Delay Rational Macromodels of Long Interconnects using Loewner Matrix

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***Abstract*—This paper presents a method to obtain delay-based macromodels of electrically long interconnects from tabulated frequency data. The proposed algorithm first extracts multiple propagation delays and splits the data into single delay regions using a time-frequency decomposition transform. Then, the attenuation losses of each region is approximated using the Loewner Matrix approach. The resulting macromodel is a combination of delay rational approximations. A numerical example is presented to illustrate efficiency of the proposed method compared to traditional Loewner where the delays are not extracted beforehand.**

***Index Terms*—Delay extraction, Loewner Matrix, rational approximation, time-frequency decomposition, transmission lines.**

I. Introduction

DUE to system complexity, process variations and non-uniformities of electrical circuits, rational macromodel approximations from tabulated measured data are often used to model high speed interconnects. Among these rational curve fitting techniques are Vector Fitting (VF) and Loewner Matrix (LM) algorithms [1], [2].

Broadly speaking, two approach exists for developing macromodels of distributed networks. One approach is to approximate the data as rational functions, however, this requires many poles to accurately approximate interconnects with significant signal delay. To second approach is based on delayed rational functions [3]–[5]. This technique extracts the propagation delays from the tabulated data, while the remaining attenuation losses are approximated using low order rational functions, leading to more compact macromodels with fewer poles when compared to the first approach.

Methodologies to obtain delayed rational functions have been proposed in [3], [6] using the VF approach for the attenuation losses approximations. However, these delayed rational function techniques have not been extended to the LM approach. In recent years, LM [2], [7] has been proposed as an alternative to VF. Unlike, what is done in VF, which relies on multiple different order approximations to determine the best order to fit the data [8], LM provides a direct mechanism to identify the order based on the magnitudes of a Singular Value Decomposition (SVD) [2]. Furthermore, for the case of multiport networks the time-domain macromodel can be realized with fewer equations when compared to VF [2], [7].

In this work, delayed rational approximations from tabulated frequency data are derived using the LM method. Similar to what is done in [3], [6], explicit delay extraction is used to extract propagation delays and partition the data into single delay regions using a time-frequency transform. From the partitioned regions, a LM method is used to approximate the attenuation losses with a low-order rational function. A numerical example is presented to illustrate efficiency of the proposed method compared to traditional Loewner where the delays of the transfer function are not extracted.

II. Macormodels with Delays and Review of General Time-Frequency Decomposition

A. Theoretical Motivation

The main objective of the proposed method is to produce a delayed rational function of the following form:

$$H(s) = \sum_{m=1}^{M} H^{(m)}(s)\mathrm{e}^{-sT_m} \tag{1}$$

where T_m is the m^{th} propagation delay and $H^{(m)}(s)$ is the delay free rational approximation corresponding to m^{th} delay. In practice, it is possible to approximate a long interconnect without the extraction of the delay terms, however this generally results in a very high number of poles, which makes the transient analysis computationally intensive. By extracting the delay, the attenuation losses can be approximated by low order rational function [3], [4], [6], [9].

B. Time-Frequency Decomposition

The delay extraction is done using the concept of the time-frequency decomposition transforms [9]. A time-frequency transform relates $H(s)$ to $F(\omega, \tau)$ with the following relation:

$$F(\omega, \tau) = \int_{-\infty}^{\infty} H(\zeta)W(\zeta - \omega)\mathrm{e}^{j\zeta\tau}\mathrm{d}\zeta \tag{2}$$

where $W(\zeta - \omega)$ is a window centred at $\zeta = \omega$ of specific width L. It is observed from (2) that if $W = 1$, then the equation becomes the standard definition of the inverse Fourier Transform (IFT). Therefore the time-frequency transform can be thought of as an IFT of $H(s)$, that only retains the frequency components in the frequency band of the filtering window W [3]. In this work, the Gabor transform [10] is used, since it provides optimal support in both the time and frequency domain. In [3] it is shown energy contents of $F(\omega,\tau)$ over time is obtained by

$$\eta(t) = \int_{-\infty}^{\infty} |F(\omega,\tau)|^2 \mathrm{d}\omega \tag{3}$$

where the propagation delays can be identified as the local maxima of the $\eta(t)$ function. The inverse of (2) is defined as:

$$H(\zeta) = \frac{1}{2\pi} \iint_{-\infty}^{\infty} F(\omega,\tau) W(\zeta-\omega) \mathrm{e}^{-j\omega\tau} \mathrm{d}\omega \mathrm{d}\tau \tag{4}$$

