time for the Legendre interpolation is negligible with respect to the interpolation table filling time. In a reality, for a typical 3D structure, the number of Green's function evaluations needed for filling the MoM matrix is well above a million where we see huge speedup coming from the proposed interpolation mechanism.

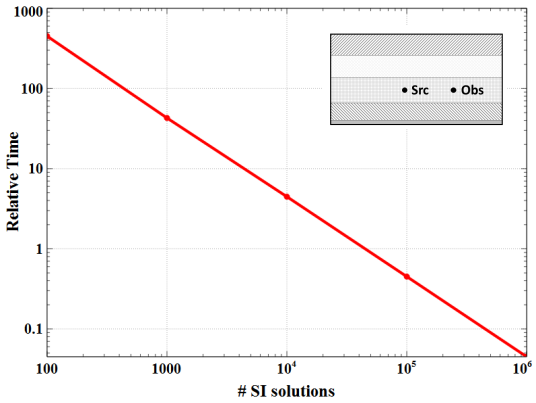

Fig 4. Relative time as a function of the SI evaluation number.

IV. CONCLUSIONS

This paper demonstrates a practical, accurate and efficient methodology for the layered media Green's function evaluation in fully 3D electromagnetic scenarios. The proposed technique uses an adaptive interpolation algorithm in conjunction with the numerical solution of the Sommerfeld Integral which is able to overcome the issues associated with semi-analytic solutions. Furthermore, thanks to a spatial error control mechanism, the derived algorithm is able to speed up the MoM matrix filling time with a predefined level of accuracy. It is worth mentioning that only the L-operator kernel is considered in this paper although the proposed approach can be easily generalized to the case of both electric and magnetic currents.

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