Using (3), the reconstruction of $H(\zeta)$ can be done by splitting the time-frequency plane into separate regions Ω_m and performing the integral (4) over each region as follows [3], [9]:

$$\begin{aligned} H(\zeta) &= \sum_k \widetilde{H}^{(m)}(\zeta) \\ \widetilde{H}^{(m)}(\zeta) &= \frac{1}{2\pi} \iint_{\Omega_m} F(\omega,\tau) W(\zeta-\omega) \mathrm{e}^{-j\omega\tau} \mathrm{d}\omega \mathrm{d}\tau \\ \bigcup_m \Omega_m &= \Re^2 \end{aligned} \tag{5}$$

The summation of each integral of (5) leads to the reconstruction of $H(\zeta)$. The time-frequency transform discussed, provides a way to extract delays from electrically long distributed networks characterized by measured or simulated data.

III. Proposed Algorithm

A. Estimation of Propagation Delays T_m

The first step of the proposed algorithm is to estimate the propagation delays, given the tabulated data H. The time-frequency representation $F(\omega,\tau)$ is computed using (2). Once the time-frequency plane is obtained, evaluating the energy content over time $\eta(t)$ using (3) provides good estimates of the propagation delays, as the time values of the local maxima [3].

In order to extract the most relevant delays, all delay terms with relative energy contributions below a user-chosen tolerance ε are not taken into account

$$\frac{\hat{n}^{(m)}}{\sum_m \hat{n}^{(m)}} < \varepsilon \tag{6}$$

The value of ε is problem dependent and is chosen such that the energy contribution of the neglected delays does not significantly affect the accuracy of the model [3], [4].

B. Time-Frequency Plane Partitioning

Once the delays are identified, the next step is to split the time-frequency plane in such a way as to get delay regions. The method used to split the plane is the same as the one proposed in [3], [4]. The partitioning for the (ω,τ) plane into Ω_m is done by choosing a point t_k between adjacent delays T_k and T_{k+1}, where the value of the energy content at that point is lower than a predefined value δ

$$\eta(\tau = t_k) < \delta \tag{7}$$

Using (7), regions Ω_m are defined to be regions between two adjacent minima t_k and t_{k+1}, expressed as follows:

$$\Omega_m \in \{(\omega,\tau):\ 0 \le \omega \le 2\pi F_{max},\ t_k \le \tau \le t_{k+1}\} \tag{8}$$

C. Estimating Attenuation Losses $H^{(m)}(s)$

Once the Ω_m regions are determined, the last step involves computing the attenuation losses rational function corresponding to each region.

The goal is to evaluate

$$\tilde{H}^{(m)}(s) \approx H^{(m)}(s) e^{-sT_m} \tag{9}$$

where T_m is the known extracted delay using (3) and $\tilde{H}^{(m)}(s)$ is obtained using (5).

To get the rational approximation $H^{(m)}(s)$ for each region, the frequency domain data is expressed as

$$\{s_k, \tilde{H}^{(m)}(s_k) e^{+s_k T_m}\} \tag{10}$$

where $k = 1, \ldots, K$, and K is the number of data points.

The Loewner Matrix method [LM] seeks to macromodel the attenuation losses $H^{(m)}(s) \approx \tilde{H}^{(m)}(s) e^{+sT_m}$ as

$$H^{(m)}(s) = \mathbf{C}(s\mathbf{E} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} + s\mathbf{Y}^{\infty} \tag{11}$$

where $\mathbf{A}, \mathbf{E} \in \mathbb{R}^{n\times n}, \mathbf{B} \in \mathbb{R}^{n\times 1}, \mathbf{C} \in \mathbb{R}^{1\times n}, \mathbf{D} \in \mathbb{R}, \mathbf{Y}^{\infty} \in \mathbb{R}$ describe the system of order n. The descriptor state space matrices $\mathbf{A}, \mathbf{B}, \mathbf{C},$ and $\mathbf{E}$ are obtain as follows.

First the given data is split into two groups, usually referred to right and left interpolation data points as [2]

$$\begin{aligned} &[s_1 \ldots s_K] = [\mu_1 \ldots \mu_{\underline{k}}] \cup [\lambda_1 \ldots \lambda_{\overline{k}}] \\ &[H^{(m)}(s_1) \ldots H^{(m)}(s_K)] = \\ &[H^{(m)}(\mu_1) \ldots H^{(m)}(\mu_{\underline{k}})] \cup [H^{(m)}(\lambda_1) \ldots H^{(m)}(\lambda_{\overline{k}})] \end{aligned}$$

Once the data is partitioned, the next step is to construct $\mathbf{V}$ and $\mathbf{W}$ vectors, defined as

$$\mathbf{V} = \begin{bmatrix} v_1 \\ \vdots \\ v_{\underline{k}} \end{bmatrix}, \mathbf{W} = \begin{bmatrix} w_1 \ldots w_{\overline{k}} \end{bmatrix}$$

where $v_i = H^{(m)}(\mu_i)$ and $w_i = H^{(m)}(\lambda_i)$. With the left data set (μ_i, v_i) and right data set (λ_i, w_i), the $\underline{k} \times \overline{k}$ Loewner and Shifted Loewner matrices are computed as follows

$$\mathbb{L} = \begin{bmatrix} \frac{v_1 - w_1}{\mu_1 - \lambda_1} & \cdots & \frac{v_1 - w_{\overline{k}}}{\mu_1 - \lambda_{\overline{k}}} \\ \vdots & \ddots & \vdots \\ \frac{v_{\underline{k}} - w_1}{\mu_{\underline{k}} - \lambda_1} & \cdots & \frac{v_{\underline{k}} - w_{\overline{k}}}{\mu_{\underline{k}} - \lambda_{\overline{k}}} \end{bmatrix} \tag{12}$$

$$\sigma\mathbb{L} = \begin{bmatrix} \frac{\mu_1 v_1 - \lambda_1 w_1}{\mu_1 - \lambda_1} & \cdots & \frac{\mu_1 v_1 - \lambda_{\overline{k}} w_{\overline{k}}}{\mu_1 - \lambda_{\overline{k}}} \\ \vdots & \ddots & \vdots \\ \frac{\mu_{\underline{k}} v_{\underline{k}} - \lambda_1 w_1}{\mu_{\underline{k}} - \lambda_1} & \cdots & \frac{\mu_{\underline{k}} v_{\underline{k}} - \lambda_{\overline{k}} w_{\overline{k}}}{\mu_{\underline{k}} - \lambda_{\overline{k}}} \end{bmatrix} \tag{13}$$

Once the Loewner and shifted Loewner are computed, the next step is to determine the order of the approximation. In order to do that, a singular value decomposition (SVD) is performed on $(s\mathbb{L} - \sigma\mathbb{L})$. Any value of s can be chosen as long as it is not the eigenvalue of the $(\sigma\mathbb{L}, \mathbb{L})$ matrix pencil [2], [7], resulting in the following expression

$$\text{SVD}(s\mathbb{L} - \sigma\mathbb{L}) = [\mathbf{Y}, \mathbf{\Sigma}, \mathbf{X}] \tag{14}$$

where $\mathbf{\Sigma}$ is a diagonal matrix containing the singular values. The order n of the approximation is chosen as the number of normalized singular values whose value exceeds a predefined threshold,

$$\begin{aligned} &\mathbf{A} = -\mathbf{Y_n}^{*}\sigma\mathbb{L}\mathbf{X_n}, \quad \mathbf{B} = \mathbf{Y_n}^{*}\mathbf{V}, \\ &\mathbf{C} = \mathbf{W}\mathbf{X_n}, \quad \mathbf{E} = -\mathbf{Y_n}^{*}\mathbb{L}\mathbf{X_n} \end{aligned} \tag{15}$$

where $\mathbf{X_n} \in \mathbb{R}^{\overline{k} \times n}$ and $\mathbf{Y_n} \in \mathbb{R}^{\underline{k} \times n}$ are constructed from the first n columns of $\mathbf{X}$ and $\mathbf{Y}$ of (14) respectively [2]. For the case when it is required to use $\mathbf{D}$ and $\mathbf{Y}^{\infty}$ terms, they are calculated as described in [7].

IV. Numerical Examples

To validate the proposed method, a transfer function (TF) with known poles, residues and delays (described in Table I) is approximated. The delay extraction is performed using a tolerance value of $\varepsilon = 1e-4$ in (6). The original estimates for the delays were $\{20.0, 70.3\}$ ns and then the estimates were refined as in [11], to obtain values of $\{20.0, 70.0\}$ ns for delay region one and two respectively. The order selection of LM is usually determined by a significant drop in the magnitude of the singular values. Fig 1, shows the plot of the normalized singular values for the regions of the delayed rational approximations and for the LM without delay extraction. Since the most significant drop for the delayed rational function happens at the beginning, an order from 4 to 6 poles is enough to accurately approximate each region. For the LM without the delay, the significant drop in the singular values occurs in the 1400 poles range. Fig 2 shows the real part of the TF and is compared with the proposed method, LM with 1410 poles and LM with 600 poles. Both the proposed $(RMS = 9.20e-3)$ and the LM with 1410 poles $(RMS = 1.14e-4)$ show accurate results, as opposed to LM with 600 poles where it is less accurate $(RMS = 3.71e-1)$. Using the proposed approach resulted in a macromodel of size 8 to 12 poles depending on the threshold, a significant improvement compared to LM without delay extraction. It should be noted, that VF can also approximate each delay region with the same order as the proposed method. However, with VF there is no way determining the order of each region beforehand.

TABLE I: Poles, Residues and Delays of the TF

Poles (GHz)	Residues (GHz)
$Delay = 20ns$	
$-0.6132 \pm j3.4551$	$-0.9877 \mp j0.0809$
$-0.3940 \pm j7.3758$	$-0.2067 \mp j0.0131$
$Delay = 70ns$	
$-1.0135 \pm j37.9655$	$-0.6787 \mp j0.1465$
$-0.5711 \pm j57.4748$	$-0.2626 \mp j0.1037$

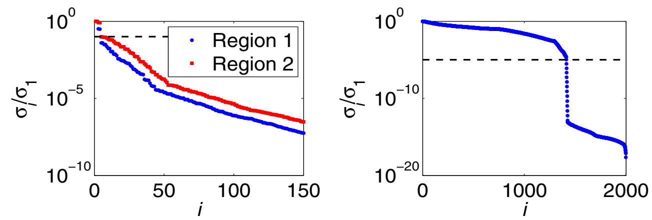

Fig. 1: Normalized singular values (left) delay LM (right) original LM.

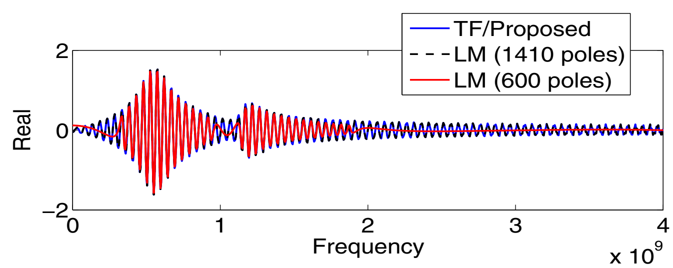

Fig. 2: Frequency Response.

References

[1] B. Gustavsen and A. Semlyen, "Rational approximation of frequency domain responses by vector fitting," *Power Delivery, IEEE Transactions on*, vol. 14, no. 3, pp. 1052–1061, Jul 1999.

[2] S. Lefteriu and A. C. Antoulas, "A new approach to modeling multiport systems from frequency-domain data," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 29, no. 1, pp. 14–27, Jan 2010.

[3] A. Chinea, P. Triverio, and S. Grivet-Talocia, "Delay-based macromodeling of long interconnects from frequency-domain terminal responses," *Advanced Packaging, IEEE Transactions on*, vol. 33, no. 1, pp. 246–256, Feb 2010.

[4] S. Roy and A. Dounavis, "Transient simulation of distributed networks using delay extraction based numerical convolution," *Computer-Aided Design of Integrated Circuits and Systems, IEEE Transactions on*, vol. 30, no. 3, pp. 364–373, March 2011.

[5] R. Mandrekar and M. Swaminathan, "Causality enforcement in transient simulation of passive networks through delay extraction," in *Signal Propagation on Interconnects, 2005. Proceedings. 9th IEEE Workshop on*, May 2005, pp. 25–28.

[6] A. Charest, M. Nakhla, R. Achar, D. Saraswat, N. Soveiko, and I. Erdin, "Time domain delay extraction-based macromodeling algorithm for long-delay networks," *Advanced Packaging, IEEE Transactions on*, vol. 33, no. 1, pp. 219–235, Feb 2010.

[7] M. Kabir and R. Khazaka, "Macromodeling of distributed networks from frequency-domain data using the loewner matrix approach," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 60, no. 12, pp. 3927–3938, Dec 2012.

[8] S. Grivet-Talocia and M. Bandinu, "Improving the convergence of vector fitting for equivalent circuit extraction from noisy frequency responses," *IEEE Transactions on Electromagnetic Compatibility*, vol. 48, no. 1, pp. 104–120, Feb 2006.

[9] S. Grivet-Talocia, "Delay-based macromodels for long interconnects via time-frequency decompositions," in *Electrical Performance of Electronic Packaging, 2006 IEEE*, Oct 2006, pp. 199–202.

[10] D. Gabor, *Theory of communication.* Institution of Electrical Engineering, 1946. [Online]. Available: https://books.google.ca/books?id=kvJSAAAAMAAJ

[11] B. Gustavsen, "Time delay identification for transmission line modeling," in *Signal Propagation on Interconnects, 2004. Proceedings. 8th IEEE Workshop on*, May 2004, pp. 103–106.